Lecture 33: Near and Far Fields of the Hertzian Dipole Antenna. Radiation Resistance.

In the previous lecture, we calculated the phasor $\bar{E}$ and $\bar{H}$ fields produced by a Hertzian dipole antenna of current $\bar{I} = \hat{a}_z I$ and length ΔL located at the origin of the coordinate system to be

$$\bar{E}(\bar{r}) = \hat{a}_r E_r + \hat{a}_\theta E_\theta \quad \text{[V/m]} \tag{1}$$

where
$$E_r = -\beta^2 \eta \frac{I\Delta L}{4\pi} 2\cos\theta \left[\frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3} \right] e^{-j\beta r} \tag{2}$$

$$E_\theta = -\beta^2 \eta \frac{I\Delta L}{4\pi} \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3} \right] e^{-j\beta r} \tag{3}$$

and
$$\bar{H}(\bar{r}) = -\hat{a}_\phi \beta^2 \frac{I\Delta L}{4\pi} \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} \right] e^{-j\beta r} \quad \text{[A/m]} \tag{4}$$

In this lecture, we will carefully examine these fields and discover interesting behavior of this Hertzian dipole antenna.

Near Fields of the Hertzian Dipole Antenna

The properties of these $\bar{E}$ and $\bar{H}$ fields in (1)-(4) are quite different depending if we observe them electrically close or electrically far from the dipole antenna. Electrical distance here is measured by βr.

As $\beta r \to 0$ we can neglect the $\frac{1}{(j\beta r)^2}$ term with respect to the $\frac{1}{(j\beta r)^3}$ term in (2) and neglect the $\frac{1}{j\beta r}$ and $\frac{1}{(j\beta r)^2}$ terms with respect to the $\frac{1}{(j\beta r)^3}$ term in (3). Additionally, for both (2) and (3) we employ the series expansion

$$e^{-j\beta r} = 1 - j\beta r - \frac{(\beta r)^2}{2} - \cdots$$

keeping only the first term as $\beta r \to 0$. After performing all three of these operations we find from (2) that

$$E_r \approx -\beta^2 \eta \frac{I\Delta L}{4\pi} 2\cos\theta \frac{1}{-j\beta^3 r^3} = \frac{I\Delta L}{j4\pi r^3 \beta/\eta} 2\cos\theta \qquad (5)$$

But

$$\frac{\beta}{\eta} = \frac{\omega\sqrt{\mu\varepsilon}}{\sqrt{\mu/\varepsilon}} = \omega\varepsilon$$

so that

$$E_r \approx \frac{I\Delta L}{j\omega\varepsilon 4\pi r^3} 2\cos\theta \qquad (6)$$

From the previous lecture, we saw that $Q = I/j\omega$ so that (6) becomes

$$\boxed{E_r \approx \frac{Q\Delta L}{4\pi\varepsilon r^3} 2\cos\theta \ \ \text{[V/m]} \ \ (\beta r \ll 1)} \qquad (7)$$

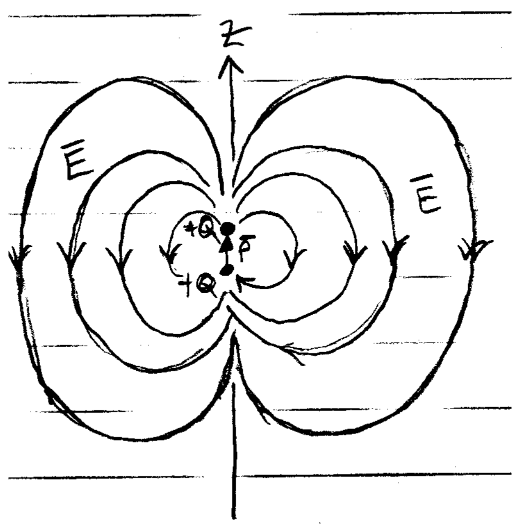
Following a similar process for E_θ in (3) we find that

$$\boxed{E_\theta \approx \frac{Q\Delta L}{4\pi\varepsilon r^3} \sin\theta \ \ \text{[V/m]} \ \ (\beta r \ll 1)} \qquad (8)$$

This electric field in (7) and (8) for the near fields of the Hertzian dipole antenna has exactly the same form as that for an electric dipole $\overline{p} = \hat{a}_z Q\Delta L$ C-m

$$\overline{E}_{\overline{p}} = \frac{p}{4\pi\varepsilon r^3}\left(\hat{a}_r 2\cos\theta + \hat{a}_\theta \sin\theta\right) \text{ V/m} \quad (9)$$

that you saw previously in EE 381 for static fields (Sadiku, Section 4.9):



Equations (7) and (8) have exactly the same form as (9). The difference is (7) and (8) are phasors while (9) is a static field.

[See Mathcad worksheet "Animated Electric Fields of the Hertzian Dipole" with small $r_{\max}$, for example $r_{\max} = 0.1$.]

Keeping the dominant term in (4) as $\beta r \to 0$ gives

$$H_\phi \approx \frac{I\Delta L}{4\pi r^2}\sin\theta \text{ [A/m]} \quad (\beta r \ll 1) \quad (10)$$

which is exactly the same form as the static magnetic field produced by a current element $I\Delta L$ using the Biot-Savart law in magnetostatics [see equation (3) in Lecture 30], though that is not proven here.

In summary, the $\bar{E}$ and $\bar{H}$ fields electrically close ($\beta r \ll 1$) to the Hertzian dipole antenna have the same form as those fields of the static problem (electric dipole, magnetic current element), but those fields of the antenna simply oscillate sinusoidally with time. These near fields of the Hertzian dipole antenna are consequently said to be **quasi-static**.

Far Fields of the Hertzian Dipole Antenna

The situation is **completely different** for the $\bar{E}$ and $\bar{H}$ fields at distances electrically far from the antenna. In the case that $\beta r \gg 1$, then from (1) through (4):

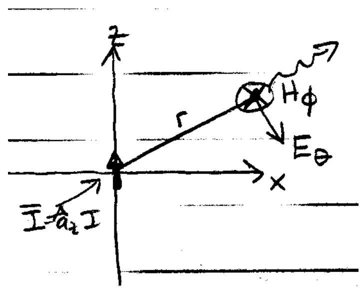
$$E_r \approx 0 \quad (\beta r \gg 1) \tag{11}$$

$$E_\theta \approx -\beta^2 \eta \frac{I\Delta L}{4\pi} \sin\theta \frac{e^{-j\beta r}}{j\beta r} = j\frac{I\Delta L}{4\pi} \beta\eta \sin\theta \frac{e^{-j\beta r}}{r} \quad (\beta r \gg 1) \tag{12}$$

$$H_\phi \approx -\beta^2 \frac{I\Delta L}{4\pi} \sin\theta \frac{e^{-j\beta r}}{j\beta r} = j\frac{I\Delta L}{4\pi} \beta \sin\theta \frac{e^{-j\beta r}}{r} \quad (\beta r \gg 1) \tag{13}$$

These far zone fields of the antenna behave very differently than the near zone fields:

- Notice that because of the $e^{-j\beta r}$ factors in (12) and (13), $\bar{E}$ and $\bar{H}$ are propagating as waves in the $+\hat{a}_r$ direction (away from the dipole antenna). These are called spherical waves.



[It is interesting to observe this phenomena in the Mathcad worksheet “Animated Electric Fields of the Hertzian Dipole” with moderate r_{max}, say r_{max} = 1.5.]

- $\bar{E} \perp \bar{H}$.

- Both $\bar{E}$ and $\bar{H}$ are perpendicular to the direction of propagation ($\hat{a}_r$) because E_r is vanishingly small with respect to the E_θ term.

- $E_\theta / H_\phi = \eta$.

- As $\omega \to 0$, E_θ and $H_\phi \to 0$.

All of these properties sound very familiar, don’t they? These are similar characteristics of uniform plane waves (UPWs). Here, though, there are two big differences. First, the far fields of this Hertzian dipole antenna are proportional to 1/*r*. They

decay in amplitude as they propagate away from the antenna. For the UPW, they didn't decay in amplitude.
The fundamental reason for this behavior is the source for a UPW is an infinite current sheet. Because of its infinite extent, the EM waves it produces don't decay as they propagate. Such a behavior, though, requires a source that supplies an infinite amount of power, which is not at all realistic.

Second, these waves propagate outward in the *r* direction, so they are called spherical waves rather than plane waves.

Power Radiated by the Hertzian Dipole Antenna

This Hertzian dipole antenna is a much more realistic source of EM waves than an infinite current sheet, and it produces a finite amount of radiated power.

We calculate this time average radiated power using the Poynting vector

$$\bar{S}_{av} = \frac{1}{2}\text{Re}\left(\bar{E} \times \bar{H}^*\right) \tag{14}$$

Substituting the far field $\bar{E}$ and $\bar{H}$ from (11)-(13) into (14) we find

$$\bar{S}_{av} = \frac{1}{2}\text{Re}\left(\hat{a}_\theta E_\theta \times \hat{a}_\phi H_\phi^*\right) = \frac{1}{2}\text{Re}\left(\hat{a}_r E_\theta H_\phi^*\right) = \hat{a}_r \frac{1}{2}\text{Re}\left(\frac{E_\theta E_\theta^*}{\eta}\right)$$

or
$$\bar{S}_{av} = \hat{a}_r \frac{1}{2} \frac{|E_\theta|^2}{\eta} \quad [\text{W/m}^2] \tag{15}$$

assuming a lossless infinite space into which the antenna radiates so that η is a real number.

Substituting (12) into (15) we find

$$\bar{S}_{av} = \hat{a}_r \frac{1}{2\eta} \left(\frac{|I| \Delta L}{4\pi} \beta \eta \sin\theta \frac{1}{r} \right)^2$$

or
$$\boxed{\bar{S}_{av} = \hat{a}_r \frac{\beta^2 \eta}{32\pi^2} |I|^2 (\Delta L)^2 \frac{\sin^2\theta}{r^2} \quad [\text{W/m}^2] \quad (\beta r \gg 1)} \tag{16}$$

This result indicates that this antenna is radiating an EM field (a wave) that is carrying time average power away from the antenna. Notice in (16) that <u>this time average power density decays as $1/r^2$</u>. (The fields decay as $1/r$.)

We can now calculate the total radiated time average power P_{av} by integrating (16) over a sphere centered on the dipole antenna with a radius in the far field of the antenna such that

$$P_{av} = \oint_s \bar{S}_{av} \cdot d\bar{s} = \int_0^{2\pi} \int_0^{\pi} \hat{a}_r S_{av,r} \cdot \hat{a}_r r^2 \sin\theta \, d\theta \, d\phi$$

$$\underbrace{=}_{(16)} \frac{\beta^2 \eta}{32\pi^2} |I|^2 (\Delta L)^2 \, 2\pi \int_0^{\pi} \frac{\sin^2\theta}{r^2} r^2 \sin\theta \, d\theta$$

The integral in this expression can easily be evaluated as

$$\int_0^{\pi} \sin^3\theta d\theta = -\frac{1}{3}\cos\theta\left(\sin^2\theta + 2\right)\Big|_0^{\pi}$$

$$= \frac{1}{3}(2) - \left(-\frac{1}{3}\right)(2) = \frac{4}{3}$$

Substituting this result gives

$$P_{av} = \frac{\beta^2\eta}{12\pi}|I|^2(\Delta L)^2 \tag{17}$$

or with $\beta = 2\pi/\lambda$ then

$$\boxed{P_{av} = \frac{\pi\eta}{3}|I|^2\left(\frac{\Delta L}{\lambda}\right)^2 \text{ [W] } (\beta r \gg 1)} \tag{18}$$

Notice that the time average radiated power in (18) is not dependent on *r*. We wouldn't expect it to be, actually. But the radiated power density in (16) varies as r^{-2}. This makes sense as the total radiated power is distributed over a larger and larger imaginary sphere of radius *r* as *r* increases.

Radiation Resistance and Equivalent Input Circuit for the Hertzian Dipole Antenna

This time average power in (18) represents power that is carried away from the terminals of the antenna by the electromagnetic wave. This power will not return to the antenna. For a generator connected to the terminals of the antenna, this effect simply looks like a resistance. Even if the antenna is made from perfectly conducting wires, there is still power "lost" to radiation.

In fact, we define a radiation resistance R_r for an antenna as a hypothetical lumped resistance that would dissipate the same amount of time average power as that radiated by the antenna.

For a resistor,

$$P_{av} = \frac{1}{2}\text{Re}\left(VI^*\right) = \frac{1}{2}R_r\left|I\right|^2 \qquad (19)$$

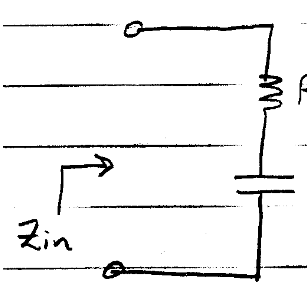
Equating (19) with (18) we find that for a Hertzian dipole antenna

$$R_r = \frac{2\pi}{3}\eta\left(\frac{\Delta L}{\lambda}\right)^2 \quad [\Omega] \qquad (20)$$

An equivalent circuit for the input terminals to the Hertzian dipole antenna includes this radiation resistance in series with a capacitive reactance that captures the near-field terminal characteristics of the dipole antenna:

See • Stutzman & Thiele, p. 47
• Schelkunoff, p. 305

(Check: this for short dipole. True for pt. dipole?)



$$X_A \approx -\frac{2\eta}{\pi\beta\Delta L}\left[\ln\left(\frac{\Delta L}{2a}\right)-1\right] [\Omega] \qquad (21)$$

where a = wire radius.

We haven't solved for this equivalent capacitance here, but that can be found in many antenna textbooks.

Example N33.1. A steel dipole antenna of length 62" and 1/8" diameter is operating at 1 MHz (an AM radio antenna, for example). Assume a Hertzian dipole antenna model.

(a) Calculate the antenna radiation resistance and Ohmic resistance.

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{10^6} = 300 \text{ m}, \quad \Delta L = 62'' = 1.575 \text{ m}$$

$$\Rightarrow \quad \frac{\Delta L}{\lambda} = \frac{1.575}{300} = 0.00525 \ll 1$$

so this antenna is electrically very short. Then using (20),

$$R_r = \frac{2\pi}{3}\cdot 376.73037\cdot\left(0.00525\right)^2 = 0.0217 \ \Omega$$

Because of the skin effect (see Lecture 9), in a wire of length L at sufficiently large frequency such that the skin depth is much less than the wire radius ($\delta \ll a$), then

$$R_{\text{ohmic}} \approx R_s\left(\frac{L}{2\pi a}\right) \ [\Omega] \qquad (22)$$

where the surface resistance R_s is defined as

$$R_s \equiv \sqrt{\frac{\pi f \mu}{\sigma}} \ [\Omega/\text{square}] \qquad (23)$$

For this antenna of length ΔL made from steel in which $\sigma = 2\times 10^6$ S/m and $\mu = \mu_0$, then

$$R_{\text{ohmic}} \approx \sqrt{\frac{\pi f \mu_0}{\sigma}}\left(\frac{\Delta L}{2\pi a}\right)$$

$$= \sqrt{\frac{\pi \cdot 10^6 \cdot 4\pi\times 10^{-7}}{2\times 10^6}} \cdot \left(\frac{1.575}{2\pi \cdot \underbrace{0.159\times 10^{-2}}_{=a}}\right) = 0.2207\ \Omega$$

(So for this example, $\delta \equiv \sqrt{2/(\omega\mu\sigma)} = 0.356$ mm while $a = 1/16$"=1.59 mm, such that $\delta = 0.224a$.)

Notice how small the radiation resistance is in comparison! It's actually ten times smaller than the Ohmic resistance of the steel wire at 1 MHz.

This turns out to be a universal characteristic of electrically small antennas: They are not efficient radiators of EM waves.

(b) Calculate the radiation efficiency e_r of this antenna. By definition, the radiation efficiency is

$$e_r \equiv \frac{R_r}{R_r + R_{\text{ohmic}}} \tag{24}$$

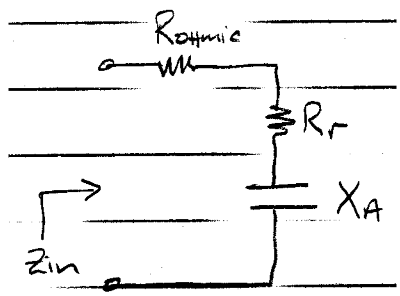
(We'll derive this expression in the next lecture in Example N34.1.)

For this specific Hertzian dipole antenna

$$e_r = \frac{0.0217}{0.0217 + 0.2207} = 8.95\%$$

which is a very low value.

(c) Calculate the input impedance. The equivalent circuit at the terminals of the antenna is



From (21):

$$X_A \approx -\frac{2\eta_0}{\pi\beta\Delta L}\left[\ln\left(\frac{\Delta L}{2a}\right) - 1\right]$$

$$= -\frac{2 \cdot 376.73037}{\pi \cdot \frac{2\pi}{300} \cdot 1.575}\left[\ln\left(\frac{1.575}{2 \cdot 0.159 \times 10^{-2}}\right) - 1\right] = -37{,}844\ \Omega$$

Therefore,

$$Z_{\text{in}} = R_{\text{ohmic}} + R_r + jX_A = 0.242 - j37{,}844\ \Omega$$

Notice the extremely large capacitive reactance in this Z_{in}. So, not only is this antenna not an efficient radiator, but it is difficult to couple energy to it from a source! Need a matching network to do this by resonating out the C using an L, for example, but then the antenna becomes narrow banded.

Not all antennas perform this poorly! One can make dipole antennas much more efficient by making them electrically longer. Approaching $\lambda/2$ in length, these antennas perform much better.

Section 9.2.1

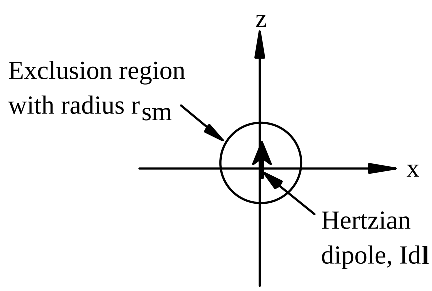
Animated Electric Fields of the Hertzian Dipole

Purpose

To simultaneously visualize the spatial and time variation of the electric fields produced by a point electric (i.e., hertzian) dipole. Two types of animation clips are generated in this worksheet. The first is an animated vector plot of the electric field and the second is an animated plot of the field lines of **E** produced by the Hertzian dipole.

Define the E field produced by the hertzian dipole

The geometry of the hertzian dipole is shown in Fig. 9.3 of the text and in the figure below:



For this worksheet, we will choose the infinite, homogeneous space surrounding the dipole to be free space and also choose the frequency such that $\beta_0 = 2\pi$:

$$\mu_0 := 4\cdot\pi\cdot 10^{-7} \qquad \varepsilon_0 := 8.854\cdot 10^{-12}$$

$$\beta_0 := 2\cdot\pi \qquad \eta_0 := \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

With $\beta_0 = 2\pi$, the wavelength is approximately 1 meter.

As stated above, the objective of this worksheet is to compute and visualize the electric fields produced by the point electric dipole – which is also called the hertzian dipole. The electric fields produced by this simple radiator in spherical coordinates are given in Equations (26) of Chap. 9 of the text as:

$$Idl := 1 \qquad r(x,z) := \sqrt{x^2 + z^2}$$

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$$E_r(x,z) := \frac{Idl\cdot\eta_0\cdot\beta_0^{\;2}\cdot z}{2\cdot\pi\cdot r(x,z)}\cdot\left[\frac{1}{\left(\beta_0\cdot r(x,z)\right)^2} - \frac{j}{\left(\beta_0\cdot r(x,z)\right)^3}\right]\cdot\exp\left(-j\cdot\beta_0\cdot r(x,z)\right)$$

$$E_\theta(x,z) := \frac{Idl\cdot\eta_0\cdot\beta_0^{\;2}\cdot|x|}{4\cdot\pi\cdot r(x,z)}\cdot\left[\frac{j}{\beta_0\cdot r(x,z)} + \frac{1}{\left(\beta_0\cdot r(x,z)\right)^2} - \frac{j}{\left(\beta_0\cdot r(x,z)\right)^3}\right]\cdot\exp\left(-j\cdot\beta_0\cdot r(x,z)\right)$$

The ϕ component of **E** is zero.

We will be constructing a vector plot of this electric field in the plane y = 0. Therefore it is necessary to convert these spherical components of **E** into rectangular coordinates. The spherical unit vectors $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are converted to cartesian coordinates using (42) in Chap. 2 as:

$$\mathbf{a_r}(x,z) := \begin{pmatrix} \dfrac{x}{r(x,z)} \\ \dfrac{z}{r(x,z)} \end{pmatrix} \qquad \mathbf{a_\theta}(x,z) := \begin{bmatrix} \dfrac{x\cdot z}{\sqrt{x^2\cdot\left(x^2+z^2\right)}} \\ -\dfrac{|x|}{r(x,z)} \end{bmatrix}$$

x components.

z components.

The x and z components of **E** (as functions of x and z) are then:

$$\mathbf{E}(x,z) := E_r(x,z)\cdot\mathbf{a_r}(x,z) + E_\theta(x,z)\cdot\mathbf{a_\theta}(x,z)$$

In the remainder of this worksheet we will generate two types of animation clips in order to observe this electric field produced (i.e., radiated) by this point electric dipole in the plane y = 0. The first animation clip is a vector plot of **E** and the second is an animation of the field lines of the electric field.

Animated vector plot of E

The first type of plot we will generate is an animation clip of the electric field in the y = 0 plane produced by this hertzian dipole.

Choose the maximum dimension of the plot and the number of points to plot in the x and z directions:

$r_{max} := 1.5$ Maximum x and z for plots (λ_0).

$npts_x := 35$ $npts_z := 35$ Number of points to plot in x and z.

$x_{start} := -r_{max}$ $x_{end} := r_{max}$ x starting and ending points (λ_0).

$z_{end} := x_{end}$ $z_{start} := x_{start}$ z starting and ending points (λ_0).

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Construct a list of x_v and z_v points at which to plot the electric field:

$$i := 0 .. npts_x - 1 \qquad x_{v_i} := x_{start} + i\cdot\frac{x_{end} - x_{start}}{npts_x - 1}$$

$$j := 0 .. npts_z - 1 \qquad z_{v_j} := z_{start} + j\cdot\frac{z_{end} - z_{start}}{npts_z - 1}$$

Now compute the x and z phasor components of **E** at every x and z point in this grid. Since the electric field is extremely large near the dipole (and infinite at the dipole), the electric field will not be computed very near the dipole. Within this circular "region of exclusion", with radius r_{sm} as shown in the figure above, the electric field will be assigned a value of zero.

$r_{sm} := 0.1\cdot r_{max}$ Radius of exclusion region (λ_0).

$$E_{px_{i,j}} := \text{if}\left(r\left(x_{v_i}, z_{v_j}\right) < r_{sm}, 0, \mathbf{E}\left(x_{v_i}, z_{v_j}\right)_0\right)$$

$$E_{pz_{i,j}} := \text{if}\left(r\left(x_{v_i}, z_{v_j}\right) < r_{sm}, 0, \mathbf{E}\left(x_{v_i}, z_{v_j}\right)_1\right)$$

For the animation clip, choose the number of points per period at which to plot the electric field:

$n_{pts\_per\_period} := 20$ Number of points to plot per period.

$\omega := \dfrac{\beta_0}{\sqrt{\mu_0\cdot\varepsilon_0}}$ $\qquad T_p := \dfrac{2\cdot\pi}{\omega}$ One time period of oscillation (s).

$t_{start} := 0$ $\qquad t_{end} := T_p$ Time to start and end plot (s).

Define the variable time in terms of the constant FRAME:

$$t_{inc} := \frac{T_p}{n_{pts\_per\_period}} \qquad time := t_{start} + FRAME\cdot t_{inc}$$

Define the functions E_x and E_z which compute the x and z time-domain components of the electric field at the matrix of x and z points at each time instant:

$$E_{x_{i,j}} := \text{Re}\left(E_{px_{i,j}}\cdot\exp(j\cdot\omega\cdot time)\right) \qquad E_{z_{i,j}} := \text{Re}\left(E_{pz_{i,j}}\cdot\exp(j\cdot\omega\cdot time)\right)$$

Now create an animation clip of this **E** field. For best results, in the "Animate" dialog box choose $To = 19$ then save the file and replay the animation in a video player that supports continuous loop playback.

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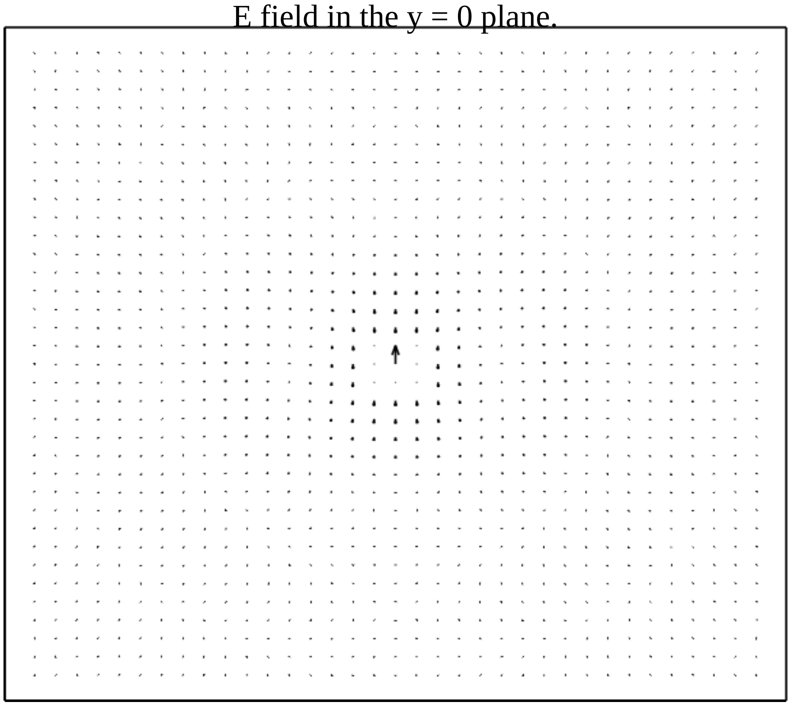
E field in the y = 0 plane.



$z_{end} = 1.500 \quad (\lambda_0)$

Time (in periods, T_p)

$\frac{time}{T_p} = 0.00$

$z_{start} = -1.500 \quad (\lambda_0)$

(E_x, E_z)

$x_{start} = -1.500 \quad (\lambda_0)$ $x_{end} = 1.500 \quad (\lambda_0)$

Note: The vector shown at the center of the plot (which is also the center of the region of exclusion) is not physical. Its purpose is simply to prevent Mathcad from distorting the animation clip by autoscaling the vector plot differently at each time step.

As we can observe in this animation clip, the **E** field is very large near the dipole and decays away quite rapidly in the spherical r direction. Consequently, very little of the electric field can be observed in this vector plot except for points near the dipole.

To better observe this electric field plot we will normalize the *phasor-domain* electric field at each point in the plot. That is, the phasor-domain electric field is assigned a magnitude of unity at every point. This normalized phasor-domain electric field is then converted to the time-domain in the usual fashion:

$$e_{norm\_x_{i,j}} := \mathrm{Re}\left(\frac{E_{px_{i,j}}}{\sqrt{E_{px_{i,j}}\cdot\overline{E_{px_{i,j}}} + E_{pz_{i,j}}\cdot\overline{E_{pz_{i,j}}}}}\cdot\exp(j\cdot\omega\cdot time)\right)$$

$$e_{norm\_z_{i,j}} := \mathrm{Re}\left(\frac{E_{pz_{i,j}}}{\sqrt{E_{px_{i,j}}\cdot\overline{E_{px_{i,j}}} + E_{pz_{i,j}}\cdot\overline{E_{pz_{i,j}}}}}\cdot\exp(j\cdot\omega\cdot time)\right)$$

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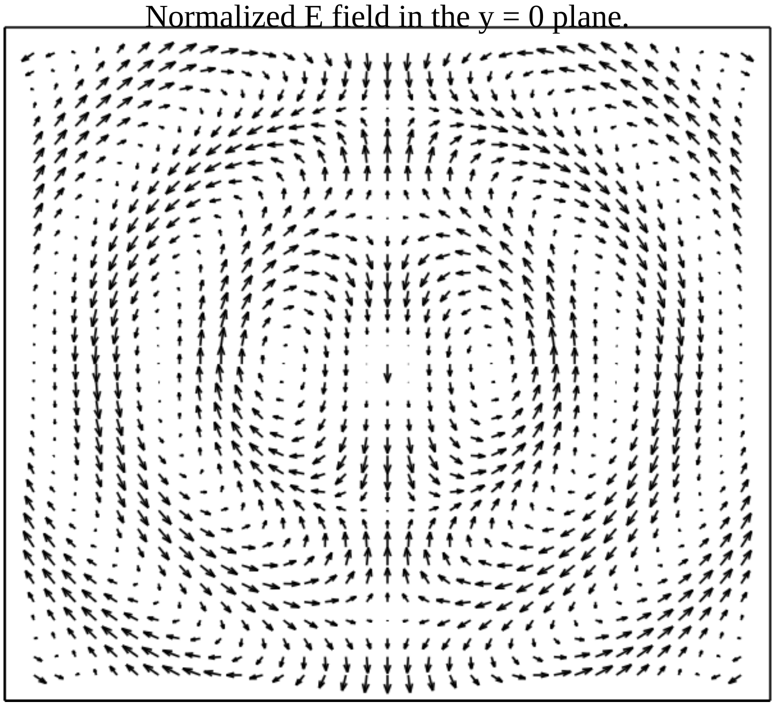
Now create an animation clip of this normalized **E** field. For best results, in the "Animate" dialog box choose To = 19 then save the file and replay the animation in a video player that supports continuous loop playback.



Normalized E field in the y = 0 plane.

$z_{end} = 1.500 \quad (\lambda_0)$

Time (in periods, T_p)

$\frac{time}{T_p} = 0.00$

$z_{start} = -1.500 \quad (\lambda_0)$

(e_{norm_x}, e_{norm_z})

$x_{start} = -1.500 \quad (\lambda_0)$ $x_{end} = 1.500 \quad (\lambda_0)$

Note: Once again, the vector shown at the center of the plot is not physical. Its purpose is simply to prevent Mathcad from distorting the animation clip by autoscaling the vector plot differently at each time step. Also, remember that in this plot the phasor-domain form of **E** has been normalized to unity at every point in the vector plot.

For a plot radius r_{max} greater than approximately 1 λ_0, propagation of the electric field away from the hertzian dipole is very evident in the animation clip. The propagation of the electric (and, simultaneously, the magnetic) field carries energy away from the dipole. This phenomenon is called *radiation*. Antennas for communication, radio, RADAR, microwave ovens, etc. are all based on this concept of electromagnetic radiation.

One topic of further exploration that you may wish to investigate is when the plot radius r_{max} is made very small, for example when $r_{max} = 0.1\ \lambda_0$. In such a case, the electric fields very near the hertzian dipole have the same pattern in space as the **electrostatic dipole**, as discussed in connection with (32) in Section 9.2.1 of the text. The difference here is that the charge oscillates from one end of the dipole to the other and therefore the electric field lines also oscillate in time. This is another example of a **quasi-static** electric field.

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Note that at time = 0 and $T_p/2$, the electric field near the dipole is zero whereas the *current* (I) in the dipole is maximum since it was assumed to vary with time as $\cos(\omega t)$. Conversely, at time = $T_p/4$ and $3T_p/4$ the electric field near the dipole is maximum but the *current* in the dipole is zero. (This is very evident when $r_{max} \le 0.1\ \lambda_0$.) Can you explain why this occurs?

Animated plot of the field lines of E

This propagation of the electromagnetic wave/field away from the hertzian dipole can alternatively be viewed by animating the *field lines* of the electric field – rather than the vector plot shown above. Choose the number of points to plot in the x and z directions:

$npts_x := 49 \qquad npts_z := 49$ Number of points to plot in x and z.

Construct a list of x_f and z_f points at which to plot the electric field:

$$i := 0 .. npts_x - 1 \qquad x_{f_i} := x_{start} + i \cdot \frac{x_{end} - x_{start}}{npts_x - 1}$$

$$j := 0 .. npts_z - 1 \qquad z_{f_j} := z_{start} + j \cdot \frac{z_{end} - z_{start}}{npts_z - 1}$$

We will generate the field lines of the electric field in the plane y = 0. The details of obtaining these field lines are not necessarily important. In short, however, the field lines of the electric field are constructed by generating a contour plot of a scalar function that is proportional to the H_ϕ produced by the hertzian dipole:

$$\text{Efieldline}(x,z) := \frac{j \cdot |x|}{r(x,z)} \cdot \left[\left[\frac{j}{\beta_0 \cdot r(x,z)} + \frac{1}{\left(\beta_0 \cdot r(x,z)\right)^2} \right] \cdot \exp\left(-j \cdot \beta_0 \cdot r(x,z)\right) \right]$$

Compute this phasor form of the field-line generating function Efieldline(x,z) at the matrix of points x_f and z_f:

$$FL_{i,j} := \text{Efieldline}\left(x_{f_i}, z_{f_j}\right)$$

Now define the time-domain form of this field-line generating function. The if statement is used primarily to prevent Mathcad from introducing distortion into the animation clip by autoscaling the contour plots differently at each time step:

$$fl_{t_{i,j}} := \text{if}\left(r\left(x_{f_i}, z_{f_j}\right) \le \frac{r_{sm}}{2}, -max_{fl}, \text{if}\left(\frac{r_{sm}}{2} < r\left(x_{f_i}, z_{f_j}\right) \le r_{sm}, max_{fl}, \text{Re}\left(FL_{i,j} \cdot \exp(j \cdot \omega \cdot \text{time}\right)\right.\right.$$

Now create an animation clip of the field lines of **E**. For best results, in the "Animate" dialog box choose $To = 19$ then save the file and replay the animation in a video player that supports continuous loop playback.

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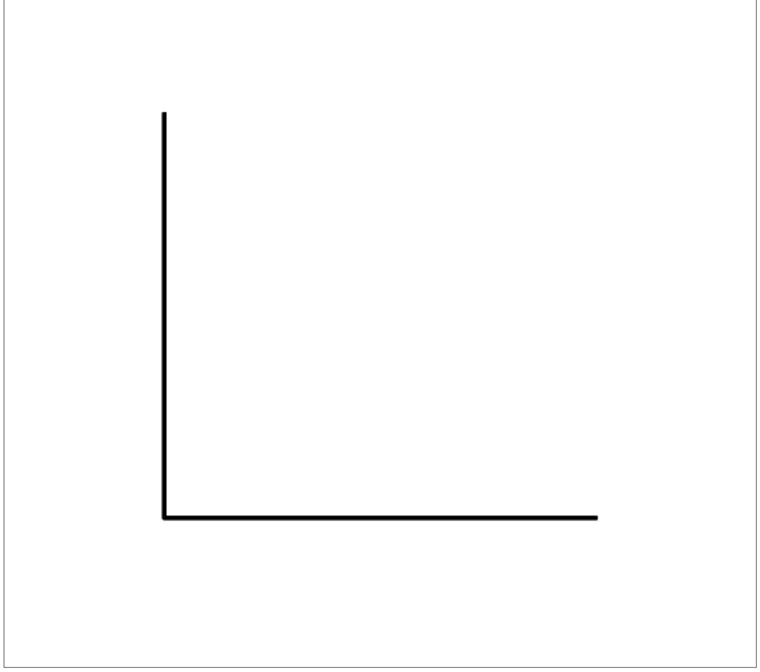
Field lines of E in y = 0 plane.



$z_{end} = 1.500 \quad (\lambda_0)$

Time (in periods, T_p)

$\frac{time}{T_p} = 0.00$

$z_{start} = -1.500 \quad (\lambda_0)$

fl_t

$x_{start} = -1.500 \quad (\lambda_0)$ $\qquad x_{end} = 1.500 \quad (\lambda_0)$

Note: The contours shown at the center of this plot (within the "exclusion region" discussed earlier in this worksheet) are not physical. The purpose for these nonphysical contour lines is simply to prevent Mathcad from distorting the animation clip by autoscaling the field-line plot differently at each time step.

Again, the contour lines shown in this plot are the *field lines* of the electric field. The vector arrows associated with these field lines are not drawn. However, you can compare this field line plot with the vector plot of **E** shown in the previous figure (and at the same instant of time) to discern the direction of the electric field. What is most important to note in this animation is the outward propagation of these field lines as time marches forward. This is another type of animation that illustrates the concept of electromagnetic *radiation* from this hertzian dipole.

One interesting thing you may wish to observe in this mesmerizing field line plot is how a "packet" of electric field lines is formed near the dipole and then "snaps off" to propagate away from the dipole. It is interesting to coordinate the times during which the packet forms and then snaps off with the time variation of the charge accumulation at the ends of the dipole.

End of worksheet.

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