Lecture 32: Hertzian Dipole Antenna.

In the previous lecture, we discussed the fundamental source-field relationship that can be used to calculate the $\bar{E}$ and $\bar{H}$ fields produced by sinusoidal steady state line currents. In that process, we first compute the phasor vector magnetic potential

$$\bar{A}(\bar{r}) = \int_{c'} \frac{\mu \bar{I}(\bar{r}')e^{-j\beta R}}{4\pi R} dl' \tag{1}$$

where $R = |\bar{r} - \bar{r}'|$, then compute the phasor magnetic field as

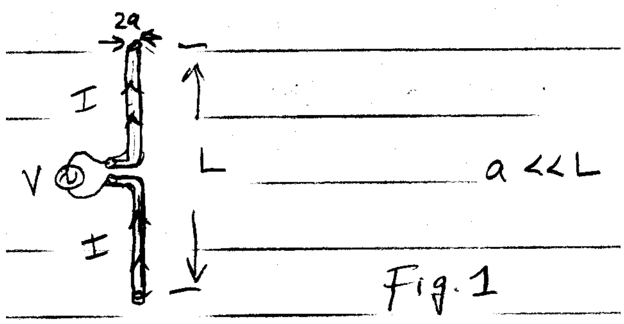
$$\bar{H}(\bar{r}) = \frac{1}{\mu} \nabla \times \bar{A}(\bar{r}) \tag{2}$$

Using Ampere's law, we determine the phasor electric field

$$\bar{E}(\bar{r}) = \frac{1}{j\omega\varepsilon} \nabla \times \bar{H}(\bar{r}) \tag{3}$$

What might be surprising to you is that one of the most difficult challenges an antenna designer faces is determining the current distribution $\bar{I}(\bar{r}')$ on a particular antenna. This is often a complicated problem and usually the only accurate solution is to use computational electromagnetics methods.

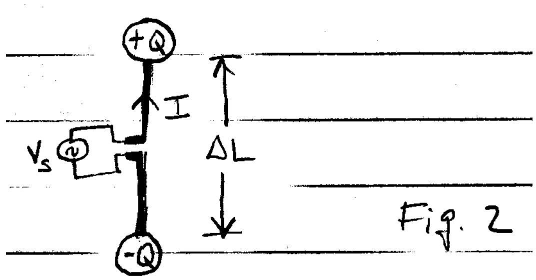
However, there is a famous class of straight wire antenna that has an accurate approximate solution for the form of the current on the antenna. This class of antenna is known as a **thin-wire dipole antenna**:



It is called a "dipole" antenna because there are two wires or poles comprising the antenna.

The form of the current on the wire is known quite accurately for thin wires $(a \ll L)$ with length L that is less than approximately 1 λ. The currents are less accurately known for longer lengths.

We will only be considering the simplest dipole antenna in this course. It is called the **Hertzian dipole**, or point dipole, antenna:



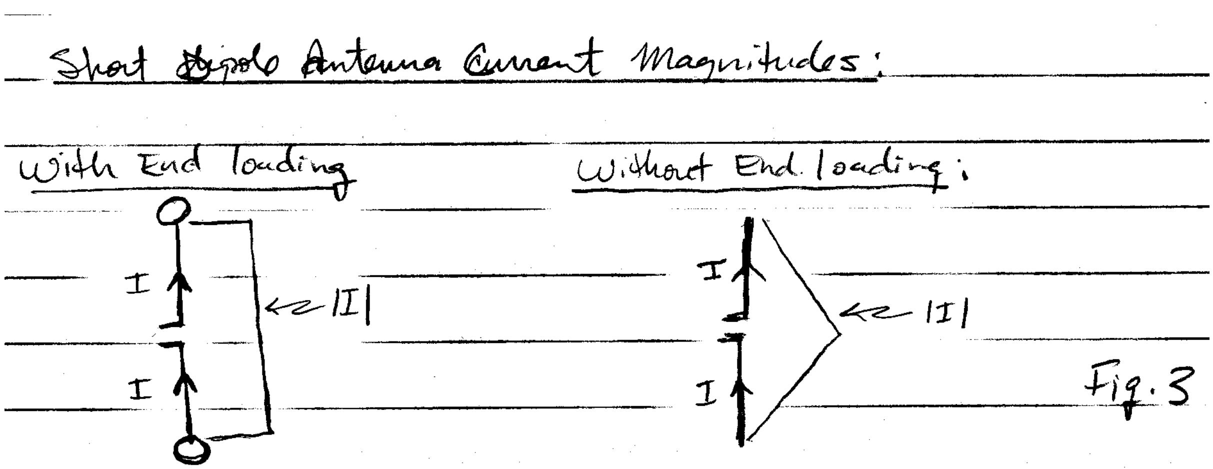
It is infinitesimally short with a uniform current distribution. Because

$$i(t) = \frac{dq(t)}{dt}$$

then for sinusoidal steady state this means that $I = j\omega Q$.

The ends of this short antenna are capacitively loaded by small metallic spheres or disks so as to create a current that is approximately uniform along its length in Fig. 2.

Without the capacitive loading the current would be approximately triangularly shaped with a maximum value at the center of the antenna and linearly decreasing to zero at both ends, as illustrated in Fig. 3:



$\bar{A}$ for a Hertzian Dipole Antenna

Following the procedure described at the beginning of page 1, we'll now determine the $\bar{E}$ and $\bar{H}$ fields produced (i.e., "radiated") by the Hertzian dipole antenna. This three step process begins with the calculation of $\bar{A}$ from (1)

$$\bar{A}(\bar{r}) = \int_{c'} \mu \bar{I}(\bar{r}') \frac{e^{-j\beta R}}{4\pi R} dl' \qquad (1)$$

The current $\bar{I}(\bar{r}')$ is that for the Hertzian dipole antenna, which we will assume is located at the center of a spherical coordinate system:

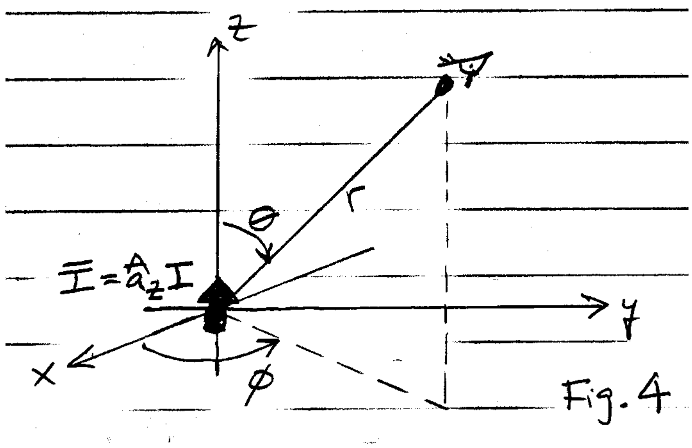


Fig. 4

Substituting for this assumed uniform current into (1) gives

$$\bar{A}(\bar{r}) = \int_{-\Delta L/2}^{\Delta L/2} \hat{a}_z \mu I \frac{e^{-j\beta R}}{4\pi R} dz' = \hat{a}_z \frac{\mu I}{4\pi} \int_{-\Delta L/2}^{\Delta L/2} \frac{e^{-j\beta R}}{R} dz' \tag{4}$$

Because this antenna is infinitesimally short then R is approximately constant over the entire range of integration. Consequently, the integral in (4) can be performed trivially leading to

$$\boxed{\bar{A}(\bar{r}) \approx \hat{a}_z \frac{\mu I \Delta L}{4\pi} \frac{e^{-j\beta r}}{r} \quad \text{[Wb/m]}} \tag{5}$$

(Notice that R has now become r.)

So we now have completed the first step and have obtained an analytical expression for $\bar{A}$. The next step is the calculation of $\bar{H}$. Because of the ϕ spherical symmetry of the dipole antenna, we'll use the spherical coordinate system to simply the mathematics.

We can convert $\bar{A}$ in (5) to spherical vector components quite easily by using the relationship

$$\hat{a}_z = \hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta \tag{6}$$

giving

$$\bar{A}(\bar{r}) = \hat{a}_r A_r + \hat{a}_\theta A_\theta + \hat{a}_\phi \cancelto{0}{A_\phi} \tag{7}$$

where
$$A_r = A_z \cos\theta = \frac{\mu I \Delta L}{4\pi} \cos\theta \frac{e^{-j\beta r}}{r} \tag{8}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I \Delta L}{4\pi} \sin\theta \frac{e^{-j\beta r}}{r} \tag{9}$$

$$A_\phi = 0 \tag{10}$$

$\bar{H}$ for a Hertzian Dipole Antenna

Because we have successfully determined $\bar{A}$ produced by the Hertzian dipole antenna, we can now determine the $\bar{E}$ and $\bar{H}$ fields, beginning with $\bar{H}$. As we saw in (2)

$$\bar{H}(\bar{r}) = \frac{1}{\mu} \nabla \times \bar{A}(\bar{r}) \tag{2}$$

Because of the ϕ symmetry, $\partial/\partial\phi \to 0$ which, using the determinant form of curl in the spherical coordinate system, gives

$$\bar{H}(\bar{r}) = \frac{1}{\mu r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & \hat{a}_\theta r & \hat{a}_\phi r\sin\theta \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & 0 \\ A_r & rA_\theta & 0 \end{vmatrix}$$

such that

$$\begin{aligned} \bar{H}(\bar{r}) &= \frac{1}{\mu r^2 \sin\theta} \left\{ \hat{a}_\phi r\sin\theta \left[\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \right\} \\ &= \hat{a}_\phi \frac{1}{\mu r} \left[\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \end{aligned} \quad (11)$$

- Using (9)

$$\frac{\partial}{\partial r}(rA_\theta) \underbrace{=}_{(9)} \frac{\partial}{\partial r}\left(-r \frac{\mu I \Delta L}{4\pi} \sin\theta \frac{e^{-j\beta r}}{r} \right) = \frac{j\beta\mu I\Delta L}{4\pi} \sin\theta e^{-j\beta r} \quad (12)$$

- Using (8)

$$\frac{\partial A_r}{\partial \theta} \underbrace{=}_{(8)} \frac{\partial}{\partial \theta}\left(\frac{\mu I \Delta L}{4\pi} \cos\theta \frac{e^{-j\beta r}}{r} \right) = -\frac{\mu I \Delta L}{4\pi} \sin\theta \frac{e^{-j\beta r}}{r} \quad (13)$$

Consequently, substituting (12) and (13) in (11), we find $\bar{H}$ to be

$$\begin{aligned} \bar{H}(\bar{r}) &= \hat{a}_\phi \frac{1}{\mu r} \left(\frac{j\beta\mu I \Delta L}{4\pi} \sin\theta e^{-j\beta r} + \frac{\mu I \Delta L}{4\pi} \sin\theta \frac{e^{-j\beta r}}{r} \right) \\ &= \hat{a}_\phi \frac{I\Delta L}{4\pi} \sin\theta \left(\frac{j\beta}{r} + \frac{1}{r^2} \right) e^{-j\beta r} \end{aligned} \quad (14)$$

Factoring out $(j\beta)^2 = -\beta^2$ from both terms in (14) we arrive at the final form of the magnetic field produced (or "radiated") by the Hertzian dipole antenna

$$\bar{H}(\bar{r}) = -\hat{a}_\phi \frac{I\Delta L}{4\pi}\beta^2 \sin\theta \left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right] e^{-j\beta r} \quad \text{[A/m]} \qquad (15)$$

$\bar{E}$ for a Hertzian Dipole Antenna

Now that $\bar{H}$ has been determined, we can solve for the $\bar{E}$ field produced by this Hertzian dipole antenna according to (3). Using the determinant form of curl in the spherical coordinate system

$$\bar{E}(\bar{r}) = \frac{1}{j\omega\varepsilon}\nabla\times\bar{H} = \frac{1}{j\omega\varepsilon}\cdot\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & \hat{a}_\theta r & \hat{a}_\phi r\sin\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0 \\ 0 & 0 & r\sin\theta H_\phi \end{vmatrix}$$

such that

$$\begin{aligned}\bar{E}(\bar{r}) &= \frac{1}{j\omega\varepsilon\cdot r^2\sin\theta}\left[\hat{a}_r\frac{\partial}{\partial\theta}\left(r\sin\theta H_\phi\right) - \hat{a}_\theta r\frac{\partial}{\partial r}\left(r\sin\theta H_\phi\right)\right] \\ &= \frac{1}{j\omega\varepsilon}\left[\hat{a}_r\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta H_\phi\right) - \hat{a}_\theta\frac{1}{r}\frac{\partial}{\partial r}\left(rH_\phi\right)\right]\end{aligned} \qquad (16)$$

Substituting for H_ϕ from (15) into (16), and simplifying, gives

$$\bar{E}(\bar{r}) = \hat{a}_r E_r + \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi \quad \text{[V/m]} \qquad (17)$$

where
$$E_r = -\frac{I\Delta L}{4\pi}\eta\beta^2 2\cos\theta\left[\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r} \tag{18}$$

$$E_\theta = -\frac{I\Delta L}{4\pi}\eta\beta^2 \sin\theta\left[\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}+\frac{1}{(j\beta r)^3}\right]e^{-j\beta r} \tag{19}$$

$$E_\phi = 0 \tag{20}$$

Summary

So this concludes the calculation of the $\bar{E}$ and $\bar{H}$ fields produced (or "radiated") by the Hertzian dipole antenna. There is a wealth of information contained in these field solutions for $\bar{E}$ in (17)-(20) and $\bar{H}$ in (15) that we will carefully pick through in the next lecture.