Lecture 31: Electromagnetic Radiation and Antennas

We've seen in the past few lectures that electromagnetic (EM) waves can transport power from one position to another without any intervening guiding structure or material, other than just space itself. While eminently useful, the uniform plane waves (UPWs) we studied are fictitious in that they are produced only by infinite current sheets.

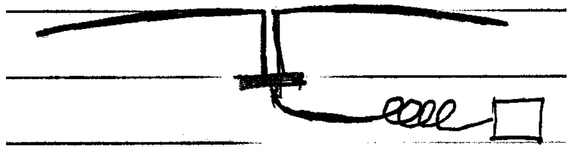
We will now begin to discuss the production of much more general types of EM fields and waves, especially those produced by certain types of simple antennas.

Antennas are devices (of finite size, of course) that serve the dedicated purpose of producing and receiving EM waves. In particular, the function of an antenna is as a **transducer** between an electrical circuit and EM waves. An antenna converts energy from an electrical circuit to an EM wave in the case of a transmitter, or converts an EM wave to oscillating electrical current in an electrical circuit if receiving.

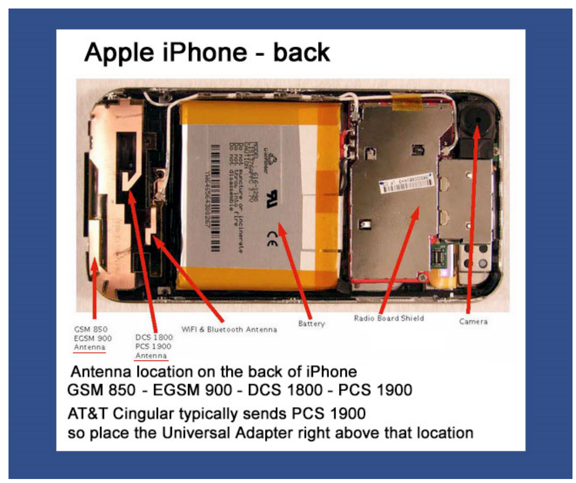
One objective for the antenna designer is to create an antenna that performs these functions efficiently.

Examples of antennas include:

- EE 322: In the lab room for this course (EP 127), you connect a coaxial cable from the output of J1 of your NorCal40A to a "dipole antenna."



- Cell phones: Some no longer have an external antenna. Rather, the antenna is built inside the phone, a so-called embedded antenna.



- Satellite TV parabolic dish antennas: Like those used by DirecTV and Dish Network.

- AM, FM, and XM radio antennas used in vehicles.

- "Whip" antennas on home Wi-Fi routers.

- NIST radio station WWVB antenna: 60-kHz signal used in consumer electronics such as wall clocks and clock radios for time synchronization. Signal transmitted from near Fort

Collins, CO. "Top loaded monopole" antenna supported by four 122-m towers:

Source-Field Relationships Review

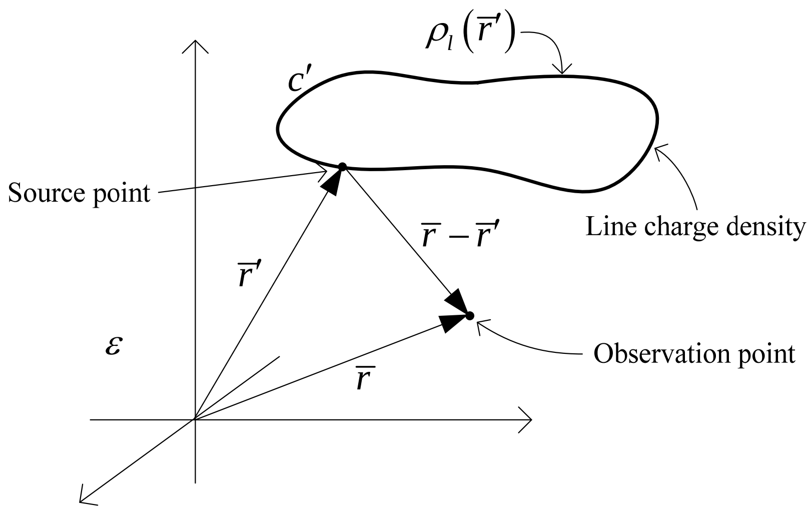
The calculation of the $\bar{E}$ and $\bar{H}$ fields radiated by an antenna is far more difficult than finding the EM fields produced by an infinite current sheet, as for UPWs. We will begin this process of calculating the EM fields produced by antennas with an assumed current distribution in space then calculate the $\bar{E}$ and $\bar{H}$ fields produced by this current. But how does one perform this calculation?

What is required for such calculations is a so-called source-field relationship. Given a distribution of current density $\bar{J}$, these relationships allow for the calculation of $\bar{E}$ and $\bar{H}$ produced everywhere in space.

You've seen source-field relationships before in EE 381 for the calculation of both electrostatic and magnetostatic problems. We'll review two of these here.

Source-Field Relationship Review #1: Scalar Potential Produced by Line Charge

The first source-field relationship we'll review is for the electrostatic potential produced by a given line charge density $\rho_l(\overline{r}')$:



The absolute electrostatic potential $\Phi_e(\overline{r})$ produced at any point $\overline{r}$ by the line charge density $\rho_l(\overline{r}')$ is given by

$$\Phi_e(\overline{r}) = \int_{c'} \frac{\rho_l(\overline{r}')}{4\pi\varepsilon|\overline{r}-\overline{r}'|} dl' = \int_{c'} \frac{\rho_l(\overline{r}')}{4\pi\varepsilon R} dl' \quad \text{[V]} \qquad (1)$$

This is the electrostatic potential at point $\overline{r}$ with respect to infinity, at which the potential is defined as zero.

Notice in (1) that there are two coordinate systems. The primed coordinate $\bar{r}'$ is for the source coordinates while the other $\bar{r}$ is for the observation (or field) coordinates. The (variable) distance between a source point and an observation point is defined as

$$R = \left| \bar{r} - \bar{r}' \right| \tag{2}$$

The form of (1) is a **superposition integral**. We can subdivide the contour c' into infinitesimal sections dl' each with total charge $\rho_l\left(\bar{r}'\right)dl' = Q$ and calculate the potential at a point $\bar{r}$ as that **due to a <u>point charge</u>** of this value according to

$$\Delta\Phi_e\left(\bar{r}\right) = \frac{Q}{4\pi\varepsilon R} = \frac{\rho_l\left(\bar{r}'\right)dl'}{4\pi\varepsilon R}$$

Adding up the partial contributions to the potential at $\bar{r}$ from all the infinitesimal line charge segments according to

$$\Phi_e\left(\bar{r}\right) = \int_{c'} \Delta\Phi_e$$

yields equation (1).

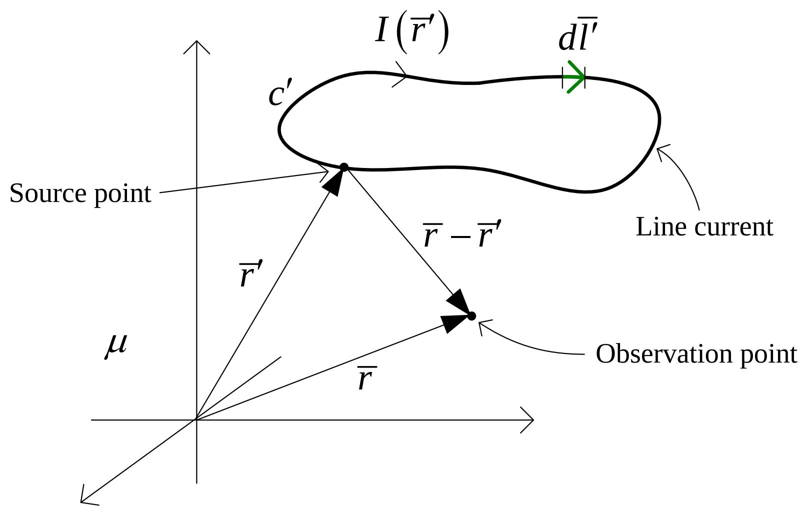
Source-Field Relationship Review #2: Magnetic Field Produced by Line Current

This first source-field relationship (1) was for a scalar source quantity producing a scalar field quantity. We'll increase the complexity a bit by next looking at vector source and vector field quantities.

In magnetostatics you studied the Biot-Savart law

$$\bar{B}(\bar{r}) = \frac{\mu}{4\pi} \oint_{c'} \frac{I(\bar{r}')\, d\bar{l}' \times \bar{R}}{R^3} \quad [\text{T}] \tag{3}$$

which is a source-field relationship for computing the magnetic flux density at observation point $\bar{r}$ produced by a closed loop of current $\bar{I}$:



You also saw this relationship (3) expressed in terms of the so-called vector magnetic potential $\bar{A}$ as

$$\bar{A}(\bar{r}) = \oint_{c'} \frac{\mu \bar{I}(\bar{r}')}{4\pi R} dl' \quad [\text{Wb/m}] \tag{4}$$

from which

$$\bar{B}(\bar{r}) = \nabla \times \bar{A}(\bar{r}) \tag{5}$$

In this latter form (4) and (5), the calculation of the field $\bar{B}$ is a two step process. First, the potential $\bar{A}$ is calculated from $\bar{I}$ in (4), then $\bar{B}$ is computed from $\bar{A}$ using (5).

Source-Field Relationship for Sinusoidal Steady State Currents

The derivation of the source-field relationships for time varying currents is fairly involved and certainly more complicated than we have time for in this class. (You will see this done in detail in an antennas course, for example.)

However, it turns out that these new source-field relationships are equations that look very similar to what you've already seen in (4) and (5)!

In particular, for a **phasor current** $\bar{I}(\bar{r}')$, it can be shown that the phasor magnetic vector potential is given by

$$\bar{A}(\bar{r}) = \int_{c'} \frac{\mu \bar{I}(\bar{r}') e^{-j\beta R}}{4\pi R} dl' \quad \text{[Wb/m]} \tag{6}$$

and $$\bar{B}(\bar{r}) = \nabla \times \bar{A}(\bar{r}) \Rightarrow \bar{H}(\bar{r}) = \frac{1}{\mu} \nabla \times \bar{A}(\bar{r}) \quad \text{[A/m]} \tag{7}$$

Note that in (6), the contour of integration no longer needs to be a closed path for time varying currents, as it does for static currents in (4). Also note that $\bar{A}$, $\bar{B}$, and $\bar{H}$ in (7) are all vector phasors.

The phasor electric field can be computed from $\bar{H}$ using Ampere's law

$$\bar{E}(\bar{r}) = \frac{1}{j\omega\varepsilon} \nabla \times \bar{H}(\bar{r}) \tag{8}$$

Notice that the phase factor $e^{-j\beta R}$ in (6) is the only difference between the static form of $\bar{A}$ in (4) and the sinusoidal steady state version in (6).

The physical origin of this factor $e^{-j\beta R}$ arises because there are no instantaneous events in electromagnetism. It takes a finite and specific amount of time for an effect to propagate and be observed elsewhere.

In other words, there is a time retardation principle in EM. For sinusoidal steady state, this creates a phase delay (and, in a lossy space, an amplitude decrease) between the source current oscillation and points farther away in space where the $\bar{E}$ and $\bar{H}$ fields are observed.