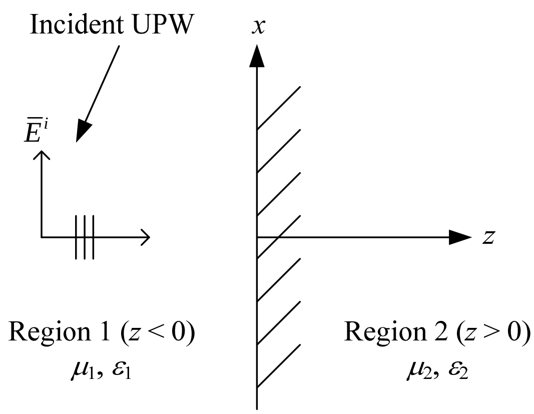
Lecture 29: UPWs Normally Incident on a Lossless Half Space.

The current sheet solution in Lecture 26 provided us with much information on the properties of uniform plane waves:

- ✓ $\bar{E} \times \bar{H}$ gives the direction of wave propagation, and it equals the power flow density, as we saw in the previous lecture.
- ✓ $\bar{E} \perp \bar{H}$, and both $\bar{E}$ and $\bar{H}$ are perpendicular to the direction of propagation.
- ✓ Ratios of perpendicular components of $\bar{E}$ and $\bar{H}$ = $\pm\eta$.
- ✓ $\lambda = 2\pi / \beta$ and $u = 1/\sqrt{\mu\varepsilon}$.

Now imagine that a UPW is "incident" on a half space as shown in the figure below. Our quest now will be to determine $\bar{E}$ and $\bar{H}$ everywhere.



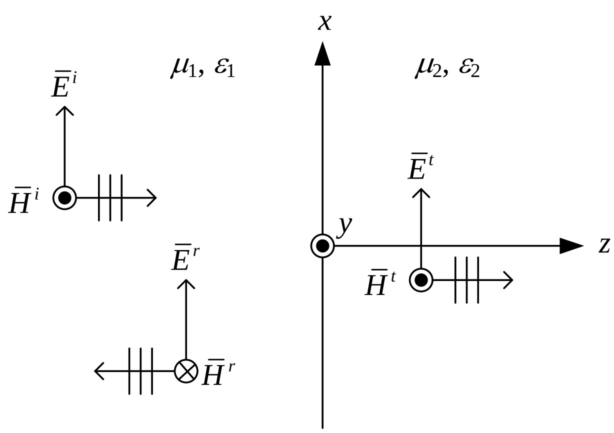
In this lecture, we will consider the reflection and transmission of UPWs normally (i.e., perpendicularly) incident on this planar boundary.

Comments

- The incident UPW is produced by an infinite current sheet located somewhere in region 1. We'll just specify an incident UPW with electric field $\bar{E}^i$ rather than solving the current sheet problem again.
- Because of the half space, we expect some "reflection" and "transmission" at the interface.
- The total field in region 1 is the sum of an incident and a reflected wave. The total field in region 2 is just the transmitted field.

Solution for Reflected and Transmitted Waves

Proceeding with the solution, we first draw the vector triplets ($\bar{E}$, $\bar{H}$, and direction) associated with each UPW (incident, reflected, and transmitted).



It is important to choose $\bar{E}^r$ and $\bar{E}^t$ in the same direction as $\bar{E}^i$. The directions of the corresponding magnetic field vectors can be found using the RHR.

Next, referring to this figure we write the solutions for $\bar{E}$ and $\bar{H}$ in each region making sure to use β_1, η_1 in region 1 and β_2, η_2 in region 2:

Region 1– $$E_{x1}(z) = E_x^i(z) + E_x^r(z) = E_1^+ e^{-j\beta_1 z} + E_1^- e^{j\beta_1 z} \quad (1)$$

$$H_{y1}(z) = H_y^i(z) + H_y^r(z) = \frac{E_1^+}{\eta_1} e^{-j\beta_1 z} - \frac{E_1^-}{\eta_1} e^{j\beta_1 z} \quad (2)$$

where $\beta_1 = \omega\sqrt{\mu_1\varepsilon_1}$ and $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$.

Region 2– $$E_{x2}(z) = E_x^t(z) = E_2^+ e^{-j\beta_2 z} \quad (3)$$

$$H_{y2}(z) = H_y^t(z) = \frac{E_2^+}{\eta_2} e^{-j\beta_2 z} \quad (4)$$

where $\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$ and $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$.

To determine the unknown coefficients E_1^- and E_2^+ (assuming E_1^+ is known) we apply the boundary conditions:

- $\bar{E}_{\tan}$ continuous at $z = 0$:

$$E_{x1}\big|_{z=0} = E_{x2}\big|_{z=0}$$

Using (1) and (3) in this boundary condition equation, then

$$E_1^+ + E_1^- = E_2^+$$

or
$$1+\frac{E_1^-}{E_1^+}=\frac{E_2^+}{E_1^+} \tag{5}$$

Similar to our analysis of TLs, we will define the ratio
$$\boxed{\Gamma \equiv \left.\frac{E_1^-}{E_1^+}\right|_{z=0}} \tag{6}$$

as the electric field **reflection coefficient** at $z = 0$ and
$$\boxed{\mathrm{T} \equiv \left.\frac{E_2^+}{E_1^+}\right|_{z=0}} \tag{7}$$

as the electric field **transmission coefficient** at $z = 0$.

Substituting these definitions into (5) we find
$$\boxed{1+\Gamma=\mathrm{T}} \tag{8}$$

- $\bar{H}_{\tan}$ continuous at $z = 0$:

$$\left.H_{y1}\right|_{z=0}=\left.H_{y2}\right|_{z=0}$$

From (2) and (4)
$$\frac{E_1^+}{\eta_1}-\frac{E_1^-}{\eta_1}=\frac{E_2^+}{\eta_2}$$
or
$$\frac{1}{\eta_1}-\frac{\Gamma}{\eta_1}=\frac{\mathrm{T}}{\eta_2} \tag{9}$$

Solving (8) and (9) for Γ and T gives
$$\boxed{\Gamma=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}} \tag{10}$$

and

$$\boxed{\mathrm{T} = \frac{2\eta_2}{\eta_2 + \eta_1}} \tag{11}$$

You probably recognize the form of (10) from your previous work with transmission lines where

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

As we can see in (10) and (11), for UPWs the amount of reflection and transmission at an interface depends on the contrast in the intrinsic impedances of the two half spaces.

We have now completed the solution for the UPW incident on a half space:

For $z \le 0$

– From (1): $$E_{x1}(z) = E_1^+\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right) \quad \text{[V/m]} \tag{12}$$

– From (2): $$H_{y1}(z) = \frac{E_1^+}{\eta_1}\left(e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z}\right) \quad \text{[A/m]} \tag{13}$$

while for $z \ge 0$

– From (3): $$E_{x2}(z) = E_1^+ \mathrm{T} e^{-j\beta_2 z} \quad \text{[V/m]} \tag{14}$$

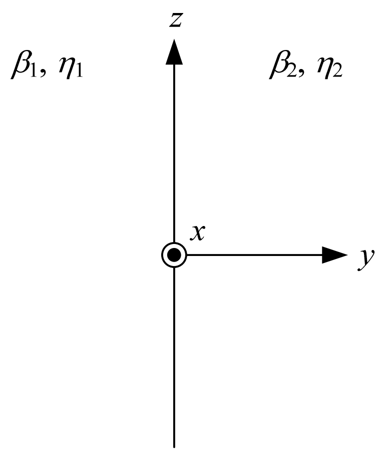
– From (4): $$H_{y2}(z) = \frac{E_1^+}{\eta_2} \mathrm{T} e^{-j\beta_2 z} \quad \text{[A/m]} \tag{15}$$

where Γ and T are given in (10) and (11).

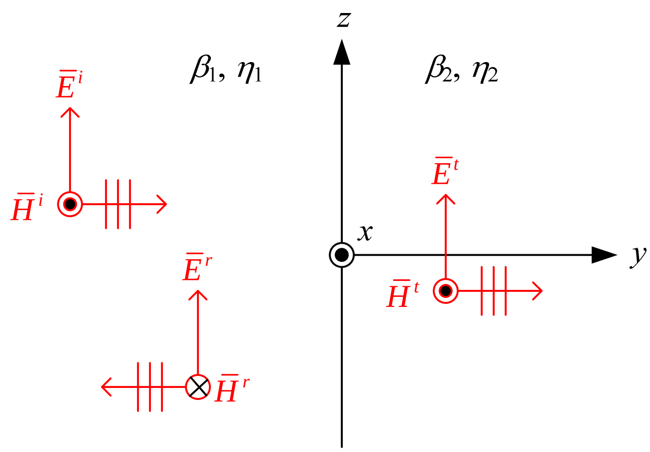
Example N29.1: A UPW is incident on the half space shown.

Given $\bar{E}^i = \hat{a}_z 2e^{-j\beta_1 y}$ V/m and $\bar{E}^t = \hat{a}_z \dfrac{6}{5} e^{-j\beta_2 y}$ V/m

Determine (a) the electric field reflection and transmission coefficients at the interface $y = 0$, and (b) the total magnetic field in each region.



The first step is to sketch the vector triplets for the incident, reflected, and transmitted UPWs:



(a) Comparing (3) and (14), we find that

$$E_2^+ = \mathrm{T}E_1^+$$

In this example, it is given that

$$\mathrm{T}E_1^+ = \frac{6}{5}$$

With $E_1^+ = 2 \Rightarrow$

$$\mathrm{T} = \frac{6}{5}\cdot\frac{1}{2} = \frac{3}{5}$$

Using (8)

$$\Gamma = \mathrm{T} - 1 = \frac{3}{5} - 1 = -\frac{2}{5}$$

(b) From the given $\bar{E}^i$ and the sketch above

$$\bar{H}^i = \hat{a}_x \frac{2}{\eta_1} e^{-j\beta_1 y} \quad \text{A/m}$$

To determine $\bar{H}^r$, first write $\bar{E}^r$ analogously from the second term in (12)

$$\bar{E}^r = \hat{a}_z \Gamma E_1^+ e^{+j\beta_1 y} = \hat{a}_z \left(-\frac{4}{5}\right) e^{+j\beta_1 y} \quad \text{V/m}$$

From this $\bar{E}^r$ and the sketch above

$$\bar{H}^r = \hat{a}_x \frac{4}{5\eta_1} e^{+j\beta_1 y} \quad \text{A/m}$$

Consequently, in region 1

$$\bar{H}_1 = \bar{H}^i + \bar{H}^r = \hat{a}_x \frac{2}{\eta_1}\left(e^{-j\beta_1 y} + \frac{2}{5} e^{+j\beta_1 y}\right) \quad \text{A/m}$$

while in region 2 using the given $\bar{E}^t$ and the sketch above

$$\bar{H}_2 = \bar{H}^t = \hat{a}_x \frac{6}{5\eta_2} e^{-j\beta_2 y} \quad \text{A/m}$$