Lecture 25: Single-Stub Tuner II – Smith Chart Solution.

We will next consider single-stub tuner analysis using the Smith chart. Before looking at this, however, we must first understand that the Smith chart can be used as an **admittance chart** as well as an impedance chart.

To see this, in Lecture 22 we derived the mapping upon which the Smith chart is based [$z(d) \leftrightarrow \Gamma(d)$] from the normalized TL impedance

$$z(d) = \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

From this, we can express the normalized TL **admittance** as

$$y(d) \equiv \frac{1}{z(d)} = \frac{1-\Gamma(d)}{1+\Gamma(d)} \quad (1)$$

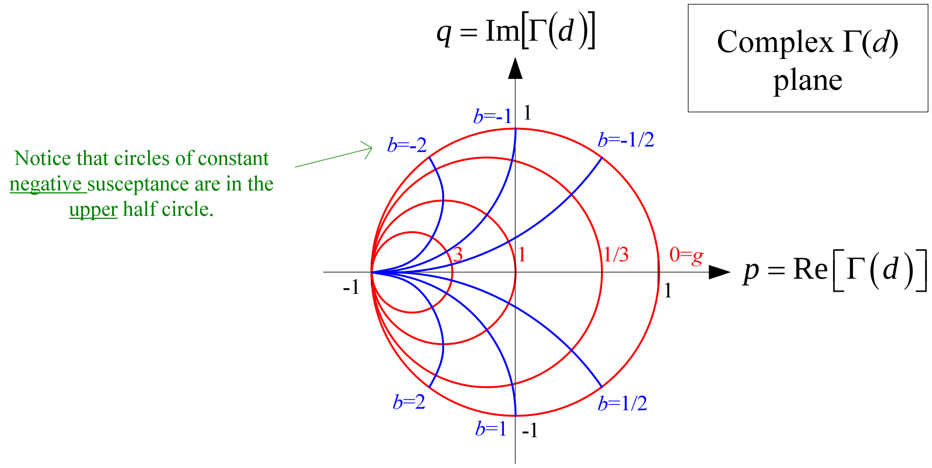
We can repeat the construction of the Smith chart with $y = g + jb$ and $\Gamma = p + jq$, as we did originally for the impedance chart. Substituting these quantities into (1) we find

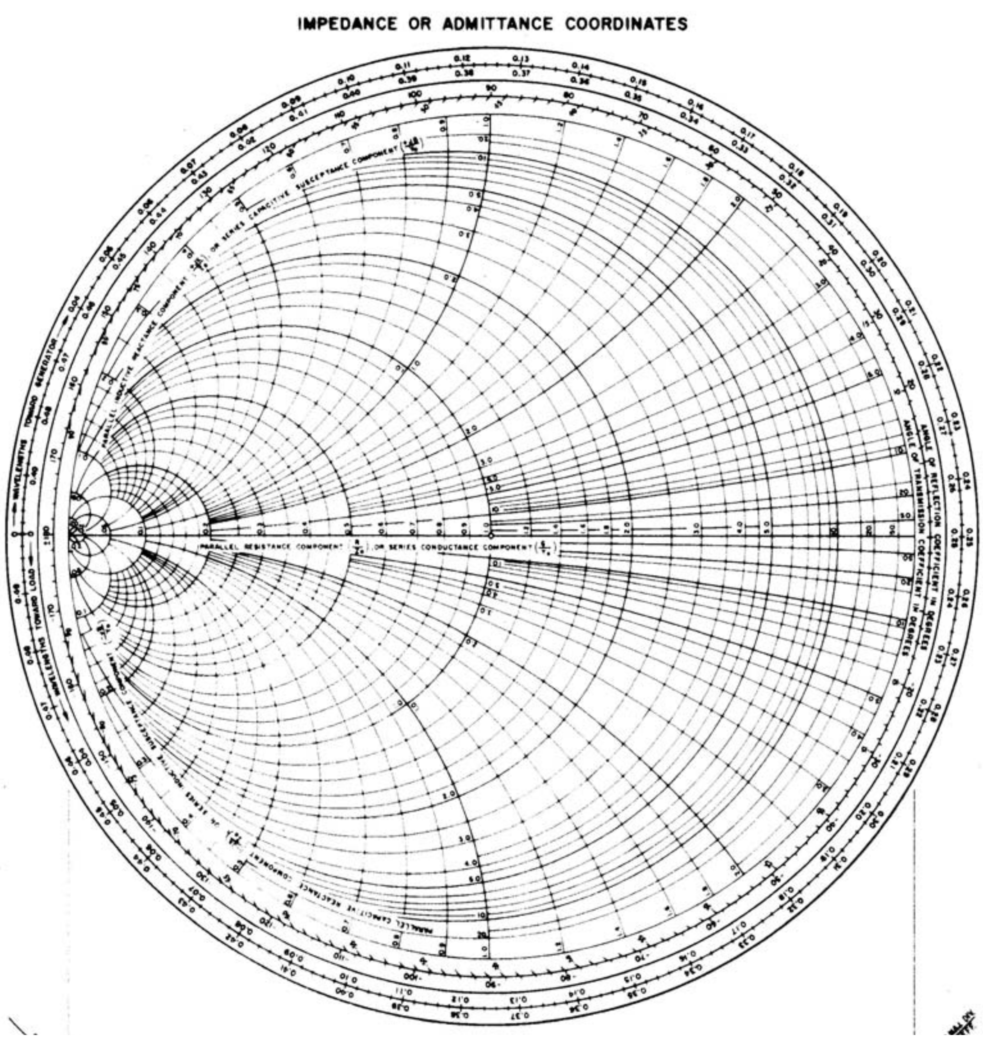
$$\left(p + \frac{g}{1+g}\right)^2 + q^2 = \left(\frac{1}{1+g}\right)^2 \quad (2)$$

and

$$(p+1)^2 + \left(q + \frac{1}{b}\right)^2 = \left(\frac{1}{b}\right)^2 \quad (3)$$

A Smith admittance chart can be constructed based on these two equations for circles in the *p*-*q* plane:





This Smith admittance chart looks very similar to the Smith impedance chart. In fact, if we rotate one chart by 180º we obtain the other.

This is actually an easily proved result. Consider the definition of the negative generalized reflection coefficient

$$-\Gamma(d) = \Gamma_L e^{-j(2\beta d+\pi)} = \Gamma_L e^{-j\left(2\beta d+\frac{\beta\lambda}{2}\right)}$$
$$= \Gamma_L e^{-j2\beta\left(d+\frac{\lambda}{4}\right)}$$

That is,

$$-\Gamma(d) = \Gamma\left(d+\frac{\lambda}{4}\right) \tag{4}$$

If we now substitute (4) into (1) we find that

$$y(d) = \frac{1+\Gamma\left(d+\frac{\lambda}{4}\right)}{1-\Gamma\left(d+\frac{\lambda}{4}\right)} = z\left(d+\frac{\lambda}{4}\right) \tag{5}$$

But what is $d+\lambda/4$? It's a half rotation around the Smith chart.

Discussion

From (5) we can deduce that:

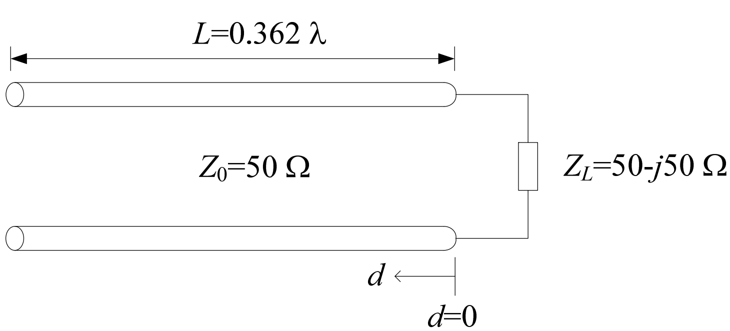
1. If $z(d)$ is known, then $y(d)$ is the point on the constant VSWR circle that is diametrically opposite the $z(d)$ point

on the Smith chart. (In this context, remember that a QWT is an impedance inverter device.)

2. The Smith chart can be used either as an impedance chart **or as an admittance chart**. Rather than keeping these two types of charts around, we can use one for either impedance or admittance calculations. The following example should help you understand this.

3. One subtlety with these mixed Smith charts is that generalized reflection coefficients are only correctly represented **on impedance charts when plotting normalized impedances** and **on admittance charts when plotting normalized admittances**. You'll read negative generalized reflection coefficients otherwise (for admittances on impedance charts and impedances on admittance charts).

Example N25.1: Use the Smith chart to compute the normalized input admittance of the TL shown below.



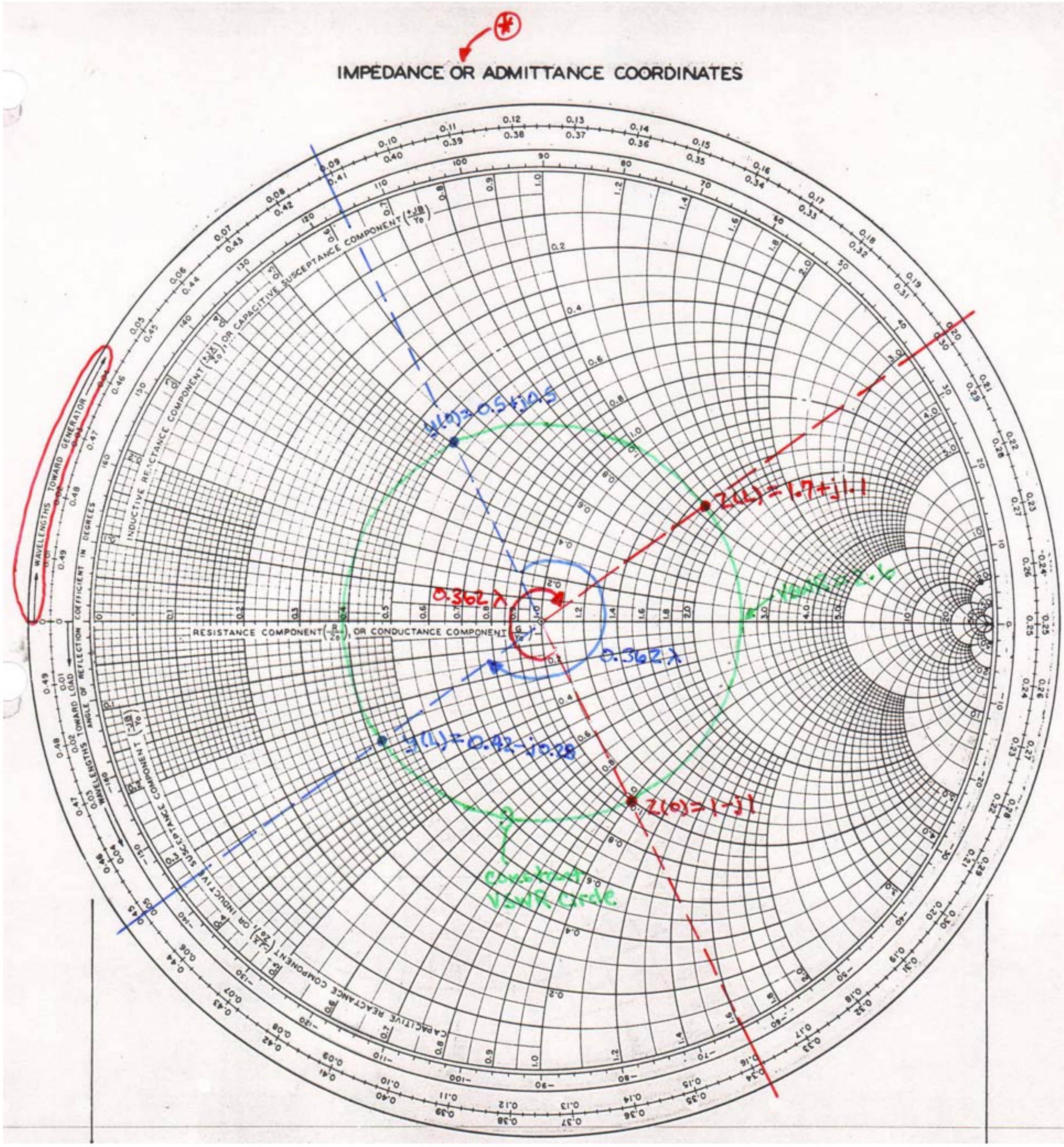
$$z(0) = \frac{Z_L}{Z_0} = \frac{50 - j50}{50} = 1 - j1 \text{ p.u.}\Omega$$

$$\Rightarrow\ y(0) = \frac{1}{z(0)} = 0.5 + j0.5 \text{ p.u.S}$$

Rotating 0.362 λ "towards generator," we can read from the Smith chart:

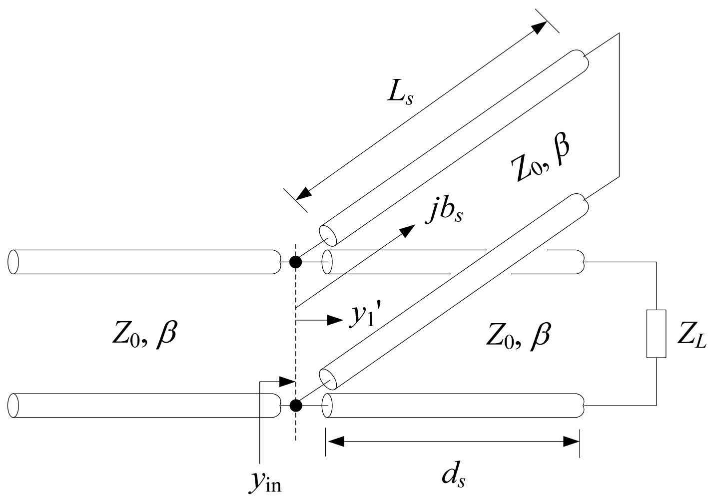
$$z(L) = 1.7 + j1.1 \quad \text{and} \quad y(L) = 0.42 - j0.28$$

[Exact: $z(L) = 1.682 + j1.103$ and $y(L) = 0.4157 - j0.2727$.]


IMPEDANCE OR ADMITTANCE COORDINATES
RESISTANCE COMPONENT, OR CONDUCTANCE COMPONENT
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
y(0)=0.5+j0.5
z(L)=1.7+j1.1
0.362λ
0.362λ
VSWR ≈ 2.6
y(L)=0.42-j0.28
z(0)=1-j1
Constant VSWR circle

Single-Stub Matching with the Smith Chart

As we saw in the previous lecture, the single-stub tuner geometry attached to a TL is



Recall that the operation of the single-stub tuner requires that

1. A length d_s is chosen such that y_1' has a **real** part = 1.
2. The imaginary part of y_1' is negated by the stub susceptance after choosing the proper length L_s.

We can perform these steps using only the Smith chart as our calculator. This process will be illustrated by an example.

Example N25.2: Using the Smith chart, design a shorted single-stub tuner to match the load $Z_L = 35 - j47.5\ \Omega$ to a TL with characteristic resistance $Z_0 = 50\ \Omega$.

The normalized load impedance and admittance are:

$$z_L = 0.70 - j0.95 \text{ p.u.}\Omega \quad \text{and} \quad y_L = 0.50 + j0.68 \text{ p.u.S}$$

Steps:

1. Locate $y_L = 0.50 + j0.68$ p.u.S on the Smith **admittance** chart.

2. Draw the constant VSWR circle using a compass.

3. Draw the line segment from the origin to y_L [this is the vector $-\Gamma(0+\lambda/4)$]. Rotate this vector **towards the source** until it intersects the **unit conductance circle**. Along this circle $\text{Re}[y(d)] = 1$.

   This is really the intersection of the constant VSWR circle for this load with the unit conductance circle.

   There will be two solutions. Both of these give $y_1' = 1 + jb_1$.

   For this example, we find from the Smith chart that

   (I) $y_1' = 1 + j1.2$

   (II) $y_1' = 1 - j1.2$

4. From these rotations we can determine d_s as

   (I) $d_s = 0.168\lambda - 0.109\lambda = 0.059\lambda$

   (II) $d_s = 0.332\lambda - 0.109\lambda = 0.223\lambda$

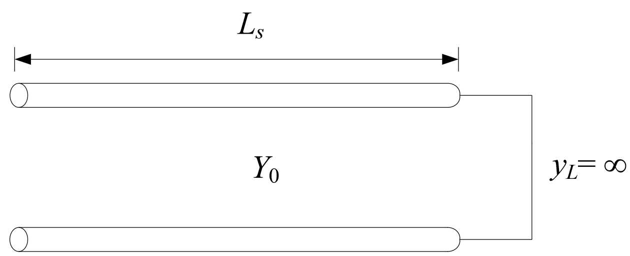
5. Next, find the stub lengths L_s.

(I) want $b_s = -1.2$

(II) want $b_s = 1.2$

When either of these two susceptances is added to y_1', then $y_{\text{in}} = 1 + j0$.

The stub lengths can also be determined directly from the Smith chart. Consider the shorted stub



On the Smith admittance chart, $y_L = \infty$ is located at p = 1, q = 0. From there, rotate "towards generator" to:

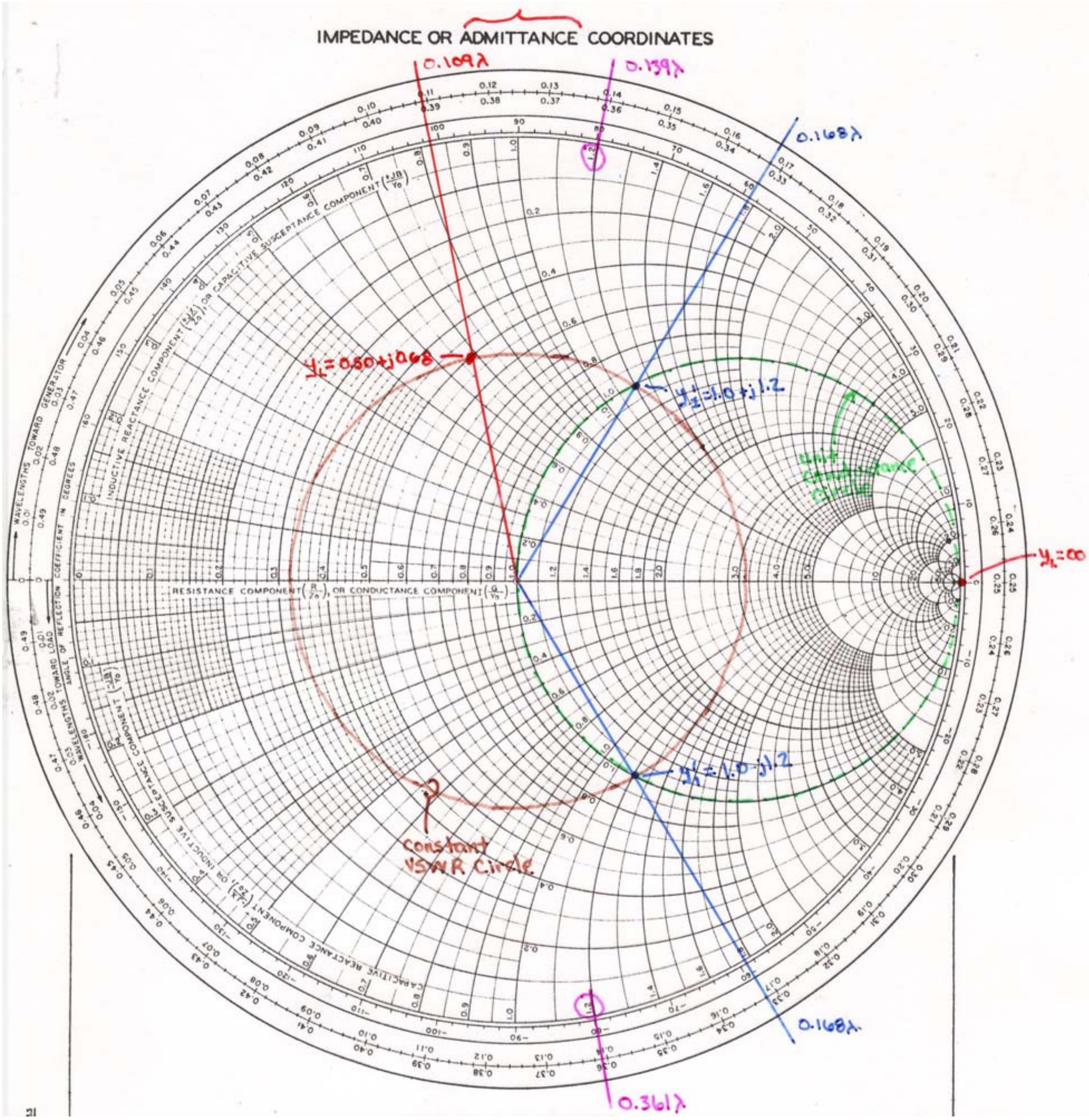
(I) $b_s = -1.2 \Rightarrow L_s = 0.361\lambda - 0.25\lambda = 0.111\lambda$

(II) $b_s = +1.2 \Rightarrow L_s = 0.25\lambda + 0.139\lambda = 0.389\lambda$

That's it. The final two solutions are:

(I) $d_s = 0.059\lambda$ and $L_s = 0.111\lambda$

(II) $d_s = 0.223\lambda$ and $L_s = 0.389\lambda$



We will check these two solutions using the results of the analytical analysis from the last lecture:

$$b_s = \pm\frac{2|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}} = \pm 1.191 \;\text{ for }\; \Gamma_L = 0.5116\measuredangle -1.367 \text{ rad.}$$

$$d_s = \frac{1}{2\beta}\left[\Theta_{\Gamma_L} - \tan^{-1}\left(\frac{b_s}{2}\right) - 2\pi\left(n \pm \frac{1}{4}\right)\right]$$
$$= \begin{cases} \lambda/(4\pi)\cdot\left[-1.904 - 2\pi(n+1/4)\right] & b_s > 0 \\ \lambda/(4\pi)\cdot\left[-0.8299 - 2\pi(n-1/4)\right] & b_s < 0 \end{cases}$$

Then

- for $b_s = 1.191$ with $n = -1$ (so that $d_s > 0$) gives $d_s = 0.2235\lambda$,
- for $b_s = -1.191$ with $n = 0$ gives $d_s = 0.05896\lambda$.

and

- for $b_s = 1.191$ gives $L_s = -\frac{1}{\beta}\tan^{-1}\left(\frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|}\right) + \frac{\lambda}{2} = 0.3888\lambda$
- for $b_s = -1.191$ gives $L_s = \frac{1}{\beta}\tan^{-1}\left(\frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|}\right) = 0.1112\lambda$