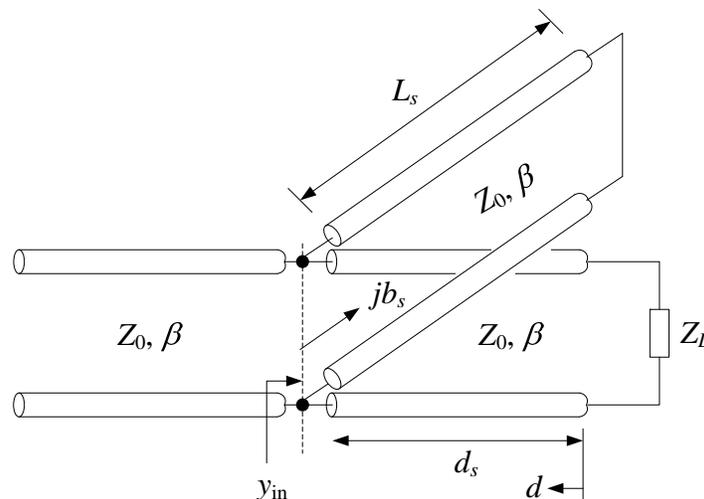


## Lecture 24: Single-Stub Tuner I – Analytical Solution.

Instead of requiring a special  $\lambda/4$ -length of TL to match a load to a TL, as discussed in the last lecture, other matching techniques can be used.

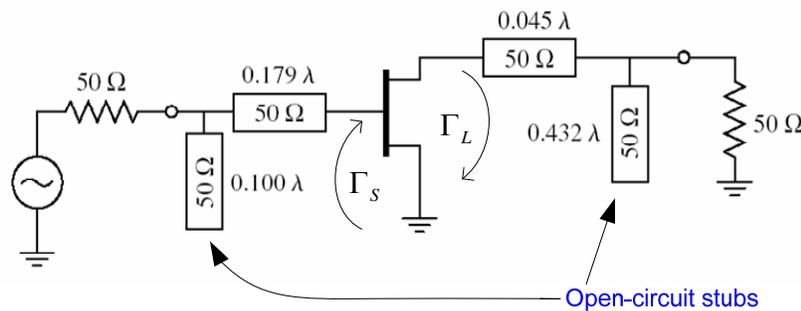
One of these is the **single-stub tuner** that we will discuss in this and the next lecture. The single-stub tuner uses a shorted (or open) section of TL attached at some position along the TL:



The shorted section is called the **stub**. All pieces of TL will be assumed to have the same  $Z_0$  and  $\beta$ , although this is not necessary.

Why a shorted section? These are easy to fabricate and the length can easily be made adjustable. In addition, no power is ideally dissipated in a shorted stub.

At microwave frequencies, microstrip or striplines are common TLs that are used for short, or open, circuited stubs. Open circuited stub tuners are used in the amplifier shown below, for example, at both the input and output ports of the transistor to create specific and desired values of  $\Gamma_S$  and  $\Gamma_L$ . The position and length of these stubs are easy to fabricate. (You'll see more of these designs in EE 481/581 *Microwave Engineering*.)



In the analysis of stub matching networks, it is **more convenient to work with admittances** rather than impedances since we are dealing with parallel connections.

The **total TL admittance**  $Y(d)$  at some position  $d$  and **TL characteristic admittance**  $Y_0$  are defined as

$$Y(d) \equiv \frac{1}{Z(d)} \text{ [S]} \quad \text{and} \quad Y_0 \equiv \frac{1}{Z_0} \text{ [S]}$$

Consequently, the normalized TL admittance is defined as

$$y(d) \equiv \frac{Y(d)}{Y_0} = \frac{1 - \Gamma(d)}{1 + \Gamma(d)} \quad (1)$$

whereby

$$\Gamma(d) = \frac{1 - y(d)}{1 + y(d)}$$

From the above TL figure and (1) we see that for a matched TL ( $d > d_s$ ) requires

$$y_{\text{in}} = 1$$

The goal of the single-stub tuner design is to derive an expression for  $y_{\text{in}}$ , then solve this expression for the parameters  $L_s$  and  $d_s$  such that  $y_{\text{in}} = 1$ .

Proceeding with the derivation of  $y_{\text{in}}$ , at  $d = 0$ :

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

while at  $d = d_s^-$ :  $\Gamma(d_s^-) = \Gamma_L e^{-j2\beta d_s^-}$

such that from (1)  $y(d_s^-) = \frac{1 - \Gamma(d_s^-)}{1 + \Gamma(d_s^-)}$  (2)

Since the stub is a short circuit, its input admittance is **purely imaginary (reactive)**. Specifically, recall from Lecture 19 that for a shorted section of TL

$$Z_{\text{in,ss}} = jZ_0 \tan(\beta L_s) \text{ } [\Omega]$$

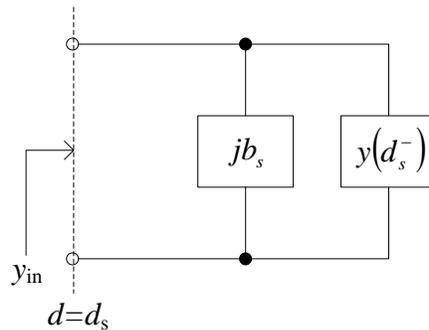
so that  $Y_{\text{in,ss}} = -jY_0 \frac{1}{\tan(\beta L_s)}$  [S]

In terms of a normalized input admittance for the shorted stub

$$y_{\text{in,ss}} \equiv \frac{Y_{\text{in,ss}}}{Y_0} = -j \frac{1}{\tan(\beta L_s)} = jb_s$$

where the shorted stub susceptance is  $b_s = -1 / \tan(\beta L_s)$ .

Therefore, at  $d = d_s$  we can construct the **equivalent circuit**:



Adding the two normalized admittances (transformed-load and stub admittances) together at  $d = d_s$  gives our desired expression for  $y_{in}$ :

$$y_{in} = y(d_s^+) = jb_s + y(d_s^-) \quad (3)$$

As discussed above, a matched TL for  $d > d_s$  requires  $y_{in} = 1$ . Requiring this in (3) gives

$$1 = jb_s + \frac{1 - \Gamma(d_s^-)}{1 + \Gamma(d_s^-)}$$

or

$$1 + \Gamma(d_s^-) = jb_s [1 + \Gamma(d_s^-)] + 1 - \Gamma(d_s^-)$$

Therefore

$$\begin{aligned} \Gamma(d_s^-) &= \frac{jb_s}{2 - jb_s} \\ &= \Gamma_L e^{-j2\beta d_s^-} \end{aligned} \quad (4)$$

This last equation comes from the definition of  $\Gamma(d_s^-)$  on page 3.

We will now solve (4) for  $d_s$  and  $L_s$  by **equating the magnitude and phase of both sides of (4)**:

- Magnitude:

$$|\Gamma_L| = \frac{|b_s|}{\sqrt{4 + b_s^2}} \quad (5)$$

- Phase:

$$\begin{aligned} \Theta_{\Gamma_L} - 2\beta d_s - n2\pi &= \angle \left\{ \frac{jb_s}{2 - jb_s} \right\} \\ &= \pm \frac{\pi}{2} - \tan^{-1} \left( -\frac{b_s}{2} \right) \quad b_s \geq 0 \end{aligned}$$

or 
$$\Theta_{\Gamma_L} = \pm \frac{\pi}{2} + \tan^{-1} \left( \frac{b_s}{2} \right) + 2\beta d_s + 2n\pi \quad b_s \geq 0 \quad (6)$$

Finally, we solve (5) and (6) for the two unknowns  $d_s$  and  $L_s$  that force  $y_{in} = 1$ :

- From (6):

$$d_s = \frac{1}{2\beta} \left[ \Theta_{\Gamma_L} - \tan^{-1} \left( \frac{b_s}{2} \right) - 2\pi \left( n \pm \frac{1}{4} \right) \right] \quad b_s \geq 0 \quad (7)$$

- From (5):

$$|\Gamma_L|^2 = \frac{b_s^2}{4 + b_s^2} \Rightarrow 4|\Gamma_L|^2 = b_s^2 (1 - |\Gamma_L|^2)$$

or 
$$b_s = \pm \frac{2|\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} \quad b_s \geq 0$$

However, for the shorted stub

$$b_s = -\frac{1}{\tan(\beta L_s)}$$

as developed earlier on page 3 of this lecture, then

$$\tan(\beta L_s) = \mp \frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|} \quad b_s \geq 0$$

Finally, from this last expression we can determine that

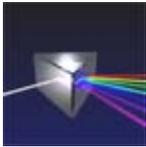
$$L_s = \begin{cases} -\frac{1}{\beta} \tan^{-1} \left( \frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|} \right) + \frac{\lambda}{2} & \text{(yields } b_s > 0) \\ \frac{1}{\beta} \tan^{-1} \left( \frac{\sqrt{1-|\Gamma_L|^2}}{2|\Gamma_L|} \right) & \text{(yields } b_s < 0) \end{cases} \quad (8)$$

Equations (7) and (8) are the final solutions we need. That is, by selecting the length of the shorted stub  $L_s$  according to (8) and the location of the stub  $d_s$  according to (7) **we can match any load  $Z_L$  to the TL for  $d > d_s$ .**

Note this is accomplished without consuming any power in the matching network!

**Example N24.1:** Match a load of  $Z_L = 25 - j50 \, \Omega$  to a TL with  $Z_0 = 50 \, \Omega$  using a short-circuited, single-stub tuner. Use the analytical solution method outlined in this lecture.

[See the VisualEM “Example C.7” worksheet for the solution. Note that the animation shows no “pulsation” of the voltage indicating a purely traveling wave towards the load. This is expected behavior on a perfectly matched TL.]



## Example C.7

# Single-Stub Tuner Calculator

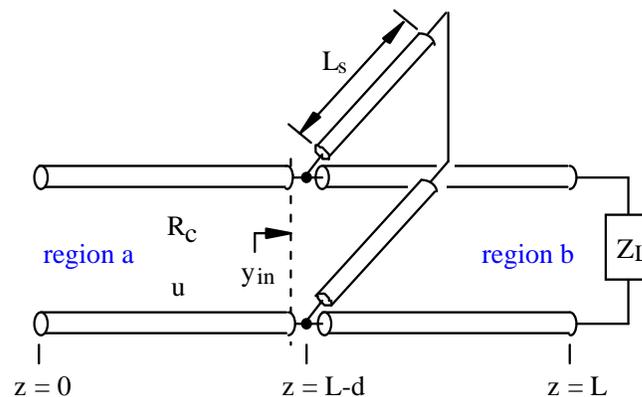


### Purpose

To design a single-stub tuner using an analytical solution and to visualize the voltage on the resulting matched transmission line (TL). This analytical solution is a complement to the Smith chart method discussed in Sect. C.4.1 of the text. After computing the design parameters for the single-stub tuner, the phasor voltage is plotting on the TL followed by an animation clip of this voltage. The purely traveling wave nature of the voltage in the matched region is evident in both types of plots.

### Enter parameters

The transmission line (TL) with the attached single-stub tuning (SST) network is depicted in the figure below:



A shorted section of TL is used to form the stub and we will assume that all sections of TL have the same properties (characteristic resistance and propagation velocity).

Choose the load impedance and the characteristic resistance and propagation velocity of the TL:

$$Z_L := 25 - j \cdot 50$$

Load impedance ( $\Omega$ ).

$$R_C := 50$$

TL characteristic resistance ( $\Omega$ ).

$$u := 300 \cdot 10^6$$

Propagation velocity (m/s).

$$f := u$$

Source frequency (Hz).

Compute the phase constant and the wavelength for this TL:

$$\omega := 2 \cdot \pi \cdot f$$

$$\beta := \frac{\omega}{u} \quad \beta = 6.283 \quad (\text{rad/m})$$

$$\lambda := \frac{2 \cdot \pi}{\beta} \quad \lambda = 1.000 \quad (\text{m})$$

The period for one time cycle of the sinusoidal voltage is:

$$T_p := \frac{1}{f} \quad T_p = 3.333 \times 10^{-9} \quad (\text{s})$$

## Background

In Section C.4.1 of the text, a technique was shown for designing a single-stub tuner (SST) matching network using the **Smith chart**. That approach gives a great deal of insight into the inner workings of such a design. What we are providing in this worksheet is an analytical solution for computing the design parameters of the SST. This solution gives the exact values for the stub length and location required in the SST design. You can also use this worksheet in conjunction with the Smith chart method to confirm your results from the latter technique.

The analytical solution proceeds by equating to unity the normalized input admittance,  $y_{in}$ , which is indicated in the figure above. When such a condition is satisfied by adjusting the stub location  $d$ , and the stub length,  $L_s$ , there will be no reflection at this junction – which necessarily means that the load and the SST matching network combination have been perfectly matched to the TL. The steps for obtaining  $d$  and  $L_s$  are somewhat tedious so only the final equations necessary for the design of a single stub tuner will be shown.

## Single-stub tuner design

The first step in the analysis of the SST is to compute the voltage reflection coefficient at the load which, from (66) in Chap. 7 of the text, is:

$$\Gamma_L := \frac{Z_L - R_C}{Z_L + R_C} \quad \Gamma_L = 0.077 - 0.615j$$

$$\text{Mag}_{\Gamma_L} := |\Gamma_L| \quad \theta_{\Gamma_L} := \arg(\Gamma_L) \quad \Gamma_L \text{ magnitude and phase.}$$

From the analytical solution for the SST, the two possible lengths of the shorted-stub tuning section are:

$$L_{s1} := \frac{1}{\beta} \cdot \text{atan} \left( \frac{\sqrt{1 - \text{Mag}_{\Gamma_L}^2}}{2 \cdot \text{Mag}_{\Gamma_L}} \right) \quad L_{s2} := -\frac{1}{\beta} \cdot \text{atan} \left( \frac{\sqrt{1 - \text{Mag}_{\Gamma_L}^2}}{2 \cdot \text{Mag}_{\Gamma_L}} \right) + \frac{\lambda}{2}$$

$$L_{s_1} = 0.0898 \quad (\text{m})$$

$$L_{s_2} = 0.4102 \quad (\text{m})$$

Also from the analytical solution, the normalized input susceptance of the shorted-stub section must be:

$$b_{s_1} := -\frac{2 \cdot \text{Mag}_{\Gamma L}}{\sqrt{1 - \text{Mag}_{\Gamma L}^2}} \quad b_{s_2} := -b_{s_1}$$

$$b_{s_1} = -1.5811$$

$$b_{s_2} = 1.5811$$

With this information, we can now compute the distance from the load,  $d$ , at which this shorted stub must be connected to the TL. There are many possible solutions for this distance. In the following few statements we select the *shortest* distance  $d$  for both solutions 1 and 2. First define the functions that compute the location of the shorted stub for solutions 1 and 2:

$$\text{loc}_1(n) := \frac{1}{2 \cdot \beta} \cdot \left[ \theta_{\Gamma L} - \text{atan}\left(\frac{b_{s_1}}{2}\right) - 2 \cdot \pi \cdot \left(n - \frac{1}{4}\right) \right] \quad \text{Stub locations for solution \#1.}$$

$$\text{loc}_2(n) := \frac{1}{2 \cdot \beta} \cdot \left[ \theta_{\Gamma L} - \text{atan}\left(\frac{b_{s_2}}{2}\right) - 2 \cdot \pi \cdot \left(n + \frac{1}{4}\right) \right] \quad \text{Stub locations for solution \#2.}$$

Now construct two lists  $\text{dist}_1$  and  $\text{dist}_2$  that contain a number of possible locations for the stubs in solutions 1 and 2:

$$n := -10..10$$

$$\text{dist}_{1_{n+10}} := \text{if}(\text{loc}_1(n) \geq 0, \text{loc}_1(n), 1000)$$

$$\text{dist}_{2_{n+10}} := \text{if}(\text{loc}_2(n) \geq 0, \text{loc}_2(n), 1000)$$

The purpose of the if statements is to reject the solutions with negative distance. The shortest distances  $d_1$  and  $d_2$  for the location of the shorted stub are then:

$$d_1 := \min(\text{dist}_1) \quad d_2 := \min(\text{dist}_2)$$

The two solutions for the single stub tuning network are:

### Solution 1

$$d_1 = 0.0631$$

$$L_{s_1} = 0.0898$$

### Solution 2

$$d_2 = 0.2067$$

$$L_{s_2} = 0.4102$$

Smallest stub distance from load (m).

Stub length (m).

This completes the design of the single-stub tuner.

### Solve for the voltage on the TL

Now that the SST has been designed, for the remainder of this worksheet we will compute and visualize the voltage that exists on the TL with this attached SST. In this section, we will compute the voltage that exists in both regions a and b (as indicated in the figure above) with an attached single-stub matching network. In the following sections, we will plot and animate this voltage.

First, choose one of the two above solutions for the SST matching network:

$$n_{\text{soln}} := 2 \quad \text{Choose a solution number: Enter either 1 or 2.}$$

We will define the total length of the TL ( $L$  in the above figure) to be equal to the sum of one wavelength (in meters) plus the distance of the stub from the load:

$$d_{\text{stub}} := d_{n_{\text{soln}}}$$

$$L := 1 \cdot \lambda + d_{\text{stub}}$$

The solution for the voltage anywhere on the TL,  $V(z)$ , is computed in the following *referenced worksheet*:

 Reference:D:\VisualEM\vem 1.2alpha\VISUALEM\vemac021.mcd(R)

The details of this solution for  $V(z)$  are not of central importance in this worksheet. However, you may wish to browse this referenced worksheet and see the procedure for solving a two-region TL problem.

One thing of interest to us here is the input impedance just to the left of the stub seen looking towards the SST. This input impedance, as computed in the referenced worksheet, is:

$$Z_{\text{in}} = 50.000 \quad (\Omega)$$

If this single-stub tuner is designed correctly, then  $Z_{\text{in}}$  must be equal to  $R_C = 50.000 \quad (\Omega)$  since such a condition will produce no reflection at this junction.

### Plot V on the TL

We will now construct the plots of the voltage on this matched TL. The first two plots are the magnitude and phase of the (phasor) voltage on this TL. The third plot will be an animation of this voltage field in space and time.

Choose the number of points at which to plot the voltage on the TL:

$$n_{\text{pts}} := 80 \quad \text{Number of points to plot the voltage.}$$

Construct a list of  $z_i$  points at which to plot the voltage:

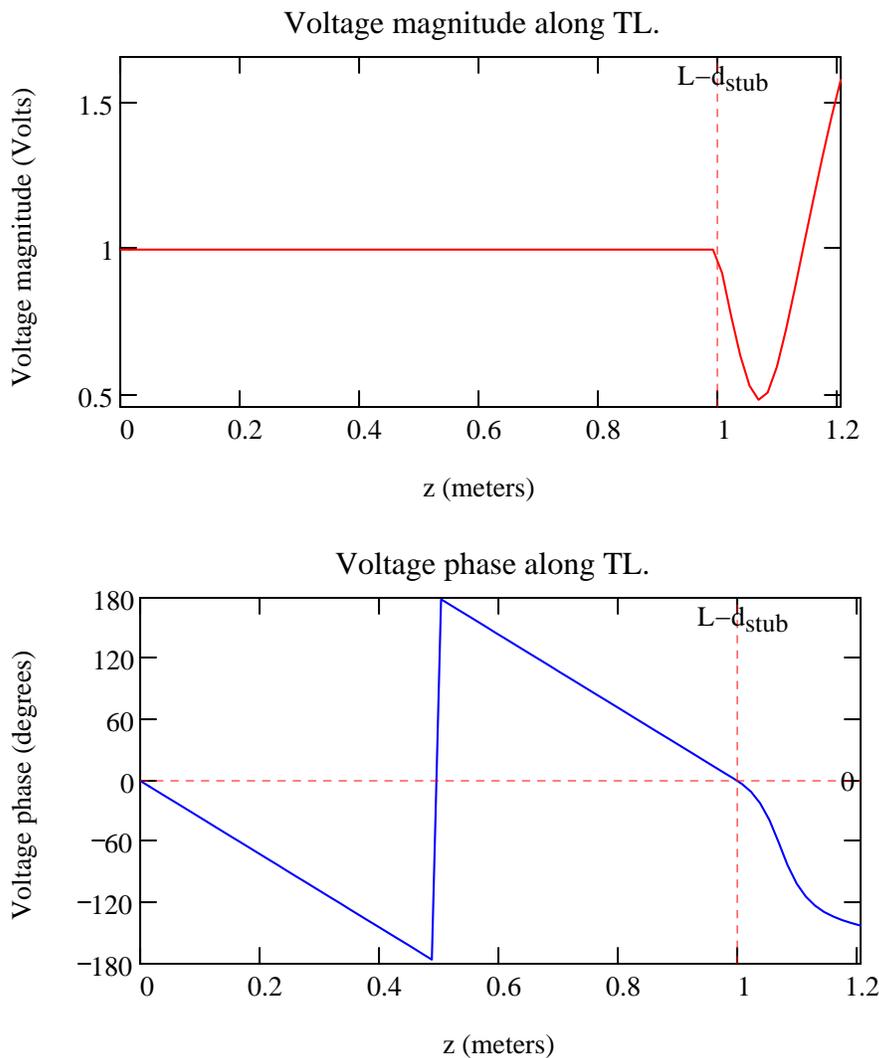
$$i := 0..npts - 1 \quad z_i := i \cdot \frac{L}{npts - 1}$$

Compute the voltage magnitude and phase at every point  $z_i$  along the TL:

$$\text{Mag}_{V_i} := |V(z_i)| \quad \text{Voltage magnitude (V).}$$

$$\Theta_{V_i} := \text{if} \left( |V(z_i)| \neq 0, \frac{\arg(V(z_i))}{\text{deg}}, 0 \right) \quad \text{Voltage phase (}^\circ\text{).}$$

Plot the voltage magnitude and phase along the TL with the single-stub matching network attached:



In both of these plots, the load is located at the far right and the shorted stub is connected to the TL at the location  $L - d_{\text{stub}} = 1.000$  (m).

The spatial variation of this voltage is very different whether we observe it between the load and the stub (region b) or past the stub (region a). In the latter region, where  $z < L - d_{\text{stub}}$ , the magnitude of the voltage is constant and the phase is linearly decreasing towards the load. These properties are the two distinguishing characteristics of a pure *traveling wave*. That is, the voltage in this portion of the TL is composed only of a voltage wave traveling towards the load. There is no reflection from the SST matching network and load combination.

Conversely, the voltage in region b varies significantly from this traveling wave behavior. This is a result of both forward and reverse propagating voltage waves in this region that gives rise to the **interference** we observe in these two plots. So, while the TL is matched for  $z < L - d_{\text{stub}}$ , the TL in the region  $L - d_{\text{stub}} < z < L$  is definitely not – nor was this a requirement in the design of the single-stub tuning network.

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### Animated plot of the voltage

The last topic of this worksheet is the generation of an animation clip to show both the time and spatial variation of the voltage on the TL.

The time-domain expression for the voltage everywhere on the transmission line can be computed from the phasor-domain form as:

$$V_t(z, t) := \text{Re}(V(z) \cdot \exp(j \cdot \omega \cdot t))$$

Choose the number of points in time at which to plot the voltage:

$$n_{\text{pts}} := 20 \quad \text{Number of points to plot in one time period.}$$

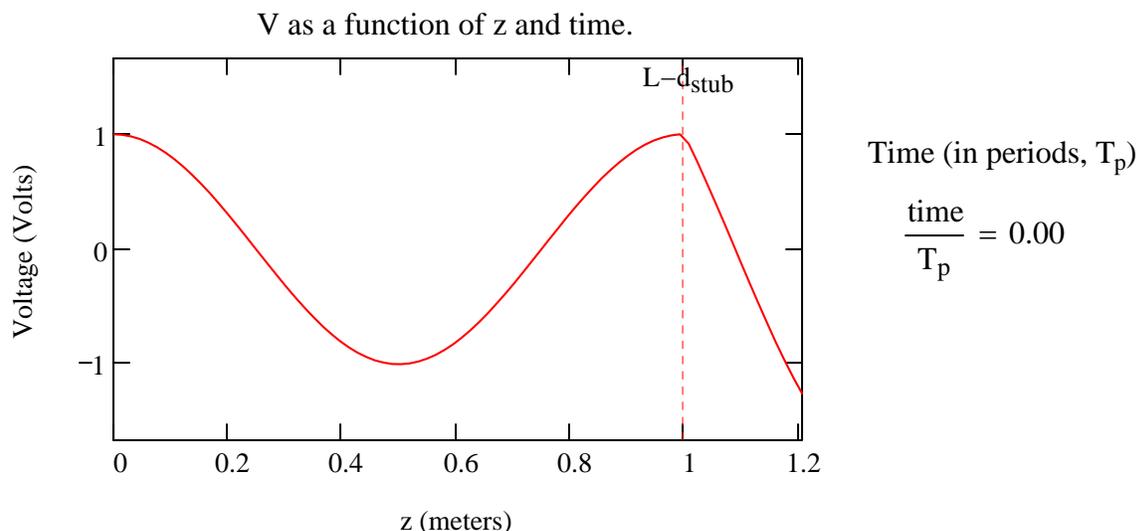
The starting and ending times for the animation clip are:

$$t_{\text{start}} := 0 \quad t_{\text{end}} := T_p \quad \text{Times to start and end plot (s).}$$

Define the variable time in terms of the constant FRAME: 

$$t_{\text{inc}} := \frac{T_p}{n_{\text{pts}}} \quad \text{time} := t_{\text{start}} + \text{FRAME} \cdot t_{\text{inc}}$$

Now create an animation clip of the voltage on the TL. For best results, in the "Animate" dialog box choose  $T_0 = 19$  then save the file and replay the animation in a video player that supports continuous loop playback. 



Once again, the load is at the far right in this plot and the shorted stub is connected to the TL at the location  $L - d_{\text{stub}} = 1.000$  (m).

As we can see in this plot for all time, the voltage is continuous across the junction at  $z = L - d_{\text{stub}}$  as it should. Note that the amplitude in the right-hand region is generally quite different than that in the left-hand (i.e., the matched) region.

Viewing the animation clip of this voltage provides a time-domain interpretation of the phasor voltage magnitude and phase plots shown earlier in this worksheet. In particular, within the matched region (region a where  $z < L - d_{\text{stub}}$ ) the magnitude of the phasor voltage was constant while the phase was linearly decreasing towards the load. We interpreted these characteristics of the magnitude and phase as those of a pure traveling wave towards the load. That interpretation of the phasor domain voltage should translate very clearly to the time-domain behavior as you view this animation clip. In particular, note that the voltage wave in this region simply travels towards the load and does *not* "pulsate" with time. (This "pulsation" is a distinguishing characteristic of **standing waves**.)

Consequently, the single stub tuner has accomplished its goal. Namely, with this matching network connected to the TL, there is no reflected voltage wave in the region  $z < L - d_{\text{stub}}$ . Congratulations, you have successfully designed a single-stub matching network!

The behavior of the voltage field in the right-hand region (region b where  $L - d_{\text{stub}} < z < L$ ) is more difficult to interpret in this animation clip. The reason for this is that the distance  $d_{\text{stub}}$  is usually only a fraction of a wavelength. Therefore, observing the standing wave nature of the voltage wave in this region is then quite difficult.

Finally, since this matching network itself cannot consume (or dissipate) energy, we can infer that all the time-average power carried by the incident voltage wave in region a is being deposited into the load.

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## Reference

D. K. Cheng, *Field and Wave Electromagnetics*, 2nd ed. Reading, MA: Addison-Wesley Publishing Company, 1989.

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End of worksheet.

