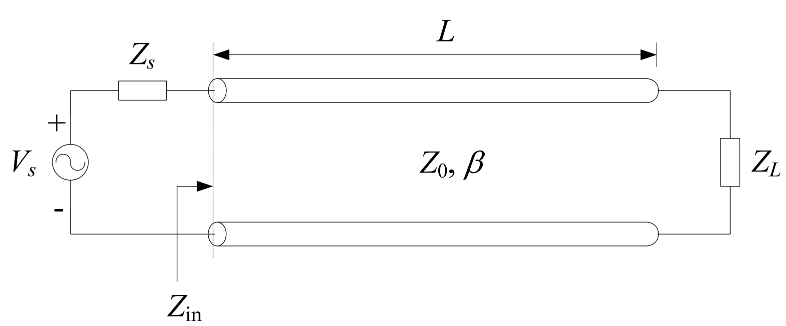
Lecture 23: TL Matching. Quarter-Wave Transformers. Resistive Pads.

Transmission lines are commonly used as components in communication systems. In this capacity, the TL functions as a conduit for an electrical signal as it propagates from one subsystem to another. In such an application, we want to transfer as much of the signal's electrical energy to its destination as possible.

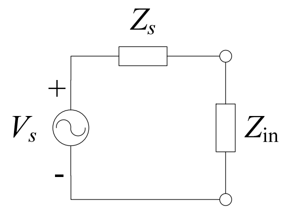
However, reflections from the load (i.e., the destination) may prevent this from happening.

Consider the sinusoidal steady state excitation of a TL:



As we have already seen in previous lectures, there will be no reflection from the load if $Z_L = Z_0$. This is called a **matched TL**. A matched TL has a VSWR = 1.

In general, though, an equivalent circuit at the input to the above TL is



According to the maximum power transfer theorem, and for a fixed Z_S, maximum power is delivered to the load on a lossless TL when $Z_{\text{in}} = Z_s^*$. Furthermore, for almost all high frequency equipment $Z_s = Z_0$.

Now, if $Z_L = Z_0$ it is straightforward to show that $Z_{\text{in}} = Z_L$. Hence, $Z_{\text{in}} = Z_s^*$ [because $Z_s^* = (Z_0)^* = Z_0$ and $Z_0 = Z_L$].

In summary, on a lossless and matched TL we have just shown that the maximum power transfer condition is met when the source impedance is Z_0.

Matching Networks

In the more common situation when $Z_L \neq Z_0$, matching a load to a TL is commonly accomplished using additional circuitry attached to the TL. This additional circuitry is called a matching network.

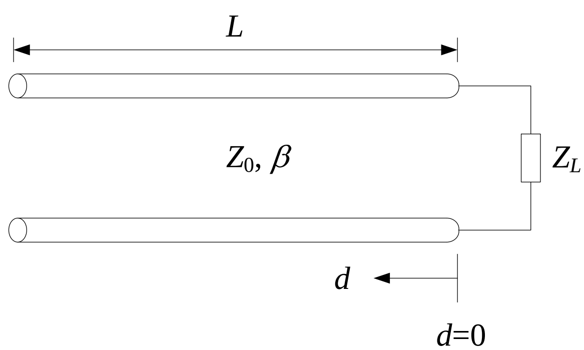
We will discuss three types of matching networks in this course:

1. Quarter-wave transformer
2. Resistive pads

3. Single-stub tuner.

Quarter-Wave Transformer

We've seen in a couple of lectures now that for a TL in the sinusoidal steady state with an arbitrary load



the total impedance at position *d* from the load is

$$Z(d) \equiv \frac{V(d)}{I(d)} = Z_0 \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

as in (9) of the previous lecture. With the generalized reflection coefficient given as

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

then

$$Z(d) = Z_0 \frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}} \qquad (1)$$

Now suppose that the length L of the TL at some frequency is exactly $\lambda/4$. Then,

$$2\beta d = 2\beta L = 2\cdot\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \pi$$

Therefore, $$e^{-j2\beta L} = e^{-j\pi} = -1$$

and (1) becomes

$$Z\left(d = \frac{\lambda}{4}\right) = Z_0 \frac{1-\Gamma_L}{1+\Gamma_L} \tag{2}$$

By definition, we know that

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

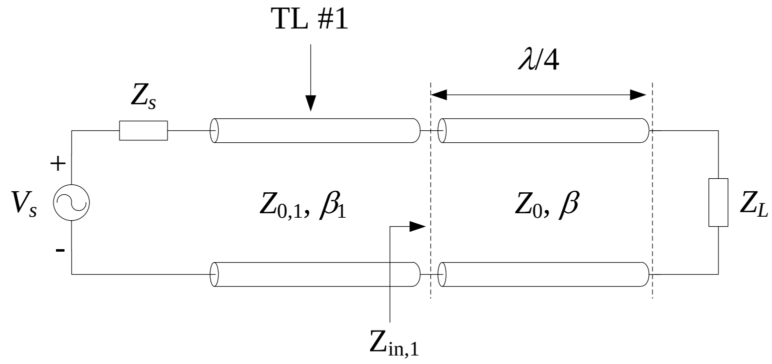
Substituting this result in (2) gives

$$Z\left(d = \frac{\lambda}{4}\right) = Z_0 \frac{Z_L + Z_0 - \left(Z_L - Z_0\right)}{Z_L + Z_0 + \left(Z_L - Z_0\right)} = \frac{Z_0^2}{Z_L}$$

or $$\boxed{Z\left(d = \frac{\lambda}{4}\right)\cdot Z_L = Z_0^2} \tag{3}$$

This result in (3) is an interesting characteristic of a TL that is exactly $\lambda/4$ long at a given frequency. We can harness this characteristic to design a matching network using a $\lambda/4$-long section of TL.

Consider the following structure comprised of an impedance load interconnected to a TL (we'll call this TL #1) through a $\lambda/4$-section of a second TL:



Imagine we wish to match an arbitrary load Z_L to the TL #1. We can use (3) to design the $\lambda/4$ section of TL by adjusting its characteristic impedance, Z_0.

In particular, to match Z_L to TL #1 we require $Z_{in,1} = Z_{0,1}$. For this example, the applicable quantities for (3) are

$$Z\left(d = \frac{\lambda}{4}\right) = Z_{in,1} = Z_{0,1}$$

Using these values in (3) gives

$$\boxed{Z_0 = \sqrt{Z_{0,1} Z_L}} \qquad (4)$$

In other words, if the $\lambda/4$ section of TL has the special characteristic impedance given in (4) – a geometrical mean of $Z_{0,1}$ and Z_L – then TL #1 will be matched to the load.

This type of matching network is called a **quarter-wave transformer** (QWT). It transforms the impedance from one value to another from the output of the $\lambda/4$ section to its input, according to (3).

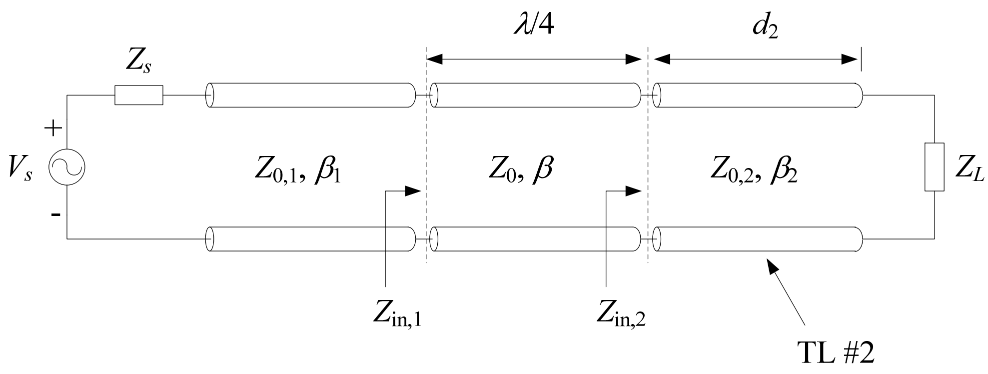
QWT Discussion

First, how practical is it to change the characteristic impedance of a TL? For coaxial cable, it is very difficult to do since Z_0 is dependent, among other quantities, on the ratio of the outer to inner radii. But at microwave frequencies, it is very easy to change Z_0 of typical TLs such as microstrip or stripline. One only needs to change the width of the lands.

Second, a matching network should not consume (much) power. In (4) we can deduce that Z_0 will generally be a complex quantity indicating that for a perfect match, the QWT will need to include a lossy TL.

However, if the load is purely resistive we can connect the QWT directly to the load and use a lossless TL.

Otherwise if Z_L is not resistive, we can insert a third TL (TL #2 in the figure below of length d_2) such that $Z_{\text{in},2}$ is real. [Think of the Smith chart (!) and the values of $Z_{\text{in},2}$ at $\left|V(d)\right|_{\text{max}}$ or $\left|V(d)\right|_{\text{min}}$.]



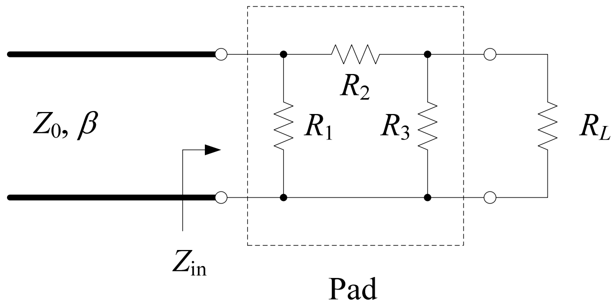
Two disadvantages of this QWT are that:

1. We would need to cut the TL to insert it, and
2. It works perfectly only for one load at one frequency. Actually, it produces some bandwidth of "acceptable" VSWR on the TL, as do all real-life matching networks.

Resistive Pads

A **resistive pad** is essentially an attenuator that provides a good deal of matching capabilities. Unfortunately, this matching capability comes at a price: reduced transmitted signal level. Moreover, these pads are sometimes relatively expensive.

A pi-structure resistive pad has the following topology:



The insertion loss (IL) of a resistive pad is defined as

$$\text{IL} = 10\log_{10}\frac{P_{L,\text{without pad}}}{P_{L,\text{with pad}}} \quad [\text{dB}] \tag{5}$$

where P_L is the time averaged power delivered to the load, R_L. It can be shown that the design equations for the pi-structure resistive pad are

$$R_1 = R_3 = \frac{R_L(1+K)}{(R_L/Z_0)K-1} \quad \text{and} \quad R_2 = (R_3 \,\|\, R_L)(K-1) \quad (6),(7)$$

where

$$K = 10^{\text{IL}/20} \tag{8}$$

A sample of a resistive pad design and its performance is shown in the *VisualEM* “Section C.4.4 and Problem C.4.13” worksheet.

Section C.4.4 and Problem C.4.13

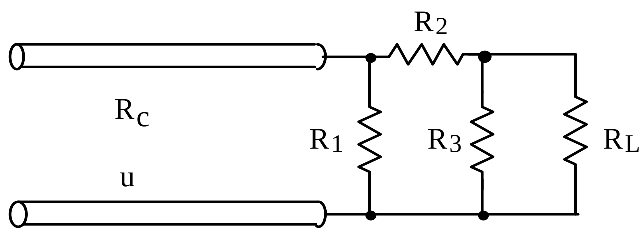
Resistive Pad Design and Performance

Purpose

To design a resistive pad matching network that has a pi topology. The design of this resistive pad follows the procedure outlined in Example C.9 of the text. Once the resistances of this pad have been obtained, the performance of this matching network is observed by plotting the VSWR on a TL that has a variable resistive load attached directly to this resistive pad.

Enter parameters

The geometry of the transmission line (TL) and the attached resistive pad matching network is shown in Fig. C.22 of the text and in the figure below:



As shown in this figure, we will assume that the pad is connected directly to a resistive load. We will design this pi-topology resistive pad, by the proper choice of the resistances R_1, R_2 and R_3, to provide a low VSWR on the TL over a wide range of load resistances, R_L.

Choose the characteristic resistance of the TL and the insertion loss in the resistive pad:

$R_C := 300$ TL characteristic resistance (Ω).

$IL := 10$ Insertion loss of the resistive pad (dB).

Design pi-topology resistive pad

Following the design procedure given in Example C.9 of the text, we will compute the resistances R_1, R_2 and R_3. We will initially choose

$R_L := R_C$

so that if the load does indeed equal R_C there will be no reflection caused by the resistive-pad matching network.

Now compute R_1, R_2 and R_3 using Equations (C-48) to (C-50) as shown in Example C.9 of the text:

$$X := 10^{\frac{IL}{20}}$$

$$R_1 := \frac{R_L \cdot (1 + X)}{\frac{R_L}{R_C} \cdot X - 1} \qquad R_3 := R_1 \qquad R_2 := (X - 1) \cdot \frac{R_3 \cdot R_L}{R_3 + R_L}$$

The pi-topology resistive pad with an insertion loss of $IL = 10.00$ (db) and matched to a TL with characteristic resistance $R_C = 300.00$ (Ω) is then:

$R_1 = 577.485$ (Ω)

$R_2 = 426.907$ (Ω)

$R_3 = 577.485$ (Ω)

This completes the design of the resistive pad.

Performance of this resistive pad design

To appreciate the matching capabilities of this resistive pad, we will now plot the **VSWR** for a TL that is attached to this network as shown in the figure above. We will plot the VSWR as a function of the load resistance value, R_L.

The impedance seen looking into the resistive pad connected to the load resistance is:

$$Z_{in}(R_{load}) := \frac{\left(\frac{R_{load} \cdot R_3}{R_{load} + R_3} + R_2\right) \cdot R_1}{\frac{R_{load} \cdot R_3}{R_{load} + R_3} + R_2 + R_1}$$

The reflection coefficient at the end of the TL can be computed using Equation (66) in Chap. 7 of the text:

$$\Gamma_L(R_{load}) := \frac{Z_{in}(R_{load}) - R_C}{Z_{in}(R_{load}) + R_C}$$

The VSWR for this TL is given from Equation (105) in Chap. 7 as:

$$\mathrm{VSWR}(R_{load}) := \mathrm{if}\left(\left|\Gamma_L(R_{load})\right| = 1, \infty, \frac{1+\left|\Gamma_L(R_{load})\right|}{1-\left|\Gamma_L(R_{load})\right|}\right)$$

We will also plot the VSWR on this TL if there is no resistive pad attached. Using (105) again, this VSWR is given as:

$$\mathrm{VSWR}_{wopad}(R_{Load}) := \frac{1+\left|\frac{R_{Load}-R_C}{R_{Load}+R_C}\right|}{1-\left|\frac{R_{Load}-R_C}{R_{Load}+R_C}\right|}$$

Now for the plot of the VSWR on the TL, choose the number of points at which to plot the VSWR and range of load resistances:

$npts := 80$ Number of points to plot VSWR.

$Rload_{start} := \frac{R_C}{5}$ $Rload_{end} := 2 \cdot R_C$ R_L starting and ending values (Ω).

Construct a list of R_L values at which to plot the VSWR:

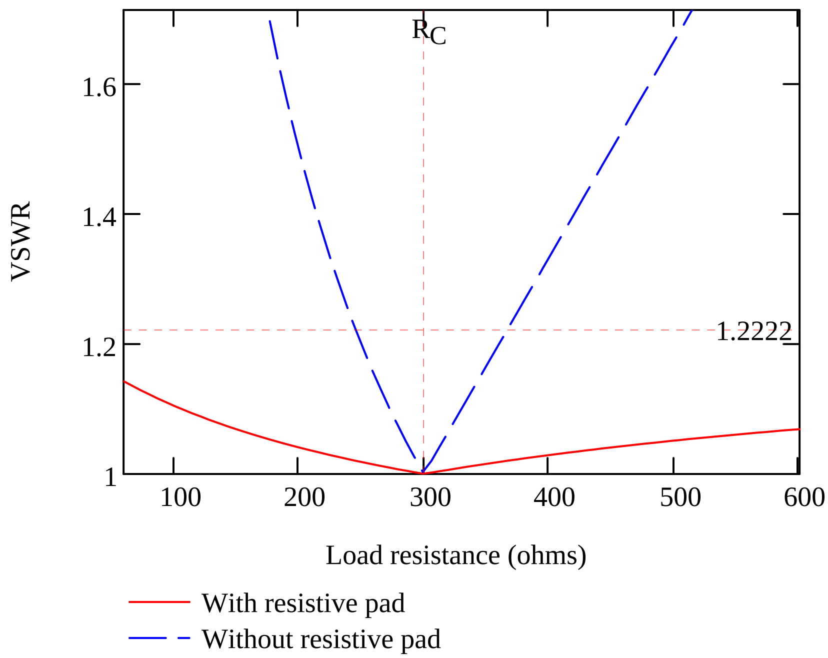
$i := 0 .. npts - 1$ $R_{load_i} := Rload_{start} + i \cdot \frac{Rload_{end} - Rload_{start}}{npts - 1}$

Compute the VSWR at the list of load resistance values both with and without the resistive pad attached to the load:

$vswr_{wpad_i} := \mathrm{VSWR}(R_{load_i})$ $vswr_{wopad_i} := \mathrm{VSWR}_{wopad}(R_{load_i})$

Plot the two VSWR curves:

VSWR of TL and variable load resistance.



For a TL with
$R_C = 300.0$ (Ω).

As we can observe in this plot, when $R_L = R_C$ there is no need for a matching network since the load is already matched to the TL. Consequently, the VSWR is unity when $R_L = R_C$ for the TL without the resistive pad attached (which is indicated by the vertical dashed line in the above plot). But at this value of load resistance ($R_L = R_C$) we can observe that with the resistive pad attached, there is also a perfect match since the VSWR = 1. In other words, this matching network has not introduced additional *mismatch* when $R_L = R_C$. This behavior was implemented into the resistive pad design as discussed near the beginning of this worksheet.

For other values of the load resistance, we can see in this plot that the resistive pad can significantly reduce the VSWR on the TL over a large range of load resistances. The VSWR = 1.2222 line shown in this plot is for a reflection coefficient, Γ, equal to -20 db – which represents 1% relative reflected power on a lossless TL.

Additional experimentation

You may wish to experiment with this worksheet and observe what happens to the bandwidth of load resistances that yield less than 1% relative reflected power if the insertion loss of the resistive pad is increased. In this case, what is the relative change in the power consumed by the pad? What happens to the bandwidth if the insertion loss is decreased? You may also wish to confirm the resistive pad designs shown in Fig. C.23a and in Example C.9.

End of worksheet.

