

Lecture 21: Lossy TLs. Dispersionless TLs. Special Cases for General TLs.

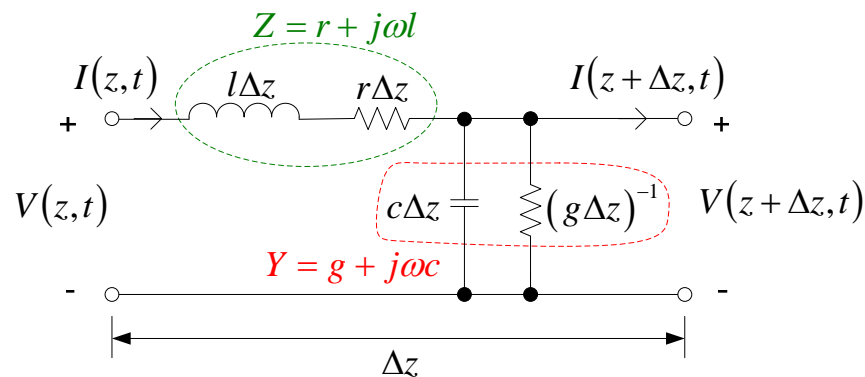
Real transmission lines – such as coaxial cables – have **losses** that will, among other effects, **attenuate the signal** as it propagates along the TL.

As we've learned, there are **two types of current** (conduction and displacement) and both of these are supported on a transmission line and are necessary for its operation.

There are loss mechanisms associated with each type of current:

- ✓ R is due to the **conductor losses** in the metal parts of the TL.
- ✓ G is due to the **losses in the dielectric** material surrounding the conductors in the TL.

The equivalent circuit for an infinitesimally short section (Δz) of such a lossy TL is:



Following a procedure very similar to that for lossless TLs, we can derive the phasor domain form of the telegrapher's equations for lossy TLs as

$$\frac{dV(z)}{dz} = -ZI(z) \quad \text{or} \quad \frac{dV(z)}{dz} = -(r + j\omega l)I(z) \quad (1)$$

$$\frac{dI(z)}{dz} = -YV(z) \quad \text{or} \quad \frac{dI(z)}{dz} = -(g + j\omega c)V(z) \quad (2)$$

The phasor form of the wave equations for $V(z)$ and $I(z)$ can be derived from these telegrapher's equations quite easily. For example, taking the derivative of (1)

$$\frac{d^2V(z)}{dz^2} + Z \frac{dI(z)}{dz} = 0$$

Substituting (2) into this equation gives the phasor-domain form of the wave equation for $V(z)$ to be

$$\frac{d^2V(z)}{dz^2} - ZYV(z) = 0 \quad (3)$$

The general solution for $V(z)$ in (3) is

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \quad (4)$$

where γ is the **propagation constant** defined as

$$\gamma = \sqrt{ZY} = \sqrt{(r + j\omega l)(g + j\omega c)} \quad [\text{m}^{-1}] \quad (5)$$

We see from this definition of γ that it is a complex quantity, so we'll define its real and imaginary parts as

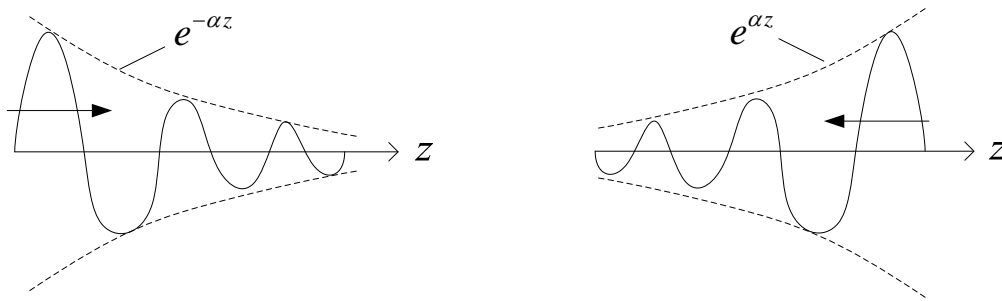
$$\gamma \equiv \alpha + j\beta \quad [\text{m}^{-1}] \quad (6)$$

where α is called the **attenuation constant** [Np/m] and β is the **phase constant** [rad/m].

Consequently, with (6) substituted in (4)

$$V(z) = V_o^+ e^{-\alpha z} e^{-j\beta z} + V_o^- e^{+\alpha z} e^{+j\beta z} \quad (7)$$

We can surmise that the **voltage signal will attenuate** as it propagates along the TL:



As shown in the text:

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{r + j\omega l}{g + j\omega c}} = R_0 + jX_0 \text{ } [\Omega] \quad (8)$$

Z_0 is the same **characteristic impedance** concept for the TL we've been using for lossless TLs. However, for a **lossy** TL we see from (8) that Z_0 is a **complex** number.

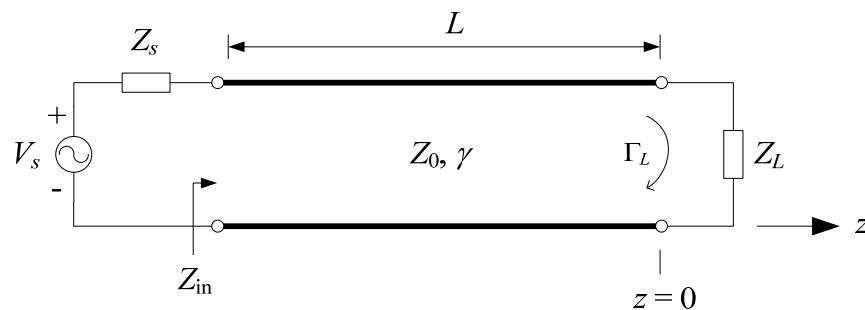
Lastly, the solution for the current waves on a lossy TL is

$$I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{+\gamma z} \quad (9)$$

The current waves attenuate as they propagate, just as the voltage waves.

$\Gamma(z)$ and Input Impedance for Lossy TLs

The generalized reflection coefficient and input impedance have proven to be useful and important concepts for transmission lines. The expressions for these two items are different for lossy transmission lines compared to lossless ones.



From (7),

$$V(z) = V_o^+ e^{-\alpha z} e^{-j\beta z} \left(1 + \frac{V_o^- e^{\alpha z} e^{j\beta z}}{V_o^+ e^{-\alpha z} e^{-j\beta z}} \right) = V_o^+ e^{-\alpha z} e^{-j\beta z} [1 + \Gamma(z)] \quad (10)$$

where

$$\Gamma(z) = \Gamma_L e^{2\alpha z} e^{j2\beta z} \quad (11)$$

is the generalized reflection coefficient for lossy TLs.

The input impedance for this lossy transmission is

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma L)}{Z_0 + Z_L \tanh(\gamma L)} \quad [\Omega] \quad (12)$$

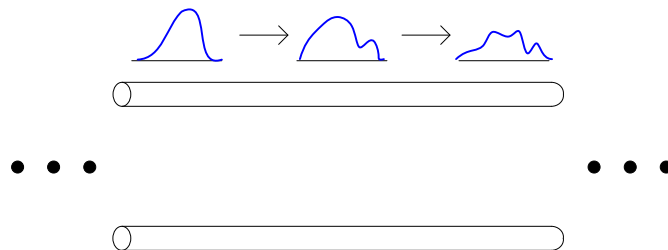
which can be derived in a fashion similar to what was done for lossless TLs in Lecture 19.

Signal Dispersion

Another non-ideal characteristic of general (lossy) TLs is **signal dispersion**. This occurs when the signal velocity is a function of frequency.

With $u = \omega / \beta$ and from (6) and (5), we can surmise that β is not simply $\omega \sqrt{lc}$. Consequently, **u is a function of frequency**.

This can be a very undesirable effect since the different frequency components of a signal will **propagate at different speeds**. This can lead to **distortion** of the signal, which gets worse the further the signal travels along the TL.



Dispersionless Transmission Lines

Oliver Heaviside (in the late 1800s) discovered the amazing fact that it is possible to design a lossy TL so that it presents **no**

signal dispersion! For this to happen, he found the PUL parameters of the TL must satisfy

$$\boxed{\frac{r}{l} = \frac{g}{c}} \quad (13)$$

When this condition is met, then α and u are not functions of frequency!

To verify this, note from (5) with $r/l = g/c$ that:

$$\begin{aligned} \gamma &= \sqrt{(r + j\omega l)(g + j\omega c)} = \sqrt{j\omega l \left(1 + \frac{r}{j\omega l}\right) j\omega c \left(1 + \frac{g}{j\omega c}\right)} \\ &= j\omega \sqrt{lc} \left(1 + \frac{r}{j\omega l}\right) = \underbrace{\sqrt{lc} \frac{r}{l}}_{\alpha} + j \underbrace{\omega \sqrt{lc}}_{\beta} \end{aligned} \quad (14)$$

Now, using this result from (14) compute the wave speed, α , and Z_0 :

$$\checkmark u = \frac{\omega}{\beta} = \frac{\omega}{\omega \sqrt{lc}} = \frac{1}{\sqrt{lc}}.$$

This is **not a function of frequency**. It's actually the same result as for lossless TLs!

✓ From (14), $\alpha = r \sqrt{\frac{c}{l}}$. Using $\frac{c}{l} = \frac{g}{r}$, then

$$\alpha = r \sqrt{\frac{g}{r}} = \sqrt{rg} \quad (15)$$

We see that α is **not a function of frequency**. That's a great result. While there is definitely some attenuation of the signal as it propagates ($\alpha \neq 0$), this attenuation is not frequency dependent. Consequently, we could position **linear amplifiers** along the channel if needed.

✓ As shown in the text, $Z_0 = \sqrt{\frac{l}{c}}$. (16)

This is the same result as for a lossless TL. It's also not a function of frequency.

Special Cases for General TLs

There are three special cases of general, lossy TLs that are worth describing in some detail. These are low loss, large reactance, and large resistance approximations for TLs.

1. **Low-Loss Approximation**. (Ref.: Paul, Whites, and Nasar, *Introduction to Electromagnetic Fields*.) In this case, it is assumed that $r \ll \omega l$ and $g \ll \omega c$ so that from (5)

$$\begin{aligned} \gamma &= \sqrt{(r + j\omega l)(g + j\omega c)} = j\omega\sqrt{lc} \left(1 + \frac{r}{j\omega l}\right)^{1/2} \left(1 + \frac{g}{j\omega c}\right)^{1/2} \\ &\approx j\omega\sqrt{lc} \left(1 + \frac{r}{2j\omega l}\right) \left(1 + \frac{g}{2j\omega c}\right) \end{aligned}$$

using the binomial expansion $\sqrt{1-z} \approx 1-z/2$. This can be simplified to

$$\gamma \approx j\omega\sqrt{lc} \left[1 + \frac{1}{2j\omega} \left(\frac{r}{l} + \frac{g}{c} \right) \right]$$

From this expression we can identify for the low-loss TL that

$$\alpha \approx \frac{r}{2} \sqrt{\frac{c}{l}} + \frac{g}{2} \sqrt{\frac{l}{c}} \text{ [Np/m]} \text{ and } \beta \approx \omega\sqrt{lc} \text{ [rad/m]} \quad (17)$$

Consequently
$$u = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{lc}} \text{ [m/s]}$$

Notice that both α and u are independent of frequency!

Next, from (8)

$$\begin{aligned} Z_0 &= \sqrt{\frac{r + j\omega l}{g + j\omega c}} = \sqrt{\frac{l}{c}} \left(1 + \frac{r}{j\omega l} \right)^{1/2} \left(1 + \frac{g}{j\omega c} \right)^{-1/2} \\ &\approx \sqrt{\frac{l}{c}} \left(1 + \frac{r}{2j\omega l} \right) \left(1 - \frac{g}{2j\omega c} \right) \\ &\approx \sqrt{\frac{l}{c}} \left[1 + \frac{1}{2j\omega} \left(\frac{r}{l} - \frac{g}{c} \right) \right] \end{aligned}$$

Furthermore, assuming that

$$\omega \gg \frac{1}{2} \left(\frac{r}{l} - \frac{g}{c} \right)$$

then this last expression can be further simplified to

$$Z_0 \approx \sqrt{\frac{l}{c}} \text{ } [\Omega] \quad (18)$$

which is the same result as for a lossless TL. Consequently, we expect the characteristic impedance of a low-loss TL to have a small and negligible imaginary component and real part equal to (18).

2. Large Reactance Approximation. In this case $\omega l \gg r$ and $g \approx 0$. From (8),

$$Z_0 \approx \sqrt{\frac{r + j\omega l}{j\omega c}} \approx \sqrt{\frac{l}{c}}$$

while from (5),

$$\begin{aligned} \gamma &\approx \sqrt{(r + j\omega l) j\omega c} = \sqrt{j\omega rc - \omega^2 lc} \\ &= j\omega\sqrt{lc} \sqrt{1 - \frac{j\omega rc}{\omega^2 lc}} = j\omega\sqrt{lc} \sqrt{1 - j \frac{r}{\omega l}} \end{aligned} \quad (19)$$

Using $\sqrt{1 - z} \approx 1 - z/2$ when $|z| \ll 1$ then

$$\begin{aligned} \gamma &\approx j\omega\sqrt{lc} \left(1 - \frac{jr}{2\omega l} \right) = j\omega\sqrt{lc} + \frac{\omega r \sqrt{lc}}{2\omega l} \\ &= j \underbrace{\omega\sqrt{lc}}_{\beta} + \frac{r}{2} \underbrace{\sqrt{\frac{c}{l}}}_{\alpha} \end{aligned} \quad (20)$$

Consequently, in the large reactance limit

$$\alpha = \frac{r}{2Z_0} \text{ [Np/m]} \quad \text{and} \quad u = \frac{\omega}{\beta} = \frac{1}{\sqrt{lc}} \text{ [m/s]} \quad (21)$$

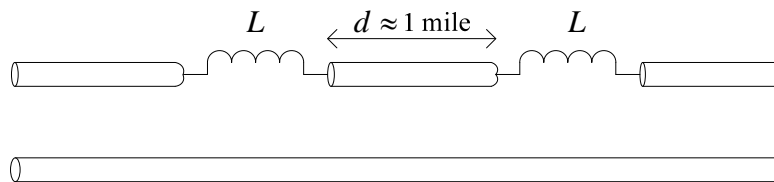
Once again, both are independent of frequency but under the large reactance approximation this might not be a low loss TL.

Rather, recall Heaviside's dispersionless line conditions from (10)

$$\frac{r}{l} = \frac{g}{c}$$

When g is small, it is difficult to make r/l small to satisfy this condition. However, with the large reactance line we have achieved nearly the same condition. Neat!

For a practical example, telephone companies achieve nearly this large reactance state by adding lumped inductor coils every mile or so:



Say $d = 1 \text{ mile} = 1,609 \text{ m} = 0.1 \lambda$. Then this implies that $0.1 \lambda = u/f = 0.67 c_0$. Therefore, $f_{\max} \approx 0.1 \cdot 0.67 \cdot 3 \times 10^8 = 20 \text{ MHz}$. Consequently, 1 mile is small with respect to wavelength for frequencies up to approximately 20 MHz.

3. Large Resistance Approximation. On the other extreme is when $r \gg \omega l$ and $g \approx 0$. In this case from (5),

$$\begin{aligned}\gamma &\approx \sqrt{(r + j\omega l) j\omega c} = \sqrt{j\omega rc - \omega^2 lc} = \sqrt{j\omega rc} \sqrt{1 - \frac{\omega^2 lc}{j\omega rc}} \\ &\approx \sqrt{j\omega rc} \sqrt{1 + j \frac{\omega l}{r}}\end{aligned}\quad (22)$$

Using that fact that

$$\sqrt{j} = \sqrt{e^{j\frac{\pi}{2}}} = e^{j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

then

$$\gamma \approx \left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \right) \sqrt{\omega rc} \quad (23)$$

From this expression, we can identify

$$\alpha \approx \sqrt{\frac{\omega rc}{2}} \text{ [Np/m]} \quad \text{and} \quad \beta \approx \sqrt{\frac{\omega rc}{2}} = \alpha \text{ [rad/m]} \quad (24)$$

Both α and u are **functions of frequency** (actually \sqrt{f}) and the TL is **highly dispersive**. This is **not a good communications “channel”**.

The first transatlantic cable was, unfortunately, an example of such a “high resistance” line and was found to be useless for communications. There is an interesting description of this on pp. 82-83 in your EE 322 text *The Electronics of Radio* by D. Rutledge.