

Lecture 2: Faraday's Law of Induction. Lenz's Law.

Last semester in EE 381 *Electric and Magnetic Fields*, you saw that in

- Electrostatics: stationary charges produce \bar{E} (and \bar{D})
- Magnetostatics: steady currents (charges in constant motion) produce \bar{B} (and \bar{H}).

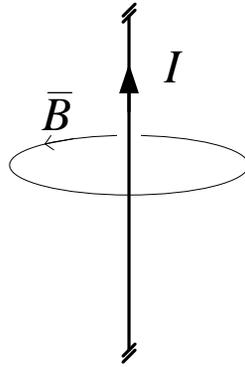
These are **two distinct theories** that were developed from two different experimentally derived laws: Coulomb's law and Ampère's force law.

Now we are going to consider **time-varying** sources (charges and currents) and their associated electric and magnetic fields. While many of the concepts we've learned in statics will still apply, two new phenomenon we will observe are:

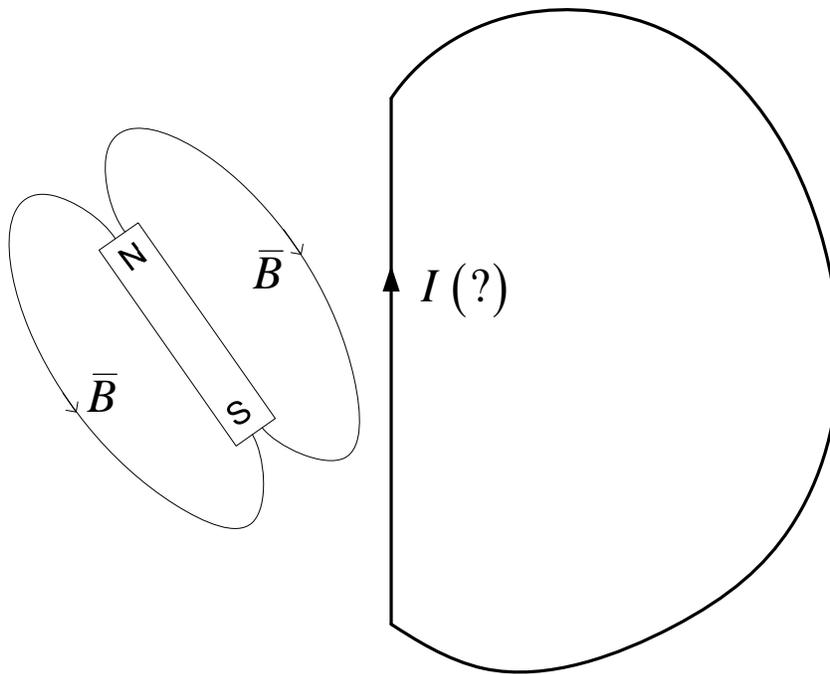
- Time varying \bar{B} produces \bar{E} , and
- Time varying \bar{E} produces \bar{B} !!

The complete electro-magnetic theory uses Coulomb's and Ampère's laws as a subset and requires one more experimentally derived law called **Faraday's law of induction**.

We'll introduce this law with the following thought experiment. You've seen in EE 381 that a steady current in a wire produces a \bar{B} :

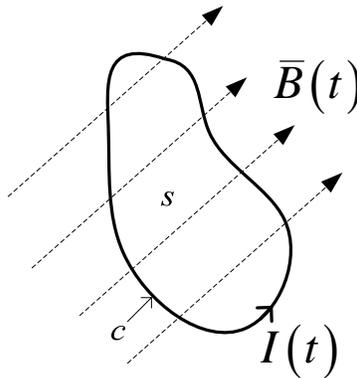


It may seem possible (by some type of “reciprocity”) that if we had a wire and a magnet, for example, that a current would be “**induced**” in the wire:



This doesn't occur, however. If it did there would be a clear violation of conservation of energy.

What Faraday (ca. 1831) and Henry showed was that a **time-varying magnetic flux** would produce (or “induce”) a current I in a closed loop!



Mathematically, **Faraday's law** states that

$$emf = -\frac{d\psi_m}{dt} \quad [\text{V}] \quad (1)$$

where

- $emf \equiv \oint_{c(s)} \vec{E} \cdot d\vec{l}$ is the net “push” causing charges to move.
- $\psi_m = \int_{s(c)} \vec{B} \cdot d\vec{s}$ is the magnetic flux through the surface s .

In words, Faraday's law (1) states that the emf generated in a closed loop is equal to the negative time rate of change of the magnetic flux “linking” the loop.

Substituting for the definitions of emf and ψ_m (1) yields an equivalent form of **Faraday's law of induction**

$$\oint_{c(s)} \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_{s(c)} \bar{B} \cdot d\bar{s} \quad (2)$$

Point Form of Faraday's Law

By applying Stokes' theorem to (2), as done in the text in Section 9.3.A, we can derive the point form of Faraday's law.

Specifically, applying Stokes' theorem to the left-hand side of (2) gives

$$\oint_{c(s)} \bar{E} \cdot d\bar{l} = \int_{s(c)} (\nabla \times \bar{E}) \cdot d\bar{s}$$

Substituting this result into (2) and combining terms gives

$$\int_{s(c)} \left(\nabla \times \bar{E} + \frac{\partial \bar{B}}{\partial t} \right) \cdot d\bar{s} = 0 \quad (3)$$

Since this result is valid for all s and c , then the integrand must vanish, leaving

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (4)$$

This is called the **point form of Faraday's law**.

Lenz's Law

Why the minus sign in Faraday's law? [For example, equations (1) and (2)?] Because of **Lenz's law**.

Lenz's law states that the \bar{B} produced by an induced current (we'll call this \bar{B}_{ind}) will be such that $\psi_{\text{ind}} (= \int_s \bar{B}_{\text{ind}} \cdot d\bar{s})$ **opposes the change** in the $\psi_m (= \int_s \bar{B} \cdot d\bar{s})$ that produced the induced current.

If this weren't the case, the \bar{B} field would **grow indefinitely** large even for the smallest induced current! Consider:

$$\begin{array}{ccc}
 \psi_m & \xrightarrow{\text{induces}} & I \xrightarrow{\text{produces}} \bar{B}_{\text{ind}} \\
 & \Downarrow & \\
 \psi_m + \psi_{\text{ind}} & \xrightarrow{\text{induces}} & I' \xrightarrow{\text{produces}} \bar{B}_{\text{ind}}' \\
 & \Downarrow & \\
 \psi_m + \psi_{\text{ind}} + \psi_{\text{ind}}' & \xrightarrow{\text{induces}} & I'' \xrightarrow{\text{produces}} \bar{B}_{\text{ind}}'' \\
 & & \text{etc.}
 \end{array}$$

We can see here that the total \bar{B} (the sum $\bar{B}_{\text{ind}} + \bar{B}_{\text{ind}}' + \bar{B}_{\text{ind}}'' + \dots$ in the right-hand side) is increasing without bound if the induced \bar{B} enhances the original \bar{B} .