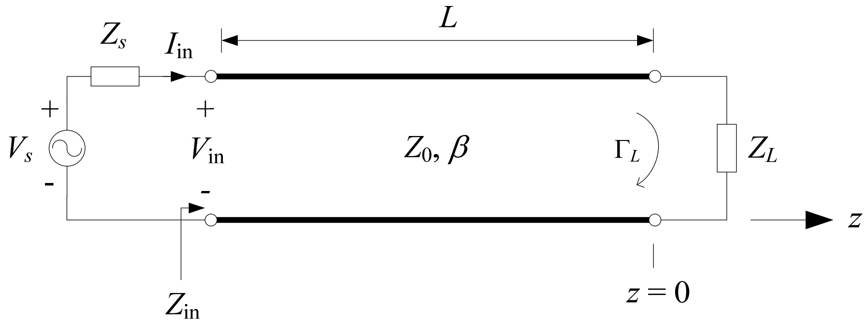
Lecture 19: Input Impedance of TLs. Excitation and the Source Conditions.

Keeping with standard circuits concepts, we can define the TL impedance at any position *z* as simply the ratio

$$Z(z) \equiv \frac{V(z)}{I(z)} = \frac{\text{total voltage at } z}{\text{total current at } z} \tag{1}$$

The total voltage and current are themselves the sum of "+" and "-" propagating waves so this ratio, in general, is not equal to Z_0.

It is helpful to have an analytical expression for the input impedance of an arbitrarily terminated TL.



Towards this objective, we saw in the previous two lectures that the voltage and current everywhere on a homogeneous TL are

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z} \tag{2}$$

and

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{+j\beta z} \tag{3}$$

We can readily construct, according to (1), an input impedance expression for a TL of length *L* by dividing (2) and (3) at *z* = -*L* for some arbitrary load reflection coefficient Γ_L at *z* = 0:

$$V(-L) = V_o^+\left(e^{+j\beta L} + \frac{V_o^-}{V_o^+}e^{-j\beta L}\right) = V_o^+\left(e^{+j\beta L} + \Gamma_L e^{-j\beta L}\right) \quad (4)$$

$$I(-L) = \frac{V_o^+}{Z_0}\left(e^{+j\beta L} - \frac{V_o^-}{V_o^+}e^{-j\beta L}\right) = \frac{V_o^+}{Z_0}\left(e^{+j\beta L} - \Gamma_L e^{-j\beta L}\right) \quad (5)$$

such that

$$Z_{\text{in}} \equiv \frac{V_{\text{in}}}{I_{\text{in}}} = \frac{V(-L)}{I(-L)} = \frac{V_o^+\left(e^{+j\beta L} + \Gamma_L e^{-j\beta L}\right)}{\frac{V_o^+}{Z_0}\left(e^{+j\beta L} - \Gamma_L e^{-j\beta L}\right)} = Z_0\frac{1+\Gamma_L e^{-j2\beta L}}{1-\Gamma_L e^{-j2\beta L}}$$

Substituting for Γ_L and simplifying gives

$$Z_{\text{in}} = Z_0\frac{Z_L + jZ_0\tan(\beta L)}{Z_0 + jZ_L\tan(\beta L)} \quad [\Omega] \quad (6)$$

This is the input impedance for a lossless TL of length L and characteristic impedance Z_0 with an arbitrary load Z_L.

It is easy to see from (6) that one function of a TL can be as an impedance transformer.

Input Impedance for Special Loads

Three special cases for the input impedance in (6) are:

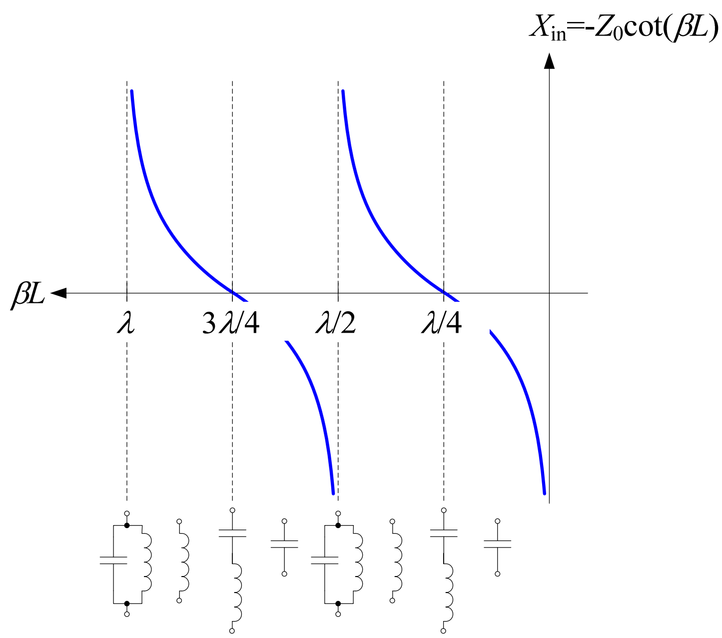
1. With an open circuit load ($Z_L = \infty$), (6) yields

$$Z_{\text{in}} = -jZ_0 \cot(\beta L) \quad [\Omega] \tag{7}$$

In other words, the input impedance is purely reactive

$$Z_{\text{in}} = jX_{\text{in}} \quad \text{where} \quad X_{\text{in}} = -Z_0 \cot(\beta L) \tag{8}$$

A plot of this input reactance is:



Notice that any value of inductive or capacitive reactance can be achieved at the input of the TL. All we need to do is adjust the electrical length (βL) of the TL to change this value of input reactance. The electrical length can be changed either by varying the signal frequency or the physical length of the TL.

This is an example of the impedance transformation capability of TLs.

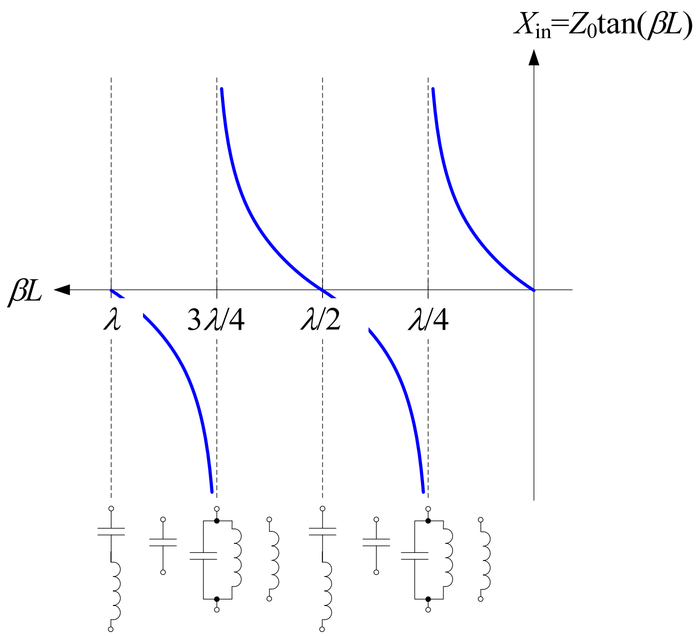
2. With a short circuit load ($Z_L = 0$), (6) yields

$$Z_{\text{in}} = jZ_0 \tan(\beta L) \quad [\Omega] \tag{9}$$

This input impedance is also purely reactive

$$Z_{\text{in}} = jX_{\text{in}} \quad \text{where} \quad X_{\text{in}} = Z_0 \tan(\beta L) \tag{10}$$

A plot of this input reactance is:



Again, we see here (as we did earlier with the open circuit load) that any value of reactive impedance can be realized at the input to the TL simply by adjusting the electrical length of the TL.

3. With the resistive load $Z_L = Z_0$, (6) yields

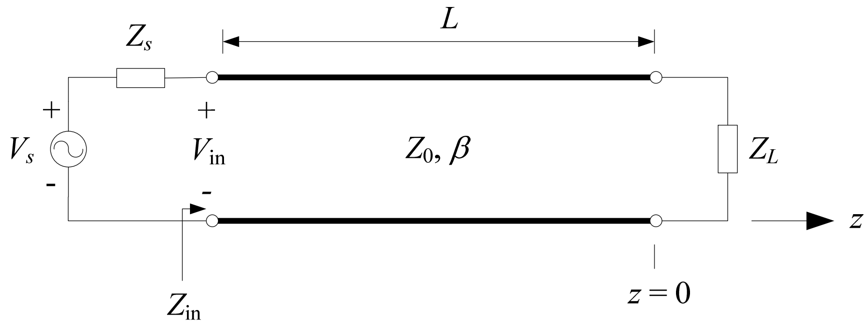
$$Z_{\text{in}} = Z_0 \quad [\Omega] \tag{11}$$

The input impedance is Z_0 regardless of the length of the TL.

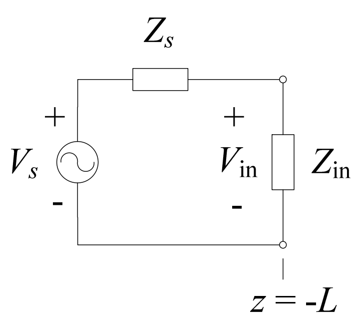
Again, note that both input impedances (7) and (9) are purely reactive, which is expected since neither type can dissipate energy, assuming lossless TLs.

Excitation of Transmission Lines

For the sinusoidal excitation of TLs,



the total voltage at $z = -L$ can be found using the input impedance of the TL and this equivalent circuit at the input:

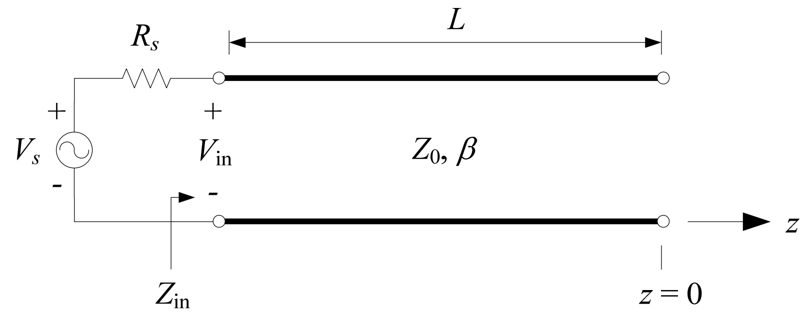


By voltage division, we can compute V_{in} from this equivalent circuit at the TL input as

$$V_{\text{in}} = \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_s} V_s \qquad (12)$$

This process is illustrated in the following example.

Example N18.1: Determine an expression for the voltage at the input to the following TL (open circuit load) assuming $R_s = Z_0$:

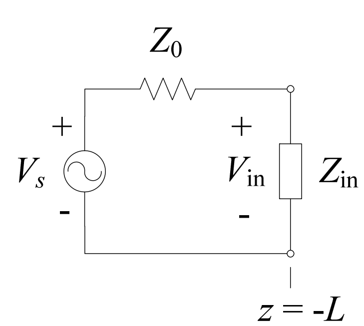


To calculate the input voltage V_{in}, we'll first determine the input impedance Z_{in}. To repeat, this is the effective impedance seen at the TL input terminals $z = -L$ seen looking towards the load.

For an open circuit load, we have from (7) that

$$Z_{\text{in}} = \frac{V(-L)}{I(-L)} = -jZ_0 \cot(\beta L) \text{ } [\Omega]$$

An equivalent circuit can now be constructed at the input to the TL by using $R_s \left(= Z_0\right)$ and Z_{in} as



From voltage division,

$$V_{\text{in}} = \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_0} V_s = \frac{-jZ_0 \cot(\beta L)}{-jZ_0 \cot(\beta L) + Z_0} V_s \tag{13}$$

which is the desired quantity.

This circuit voltage V_{in} is **also** the voltage on the TL at $z = -L$. That is, from (4) above with $\Gamma_L = +1$

$$V(z = -L) = 2V_o^+ \cos(\beta L) \tag{14}$$

Since $V_{\text{in}} = V(z = -L)$, we can equate these two voltages (13) and (14) giving

$$2V_o^+ \cos(\beta L) = \frac{-jZ_0 \cot(\beta L)}{-jZ_0 \cot(\beta L) + Z_0} V_s$$

More often than not, expressions of this type are used to determine V_o^+ in terms of V_s and R_s. Once V_o^+ is known, the voltage and current anywhere on the TL can be calculated since from (2) and (3)

$$V(z) = V_o^+ \left(e^{-j\beta z} + \Gamma_L e^{+j\beta z} \right) \tag{15}$$

and

$$I(z) = \frac{V_o^+}{Z_0} \left(e^{-j\beta z} - \Gamma_L e^{+j\beta z} \right) \tag{16}$$