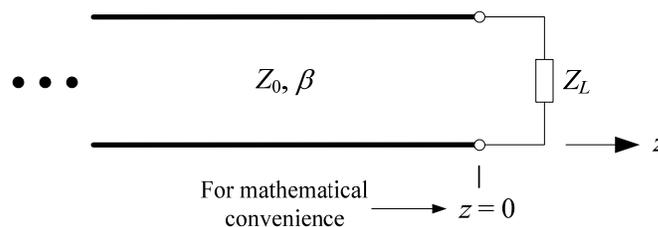


Lecture 18: Termination of TLs. Load Reflection Coefficient.

In the last lecture, we considered for the first time TLs in the sinusoidal steady state. We will now consider the **termination** of TLs that are excited by sinusoidal steady state sources.



Your text places $z = 0$ at the source end of the TL. Here, we're placing $z = 0$ at the load for convenience. While some intermediate formulas will be different by doing this, the end results will be the same.

Adding terminations produces **reflections** so that the total voltage and current anywhere on the TL are **sums of forward and reverse propagating waves**. From results in the previous lecture, the voltage and current on the TL will have the form

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z} \quad (1)$$

and

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{+j\beta z} \quad (2)$$

The “lumped load” Z_L that terminates the TL is considered a **boundary condition** for the voltage and current in (1) and (2):

$$V(z=0) = I(z=0)Z_L \quad (3)$$

Therefore, we can solve for V_o^- in terms of V_o^+ by applying this boundary condition as:

- from (1): $V(z=0) = V_o^+ + V_o^-$
- from (2): $I(z=0) = \frac{1}{Z_0}(V_o^+ - V_o^-)$

Forming the ratio of these quantities gives

$$\frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_0 = Z_L$$

As with time domain TL analysis, we will solve for V_o^-/V_o^+ , and define this ratio as the **voltage reflection coefficient at the load** ($z=0$), we find

$$\Gamma_L \equiv \left. \frac{V_o^-}{V_o^+} \right|_{z=0} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (4)$$

This expression for Γ_L is very similar to what we derived for Γ_L and time varying TL analysis. Here, however, Γ_L is a complex number, as is Z_L in general. (Is Γ_L a phasor?)

Example N18.1: For an open circuit load on the TL shown above, compute the load reflection coefficient and sketch the voltage and current magnitude on the TL.

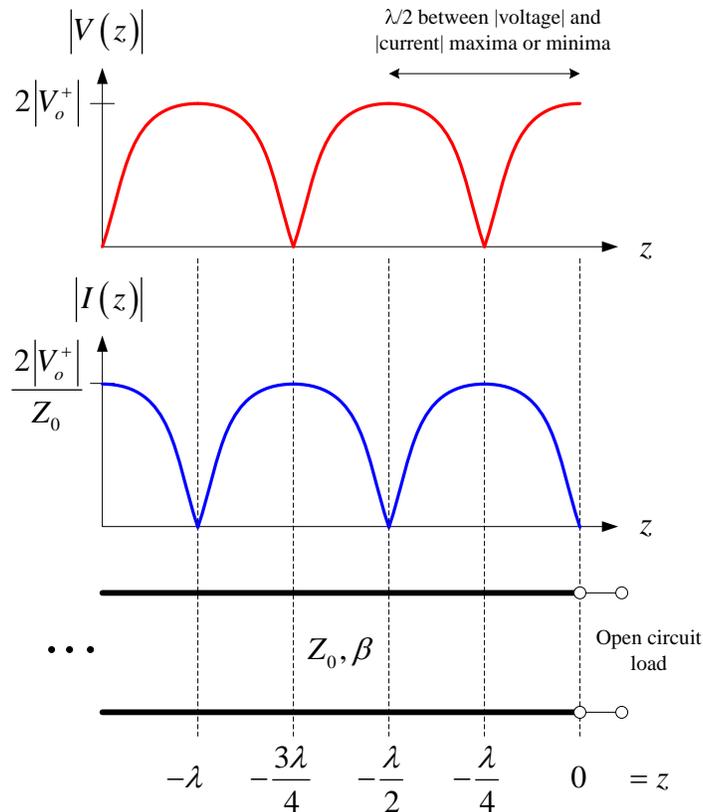
For an open circuit load $Z_L = \infty$ (i.e., an extremely large impedance), so that from (4) $\Gamma_L = +1$. With this value of Γ_L , then from (1) and (2) the solutions for $V(z)$ and $I(z)$ are

$$V(z) = V_o^+ \left(e^{-j\beta z} + \underbrace{\frac{V_o^-}{V_o^+}}_{=\Gamma_L} e^{j\beta z} \right) = 2V_o^+ \cos(\beta z) \quad (5)$$

$$\text{and } I(z) = \frac{V_o^+}{Z_0} \left(e^{-j\beta z} - \underbrace{\frac{V_o^-}{V_o^+}}_{=\Gamma_L} e^{j\beta z} \right) = -\frac{j2V_o^+}{Z_0} \sin(\beta z) \quad (6)$$

These two equations (5) and (6) are **not traveling waves**. So, where has the traveling wave behavior in $V(z)$ and $I(z)$ gone? **The interference between the incident and reflected waves produces standing waves**, such as these.

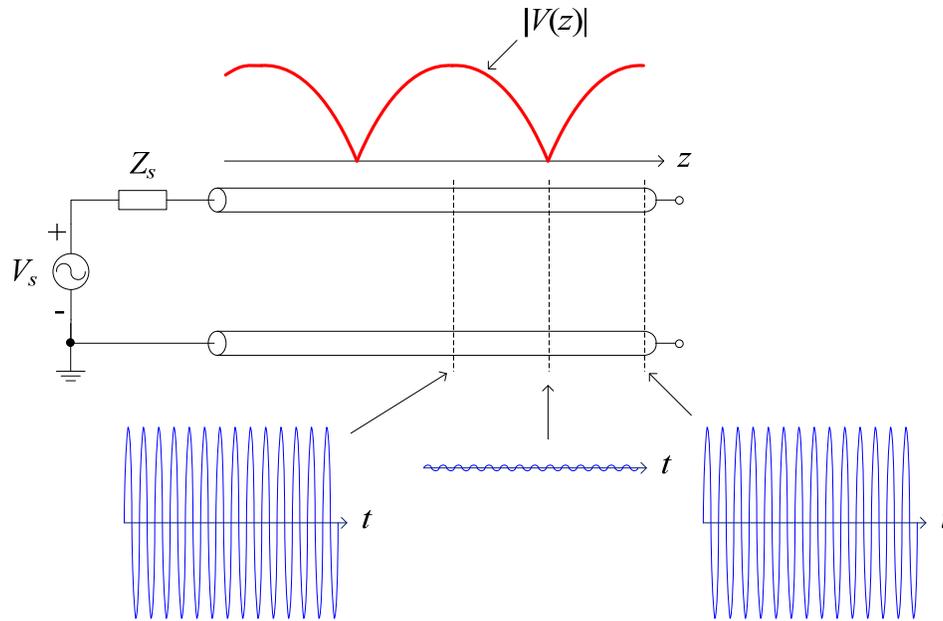
$|V(z)|$ and $|I(z)|$ are shown here for the open circuit load:



(Fig. 1)

These phasor voltage and current magnitudes vary noticeably along TLs provided the TL length is greater than about 0.05λ or so.

Remember, though, that we are plotting the **magnitude** of phasor voltages and currents. The voltage and current **oscillate** as functions of time with amplitudes equal to $|V(z)|$ and $|I(z)|$, respectively, at each point along the TL:



Notice that the “distance” in Fig. 1 is measured in wavelengths. This is common in EM work. The quantity “ βz ” in (5) and (6) is called the **electrical distance**. (It’s not really distance since its units are radians.)

One method for determining β of TLs in the lab is to connect a short circuit load and probe for the **voltage nulls**. Defining the distance between adjacent nulls as d then

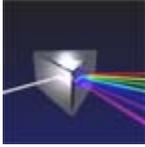
$$d = \frac{\lambda}{2} \quad \text{so that} \quad \lambda = 2d$$

But, from (17) in the previous lecture

$$\beta = \frac{2\pi}{\lambda}$$

Therefore,

$$\beta = \frac{\pi}{d} \quad [\text{rad/s}] \quad (7)$$



Example 7.5



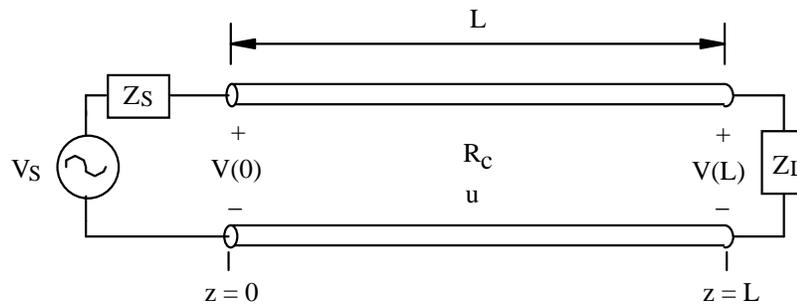
Sinusoidal Voltage and Current Waves on a Lossless Transmission Line

Purpose

To compute and visualize the voltage and current on a lossless transmission line (TL) that is driven by a sinusoidal voltage source. Once the functions for the voltage and current have been obtained, phasor-domain plots of the voltage and current are generated. These are followed by animated plots of the voltage on the TL as well as the forward and reverse propagating voltage waves. The concept of a standing wave is also discussed.

Enter the voltage source and transmission line parameters

A pictorial representation of the lossless transmission line (TL) is shown in Fig. 7.21a in the text and in the figure below:



Choose the parameters for the TL including its length, characteristic resistance and its propagation velocity:

$L := 10$ Length of the TL (m).

$R_C := 50$ TL characteristic resistance (Ω).

$u := 200 \cdot 10^6$ Propagation velocity (m/s).

Now choose the parameters for the source and the load:

$V_S := 100 \cdot \exp(j \cdot 0 \cdot \text{deg})$ Phasor open-circuit source voltage (V).

$f := 26 \cdot 10^6$ Source frequency (Hz).

$Z_S := 50 + j \cdot 0$ Source impedance (Ω).

$Z_L := 100 + j \cdot 50$ Load impedance (Ω).

Compute the radian frequency, ω , the period, T_p , and phase constant, β , of the voltage and current waves on this TL:

$$\omega := 2 \cdot \pi \cdot f$$

$$T_p := \frac{1}{f} \quad T_p = 3.846 \times 10^{-8} \quad (\text{s})$$

$$\beta := \frac{\omega}{u} \quad \beta = 0.817 \quad (\text{rad/m})$$

Determine functions for the voltage and current everywhere on the TL

The first step in this analysis of this TL is to develop the expression for the voltage and current everywhere on the TL. Using Equation (66) in Chap. 7 of the text, the voltage reflection coefficient at the load is:

$$\Gamma_L := \frac{Z_L - R_C}{Z_L + R_C} \quad \Gamma_L = 0.4000 + 0.2000j$$

The (generalized) voltage reflection coefficient is given in (68) of Chap. 7:

$$\Gamma(z) := \Gamma_L \cdot \exp[j \cdot 2 \cdot \beta \cdot (z - L)]$$

from which we can compute the reflection coefficient at the input to the TL as:

$$\Gamma(0) = -0.4412 + 0.0733j$$

Using $\Gamma(0)$, the input impedance looking into the TL at $z = 0$ is then from (62) of Chap. 7:

$$Z_{in} := R_C \cdot \frac{1 + \Gamma(0)}{1 - \Gamma(0)} \quad Z_{in} = 19.209 + 3.521j \quad (\Omega)$$

In this worksheet, we wish to determine the voltage and the current everywhere on the TL. The expressions for the voltage and current are given in (60) of Chap. 7 of the text:

$$V(V_{m_plus}, z) := V_{m_plus} \cdot \exp(-j \cdot \beta \cdot z) \cdot (1 + \Gamma(z))$$

$$I(V_{m_plus}, z) := \frac{V_{m_plus}}{R_C} \cdot \exp(-j \cdot \beta \cdot z) \cdot (1 - \Gamma(z))$$

What remains to be determined is the constant V_{m_plus} . As discussed in Example 7.5, this constant can be found by applying the boundary conditions for the voltage at the input to the transmission line. The result of this analysis is that:

$$V_{in} := \frac{Z_{in}}{Z_{in} + Z_S} \cdot V_S \quad V_{in} = 27.942 + 3.666j \quad (\text{V})$$

$$V_{m_plus} := \frac{V_{in}}{1 + \Gamma(0)} \quad V_{m_plus} = 50.000 \quad (\text{V})$$

The highlighted values above can be compared to the corresponding quantities in Example 7.5.

This completes the solution for this sinusoidal excitation of the lossless TL. The voltage and current can now be computed anywhere on the TL from the functions $V(V_{m_plus}, z)$ and $I(V_{m_plus}, z)$, respectively, which are defined above.

Plot the phasor voltage and current

We will now plot the magnitude and phase of the (phasor) voltage and current everywhere on the TL. Choose the number of points at which to plot the voltage and current:

npts := 80 **Number of points to plot in z.**

z_start := 0 **z_end := L** **z starting and ending points (m).**

Construct a list of z_i points at which to plot the voltage and current:

$$i := 0..npts - 1 \quad z_i := z_{start} + i \cdot \frac{z_{end} - z_{start}}{npts - 1}$$

Compute the voltage and current magnitude and phase at every position z_i along the TL:

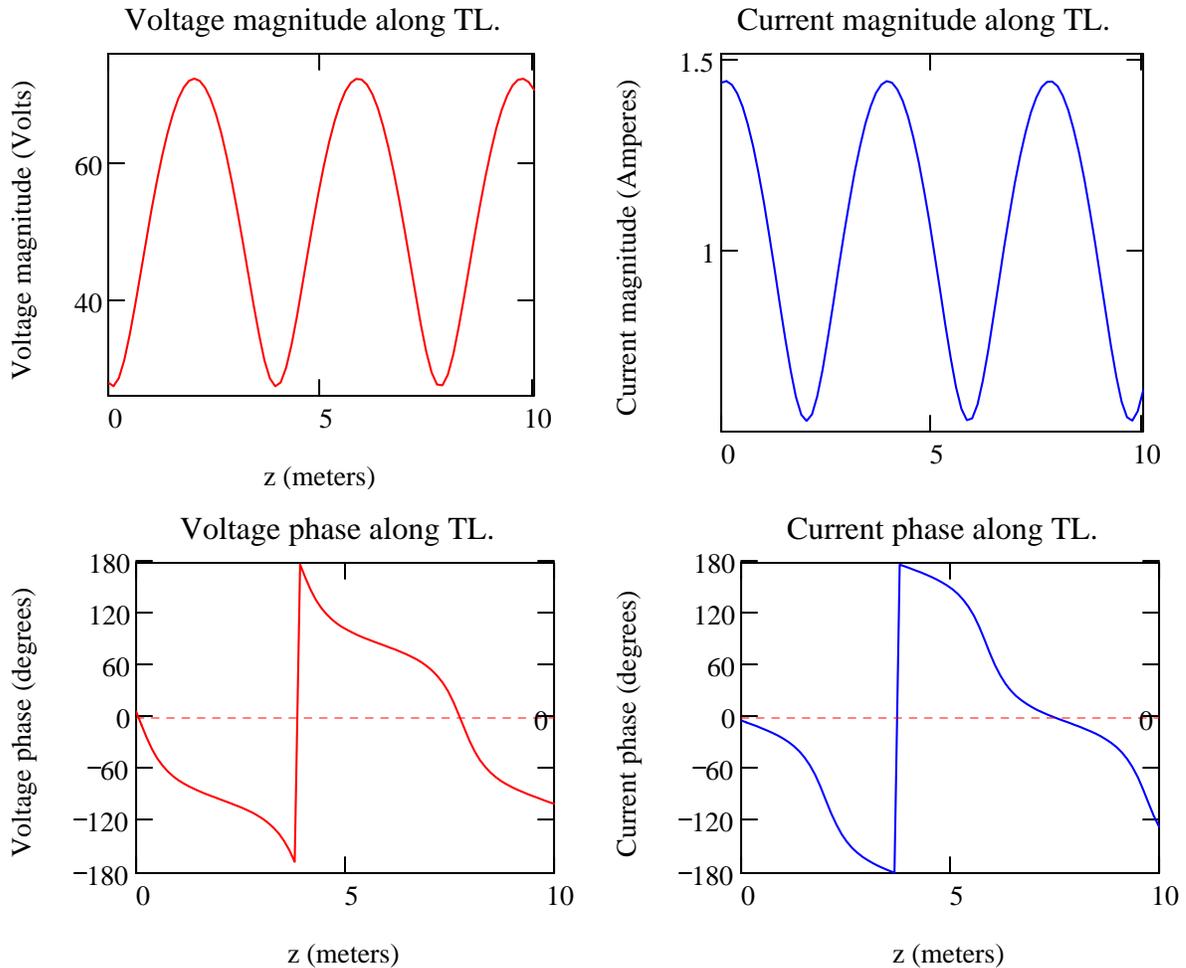
$$\text{Mag}_{V_i} := |V(V_{m_plus}, z_i)| \quad \text{Voltage magnitude (V).}$$

$$\text{Mag}_{I_i} := |I(V_{m_plus}, z_i)| \quad \text{Current magnitude (A).}$$

$$\Theta_{V_i} := \text{if} \left(|V(V_{m_plus}, z_i)| \neq 0, \frac{\arg(V(V_{m_plus}, z_i))}{\text{deg}}, 0 \right) \quad \text{Voltage phase (}^\circ\text{).}$$

$$\Theta_{I_i} := \text{if} \left(|I(V_{m_plus}, z_i)| \neq 0, \frac{\arg(I(V_{m_plus}, z_i))}{\text{deg}}, 0 \right) \quad \text{Current phase (}^\circ\text{).}$$

Now plot the magnitude and phase of the voltage and current along the TL:



The source end of the TL is at the left in all of these plots. It is quite clear from these plots that the voltage and current are both functions of position and vary (periodically) along the TL for the parameters given in Example 7.5.

Check the terminal boundary conditions

We can check that the ratio of the voltage and current at the input to the TL and at the load end satisfy the boundary conditions:

$$Z(z) := \frac{V(V_{m_plus}, z)}{I(V_{m_plus}, z)} \quad \text{Total impedance at position } z \text{ along the TL } (\Omega).$$

At the input to the TL ($z = 0$), $Z(0)$ must equal Z_{in} as computed earlier in this worksheet:

$$Z(0) = 19.209 + 3.521j \quad (\Omega) \quad Z_{in} = 19.209 + 3.521j \quad (\Omega)$$

which it does. At the load end of the TL, $Z(L)$ must equal the load impedance as chosen at the beginning of the worksheet:

$$Z(L) = 100.000 + 50.000j \quad (\Omega) \quad Z_L = 100.000 + 50.000j \quad (\Omega)$$

which it does. These checks give us an increased confidence that the analysis of the TL is correct.

Animated plot of the voltage

With the analysis of the TL voltage and current completed, we will now generate an animation clip of the time and spatial variation of the voltage on the TL for the sinusoidal source voltage.

The time-domain voltage everywhere on the transmission line can be computed from the phasor-domain form as:

$$V_t(V_{m_plus}, z, t) := \text{Re}(V(V_{m_plus}, z) \cdot \exp(j \cdot \omega \cdot t))$$

Choose the number of time periods to plot the voltage:

$$n_{\text{periods}} := 3 \quad \text{Number of time periods to plot.}$$

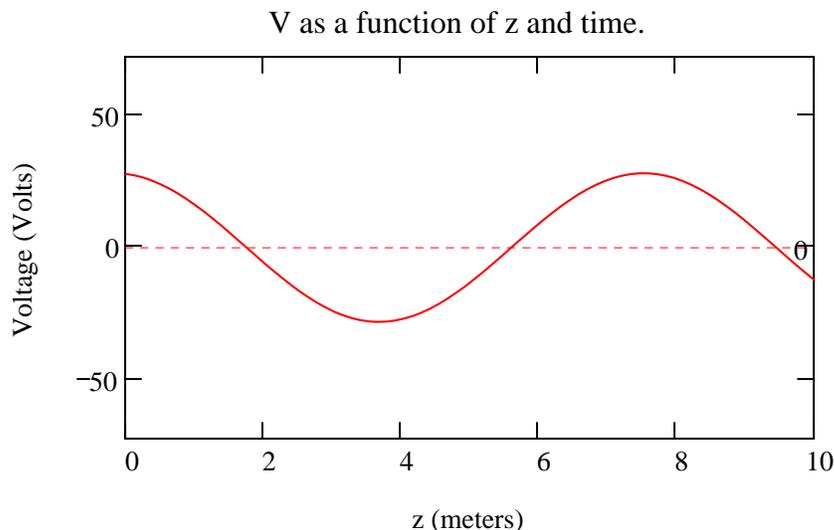
$$n_{\text{pts_per_period}} := 20 \quad \text{Number of points to plot per period.}$$

$$t_{\text{start}} := 0 \quad t_{\text{end}} := n_{\text{periods}} \cdot T_p \quad \text{Time to start and end plot (s).}$$

Define the variable time in terms of the constant FRAME: 

$$t_{\text{inc}} := \frac{T_p}{n_{\text{pts_per_period}}} \quad \text{time} := t_{\text{start}} + \text{FRAME} \cdot t_{\text{inc}}$$

Now generate the animated plot of the voltage on the TL. For best results, in the "Animate" dialog box, choose $T_0 = 60$ 



Time (in periods, T_p)

$$\frac{\text{time}}{T_p} = 0.00$$

This voltage exists on a TL with $L = 10$ (m), $R_C = 50$ (Ω) and $Z_L = 100.0 + 50.0j$ (Ω). The voltage source is located at the left-hand edge and the load at the right-hand edge of this plot.

We can see in the animation that the voltage *amplitude* increases to some value and then decreases to another value as the voltage propagates towards the load. Furthermore, the period of this oscillation is one half that of the voltage source. This behavior of the total voltage on the TL is due to the interference of the forward (+z) and reverse (-z) propagating voltage waves. This "pulsation" of the voltage wave is entirely equivalent to interfering plane waves as illustrated in the **Example 6.8** worksheet and discussed in Section 6.7 of the text. We shall see later in Section 7.3.2 of the text that it is possible to **compute** the *ratio* of these maximum and minimum voltage amplitudes in terms of only the load reflection coefficient. (This ratio is called the voltage standing wave ratio or **VSWR** for short.)

Even though there is interference of the forward and reverse propagating voltage waves, there is still net motion of the wave towards the load as is evident in the animation clip. This is due to the fact that net energy is being delivered to the load and subsequently dissipated as heat in the load impedance.

You may wish to experiment with other load impedances and observe the effect on this voltage wave. Particularly notable cases include: $Z_L = R_C$, $Z_L = 0$ and $Z_L = 10^6$ (an open-circuit load). In the first case, you will observe that the "pulsation" of the wave has disappeared entirely. Can you explain why? In the latter two cases, you will observe that while the voltage wave still "pulsates", there is **no net motion** of the voltage wave to the left or to the right. Does this make sense? Also observe that the minimum voltage *amplitude* is zero for these two cases. This will be of use to you later in Sections 7.3.1 and 7.3.2 of the text.

Animate forward, reverse and total voltage waves

The total voltage everywhere on the TL that we observed in the last animation clip is actually the sum of **forward and reverse** propagating waves. That is, a sum of a voltage wave that is traveling towards the load (the "forward" wave) and another wave that is traveling towards the source (the "reverse" wave). The final plot we will generate in this worksheet is an animation clip illustrating these two component waves as well as their sum.

From (56a) in Chap. 7 of the text, the phasor form of the total voltage at any point on the TL is the sum of the forward and reverse propagating waves as:

$$V(z) = V_m^+ e^{-j\beta z} + V_m^- e^{+j\beta z}$$

We have previously determined the constant V_{m_plus} . To find the constant V_{m_minus} , we apply the boundary condition (64), from Chap. 7 of the text, at the load such that:

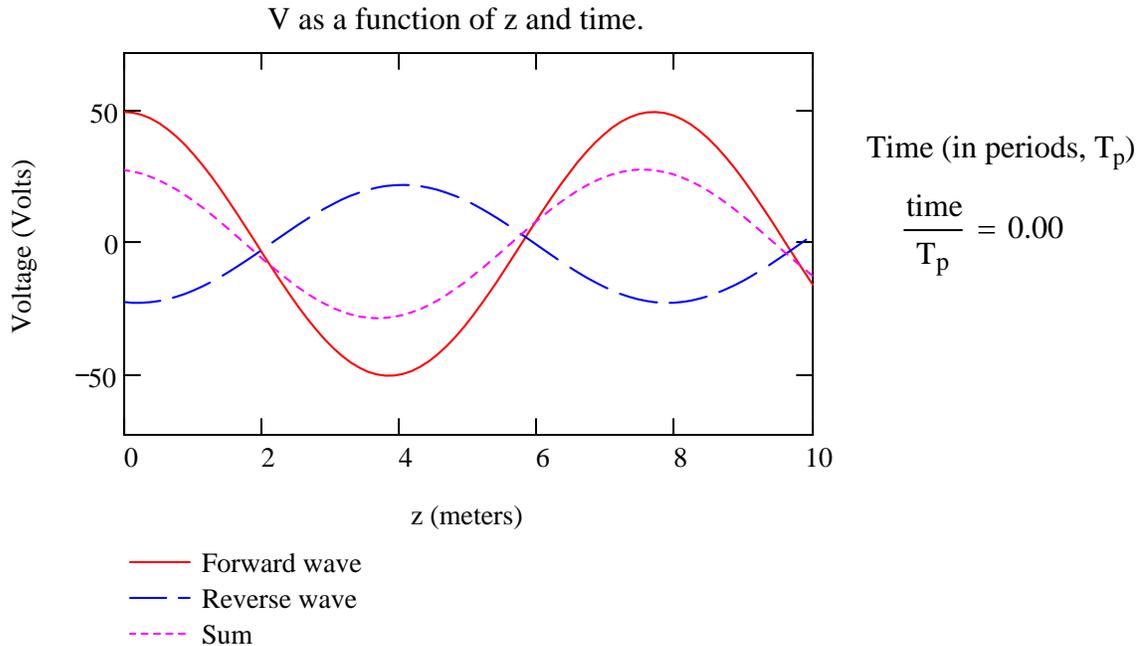
$$V_{m_minus}(V_{m_plus}) := \Gamma_L \cdot V_{m_plus} \cdot \exp(-j \cdot 2 \cdot \beta \cdot L)$$

Therefore, the time-domain forms of the forward and reverse propagating waves are, respectively:

$$V_{t_forward}(V_{m_plus}, z, t) := \text{Re}(V_{m_plus} \cdot \exp(-j \cdot \beta \cdot z) \cdot \exp(j \cdot \omega \cdot t))$$

$$V_{t_reverse}(V_{m_plus}, z, t) := \text{Re}(V_{m_minus}(V_{m_plus}) \cdot \exp(j \cdot \beta \cdot z) \cdot \exp(j \cdot \omega \cdot t))$$

Now generate the animated plot of the voltage on the TL. For best results, in the "Animate" dialog box, choose **To = 60**



This voltage exists on a TL with $L = 10$ (m), $R_C = 50$ (Ω) and $Z_L = 100.0 + 50.0j$ (Ω) where the voltage source is located at the left-hand edge of this plot and the load at the right-hand edge.

This animation clip is particularly informative in that we can see quite clearly that the **solid** line a voltage wave propagating to the right (towards the load) while the **dashed** line is propagating to the left (towards the source). The sum of these two waves is the **dotted** line which is the total voltage everywhere on the TL as a function of time. This sum waveform is identical to the voltage waveform that was animated in the previous section of this worksheet. You can compare these two plots either as separately running animation clips or at the initial time at which the animation begins.

If you carefully observe this animation clip, you can develop a deep appreciation for why the voltage on the TL increases to its maximum amplitude at one time instant and then decreases to its minimum amplitude at another time (with these two times separated by $T_p/2$). The maximum amplitude of the voltage wave occurs when the forward and reverse waves add "in phase" while the minimum amplitude occurs when these two waves add "out of phase". This statement should be very clear when you view the animation clip! Enjoy!

End of worksheet.

