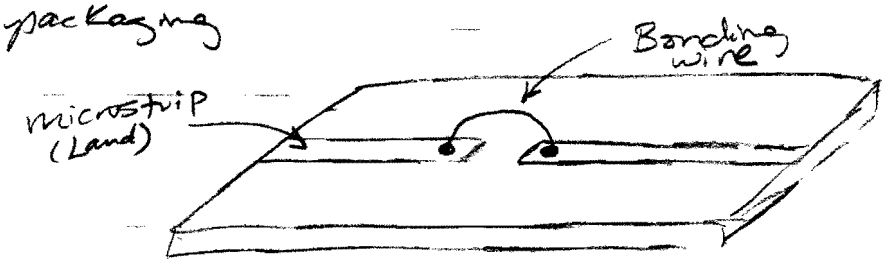
Whites EE 382

Lecture 16 – Transient Response of TL's w/ Reactive Elements. Time Domain Reflectometry (TDR)

Until now, only resistive elements have been considered as loading elements for our time domain analysis of TL's. That is, the effective Thevenin equivalent ckts for the excitation and load of the TL have been purely resistive. No energy storage elements – other than the TL itself – have been incorporated into the analysis.

This is often not the case and these two circuits (source & load) may have reactive components to them. In this lecture we will investigate some simpler examples of reactive loading of TLs in the time domain (transient response). Such reactive loading is quite common:

- → Capacitive input to transistors, gates.
- → Inductive loading due to bond wires in IC packaging

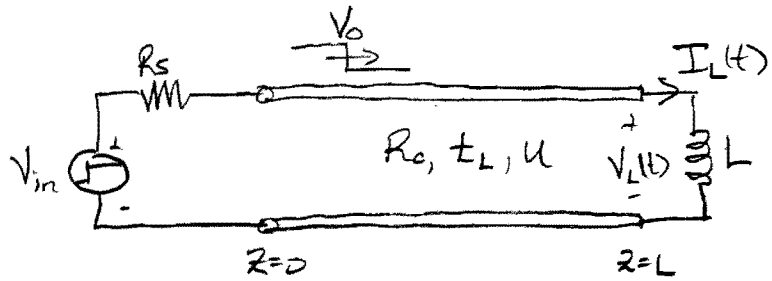


- → High speed bus circuits have capacitive loading by backplanes (plug-in card interface w/ backplane)
- → Parasitic inductance of leads.

We'll first consider a relatively simple example of a TL terminated by an ideal inductor. Then, we'll introduce TDR and some time domain "signatures" of other reactive loading.

Transient Response of TL w/ Inductive Termination

Consider a TL that is excited by a unit step source and terminated with an inductor L:



For resistive terminations, we've learned that the reflected voltage and current waves will have the same temporal shape as the source. That is, the reflected voltage & currents will be unit step changes propagating along the TL.

This is not the case with reactive terminations. The reflected voltage and current from L below will not be a unit step.

When the incident unit step voltage $V_1^+(z,t) = V_0$ (1)

reaches the inductor, L acts as an <u>open ckt load</u> since the <u>current</u> <u>cannot</u> <u>change</u> <u>instantaneously</u> through an inductor. $[I_L(t=t_L) = 0]$.

Thus, this incident unit step voltage is initially reflected by an O.C. load w/ $\Gamma_L = +1$ & $V_L(t=t_d) = 2V_0$.

After this, though, current begins to flow through the inductor, the voltage decreases and the reflection coefficient changes from $\Gamma_L = +1$ exponentially in time down to a value of -1 because eventually the voltage across the inductor becomes ZERO and acts as a <u>short circuit</u>.

To solve for this time evolution of the inductor voltage, we need to simultaneously solve the TL equations; the differential eqn for the inductor load:

$$V_L(t) = L\frac{d I_L(t)}{dt} \qquad (2)$$

The solutions for the voltage and current on the TL are already known to us:

$$V(z,t) = V^+\left(t - \frac{z}{u}\right) + V^-\left(t + \frac{z}{u}\right) \qquad (3)$$

and

$$I(z,t) = \frac{1}{R_c} V^+\left(t - \frac{z}{u}\right) - \frac{1}{R_c} V^-\left(t + \frac{z}{u}\right) \qquad (4)$$

At the load, the TL voltage (3) must equal the inductor voltage in (2). Similarly for the TL current in (4); the inductor current in (2). Sub. (3) & (4) into (2) gives

$$V^+\left(t - \frac{L}{u}\right) + V^-\left(t + \frac{L}{u}\right) = \frac{L}{R_c}\frac{d V^+(t - \frac{L}{u})}{dt} - \frac{L}{R_c}\frac{d V^-(t + \frac{L}{u})}{dt} \qquad (5)$$

or

$$\frac{d V^-(t + \frac{L}{u})}{dt} + \frac{R_c}{L} V^-\left(t + \frac{L}{u}\right) = \frac{d V^+(t - \frac{L}{u})}{dt} - \frac{R_c}{L} V^+\left(t - \frac{L}{u}\right) \qquad (6)$$

This is a 1st order diff. eq in time for V^- since V^+ (the incident voltage) is presumed to be known.

(2) We will make an important assumption here: We'll assume the TL is so long that the reflected wave from the load doesn't have time to reflect off the source and arrive back at the load in the time frame of interest to us.

Under this assumption then, the incident voltage in (6) is constant with time s.t. $V^+(t-\frac{L}{u}) = V_0$ (7)

and $$\frac{dV^+(t-\frac{L}{u})}{dt} = \frac{dV_0}{dt} = 0 \quad (8)$$

giving from (6)

$$\frac{dV^-(t+\frac{L}{u})}{dt} + \frac{R_c}{L} V^-(t+\frac{L}{u}) = -\frac{R_c V_0}{L} \quad (9)$$

The solution is $$V^-(t+\frac{L}{u}) = C e^{-\frac{R_c}{L}t} - V_0 \quad (10)$$

where C is a constant that can be found by applying the boundary condition that the current is zero through the inductor at time $t = t_L$, when the unit step first arrives at the load s.t. $\Gamma_L = +1 \Rightarrow V^-(t_L + \frac{L}{u}) = V^+(t_L - \frac{L}{u}) = V_0$ (11)

Sub into (10): $$V_0 = C e^{-\frac{R_c}{L}t_L} - V_0$$

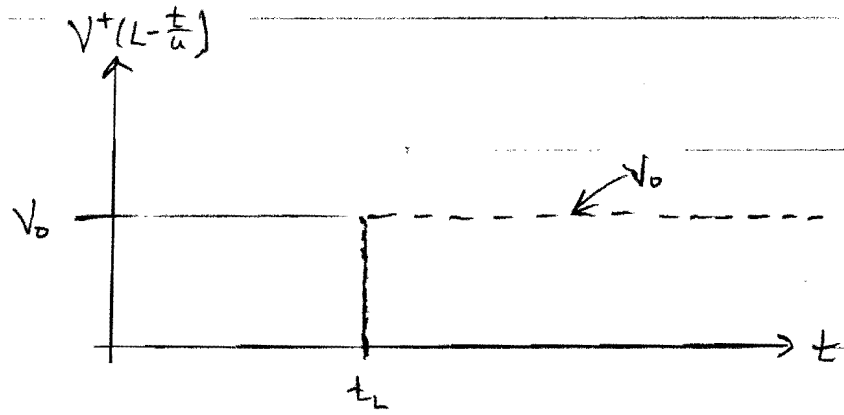
or $$C = 2V_0 e^{+\frac{R_c}{L}t_L} \quad (12)$$

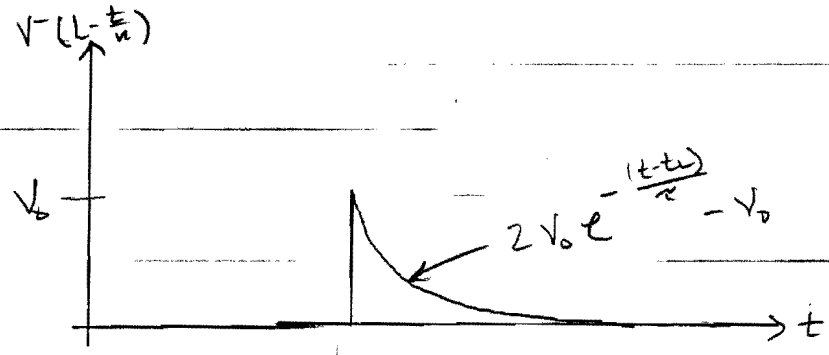
Sub C into (10) gives our final solution:

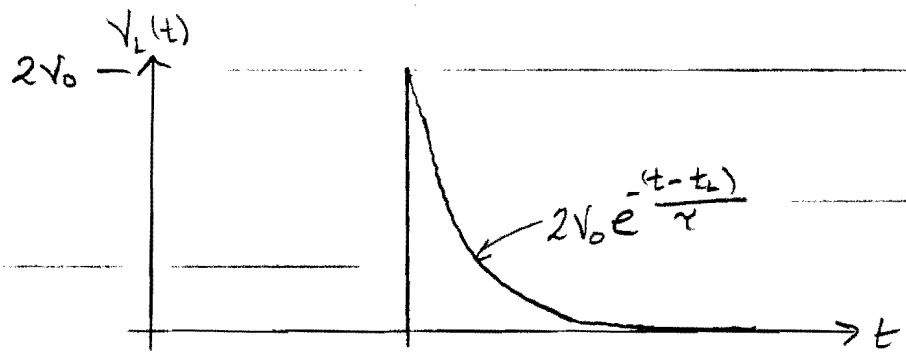
$$V^-(t+\frac{L}{u}) \begin{cases} = 2V_0 e^{-\frac{R_c}{L}(t-t_L)} - V_0 & t \geq t_L \\ = 0 & \text{otherwise} \end{cases} \quad (13)$$

The total voltage across the inductor load is found from (3) at $z = L$ substituting (7) ; (13):

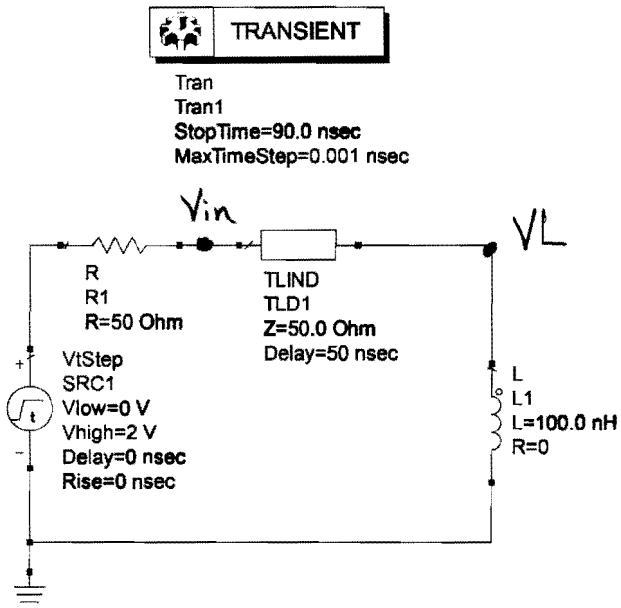
$$V_L(t) = V(L,t) \begin{cases} = 2V_0 e^{-\frac{R_c}{L}(t-t_L)} & t \geq t_L \\ = 0 & \text{Otherwise} \end{cases} \qquad (14)$$

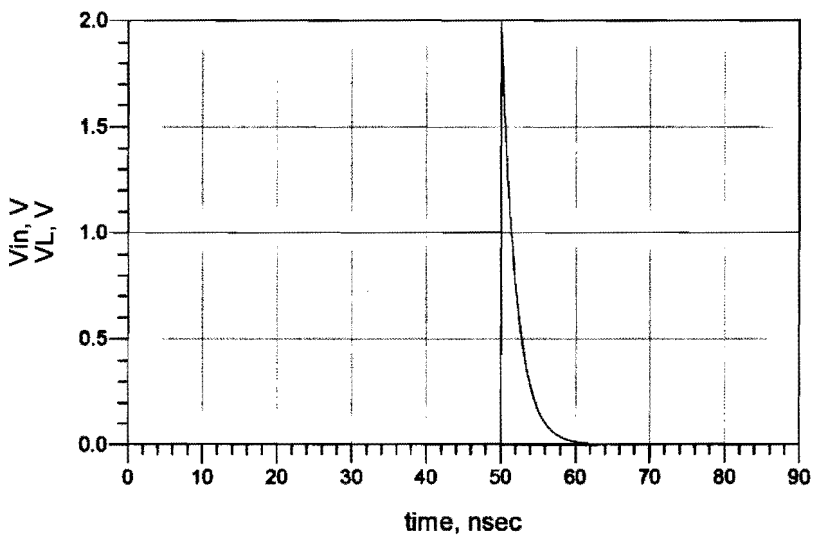






Example N16.1: Using ADS, plot the time domain voltage across a 100 nH inductor loading a TL with 50-Ω characteristic resistance and a one way time delay of 50 ns. The TL is excited by a unit step voltage source of amplitude 2 V that is matched to the TL.





Time Domain Reflectometry (TDR)

Time domain reflectometry is a very common method of using the transient response of a TL (measured at the input) to deduce some unknown characteristics of a TL such as

- The length of the TL
- The location of any breaks in the TL
- Locations of any undesired parasitic capacitances or inductances along the TL or at the load end.
- Determining the inductance of a bonding wire
- Determining the value of a load resistance

etc...

A TDR system is typically comprised of a very fast rise time pulse generator ($\lesssim$ 50 ps), a display oscilloscope, and software that allows the calculation of various quantities of interest. The TDR typically measures & displays the voltage at the input to the TL.

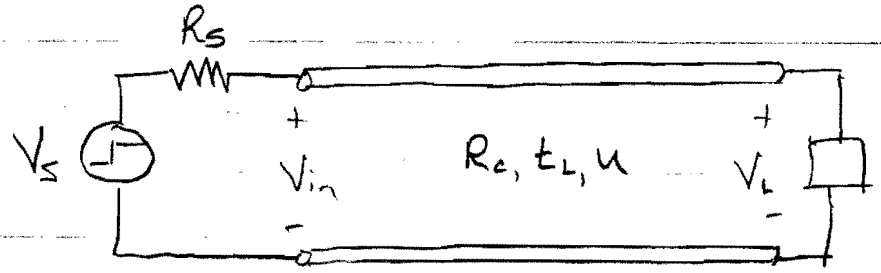


Fig. 1

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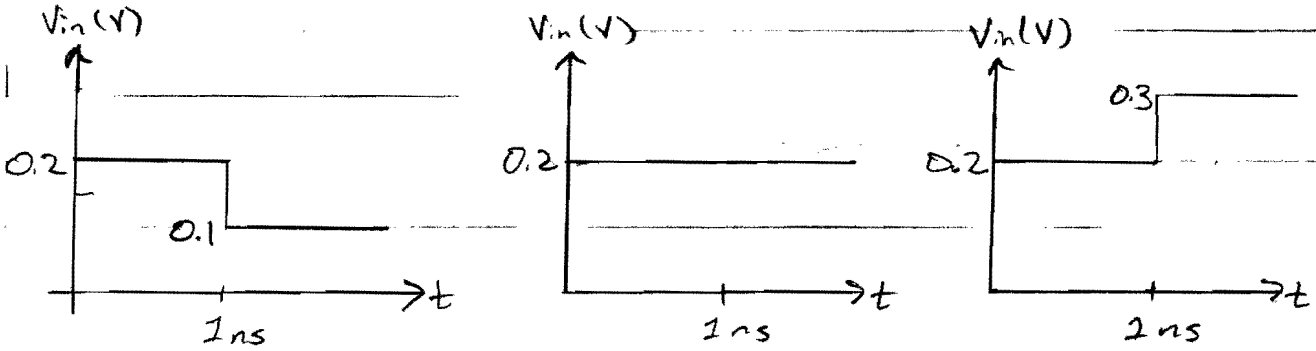
For a resistive load, the TDR can be used to determine the unknown load resistance, as illustrated in the following example.

Example N16.2 A TDR is connected to a long (50-Ω) TL shown in Fig. 1 where the load is a pure resistance. In three different configurations with different loads, the following three input voltages were measured. Determine the load resistances in all three cases.



(a) (b) (c)

Fig. 2

Using the bounce diagram on p.5 of Lecture 14, the input voltage after only 1 returned reflection from the load is

$$V_{in} = V_1^+ + \Gamma_L V_1^+ + \Gamma_S \Gamma_L V_1^+$$

but $\Gamma_S = 0$ $\therefore$ $V_{in} = V_1^+ + \Gamma_L V_1^+$ (15)

We deduce from these plots that $V_1^+ = 0.2\ V$. ($\therefore V_s = 0.4V$)

After 1st reflection, using (15):

(9) $V_{in} = 0.1 = 0.2 + \Gamma_L\, 0.2$

$$\therefore \quad \Gamma_L = \frac{-0.1}{0.2} = -0.5$$

w/ $\Gamma_L = \dfrac{R_L - R_c}{R_L + R_c} \Rightarrow \dfrac{R_c}{R_L} = \dfrac{1-\Gamma_L}{1+\Gamma_L}$

$$\text{or} \quad R_L = R_c \frac{1+\Gamma_L}{1-\Gamma_L} \qquad (16)$$

$$\therefore \quad \underline{\underline{R_L}} = 50 \cdot \frac{1-0.5}{1+0.5} = \frac{50}{3} = \underline{\underline{16.67\ \Omega}}$$

With TDR, this value of the unknown load resistance was easily measured.

(b) After 1st reflection at source, no change in V_{in}.
Using (15) with $V_{in} = 0.2\ V$:

$$0.2 = 0.2 + \Gamma_L \cdot 0.2$$

$$\text{or} \quad \Gamma_L = 0$$

From (16): $\underline{\underline{R_L}} = R_c = \underline{\underline{50\ \Omega}}$ matched load.

(c) After 1st reflection at source, $V_{in} = 0.3\ V$.
Using (15):

$$0.3 = 0.2 + \Gamma_L \cdot 0.2$$

$$\text{or} \quad \Gamma_L = \frac{0.1}{0.2} = \frac{1}{2}$$

From (16): $\underline{\underline{R_L}} = R_c \dfrac{1+\frac{1}{2}}{1-\frac{1}{2}} = 50 \cdot 3 = \underline{\underline{150\ \Omega}}$

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Fig. 3

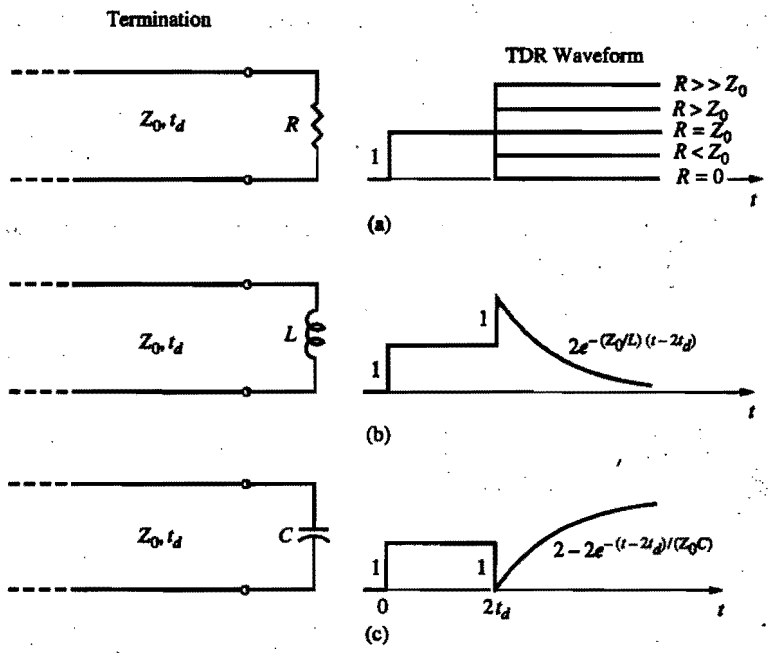

FIGURE 2.32. TDR signatures produced by simple terminations. Source-end TDR voltage signatures for purely (a) resistive, (b) inductive, and (c) capacitive terminations. In terms of excitation by a step voltage source of amplitude V_0 and source resistance R_s, the TDR traces shown are drawn for $V_0 = 2$ V and $R_s = Z_0$. (This figure was adapted from Figure 5 of B. M. Oliver, Time domain reflectometry, *Hewlett-Packard Journal*, 15 (6), pp. 14–9 to 14–16, February 1964. ©Hewlett-Packard Company 1964. Reproduced with permission.)

(U.S. Inan and A. S. Inan, *Engineering Electromagnetics*, Addison Wesley Longman, 1999.)

TDR with Reactive Elements

TDR can also be used to ascertain the presence of parasitic inductances & capacitances if the waveform has shapes that vary from the purely resistive load, as shown in Fig 3.

In the case that the load is purely inductive or purely capacitive, can actually use the TDR signal to determine L or C from the time constant measurement or from the area under the curve; as illustrated in Fig. 4.

TDR can also be used to diagnose lumped loading along a TL, rather than just at load. See Fig. 5.

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Fig. 4

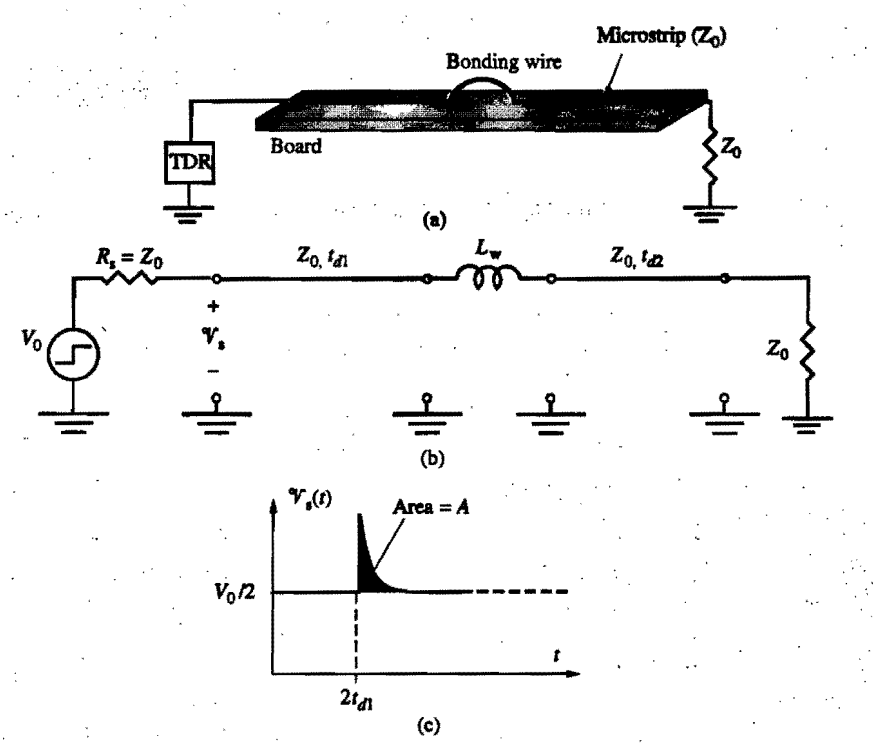

FIGURE 2.33. Measurement of bonding-wire inductance using a TDR system. (a) Actual circuit configuration. (b) Equivalent transmission line circuit model. (c) Measured TDR waveform.

(U.S. Inan and A. S. Inan, *Engineering Electromagnetics*, Addison Wesley Longman, 1999.)

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Fig. 5

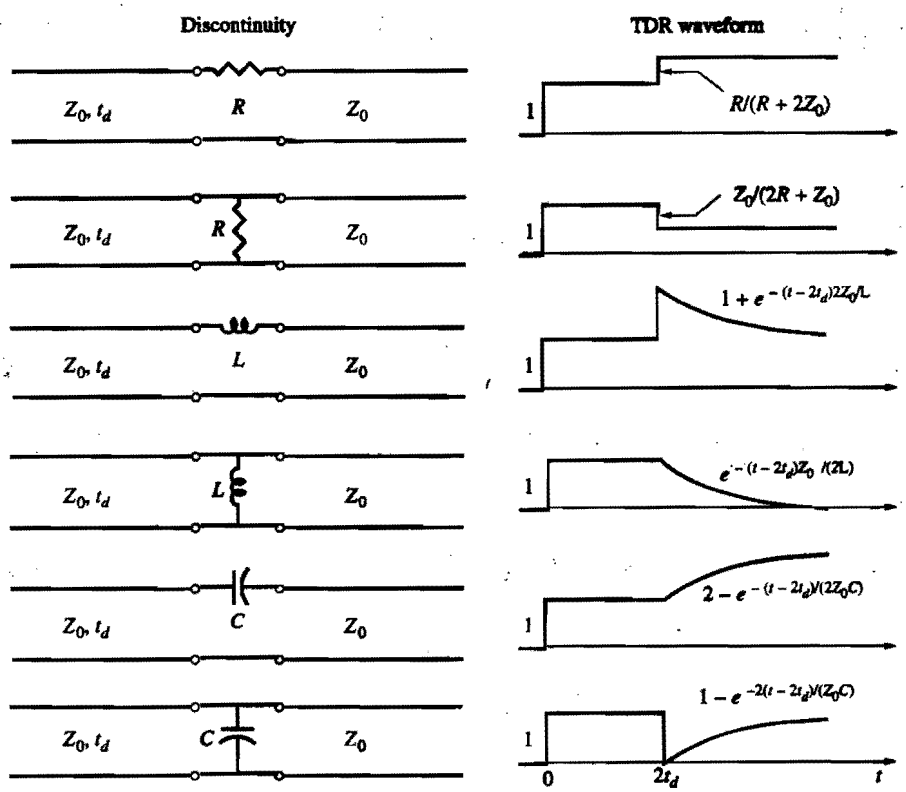

FIGURE 2.34. TDR signatures produced by simple discontinuities. Source-end TDR voltage signatures for shunt or series purely resistive, inductive, and capacitive discontinuities. In terms of excitation by a step voltage source of amplitude V_0 and source resistance R_s, the TDR voltage waveforms shown are drawn for $V_0 = 2$ V and $R_s = Z_0$, and one-way travel time t_d from the source to the disconuity. (This figure was adapted from Figure 6 of B. M. Oliver, Time domain reflectometry, *Hewlett-Packard Journal*, 15(6), pp. 14–9 to 14–16, February, 1964. ©Hewlett-Packard Company 1964. Reprinted with permission.)

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