

Lecture 15: Pulse Propagation on TLs.

The bounce diagram can be used to analyze the transient response of a TL to other types of excitations besides the unit-step, which is all we've considered to this point.

For example, another very useful excitation type is the **pulse input**:

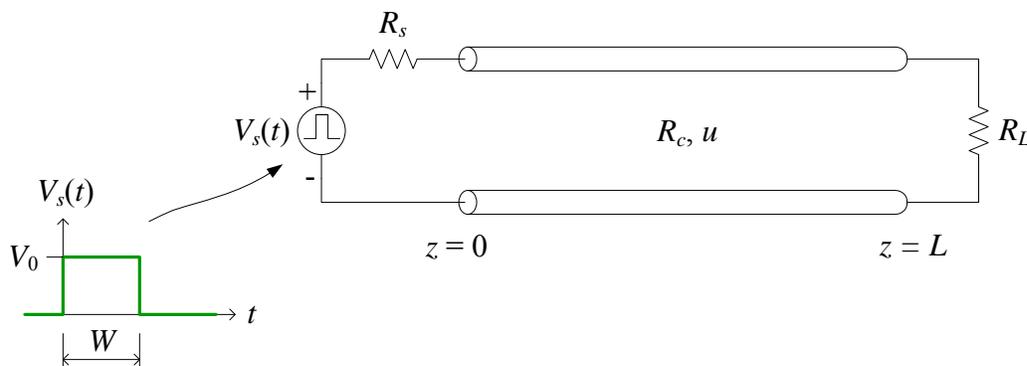
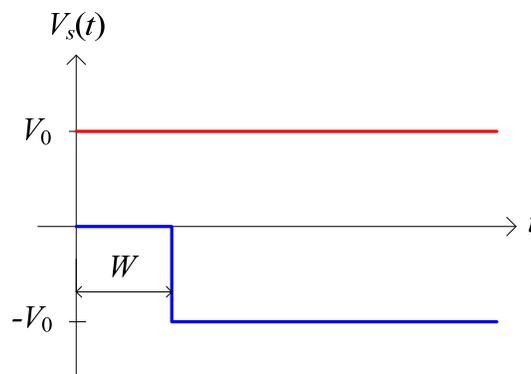


Fig. 1

We can easily use a bounce diagram to solve for the voltage (or current) on the TL if we consider this pulse input to be the **sum of two unit-step voltages**:



The sum of these two unit-step voltages gives the pulse shown in Fig. 1.

Use Bounce Diagram for $V(z,t)$ vs. t at a Fixed z

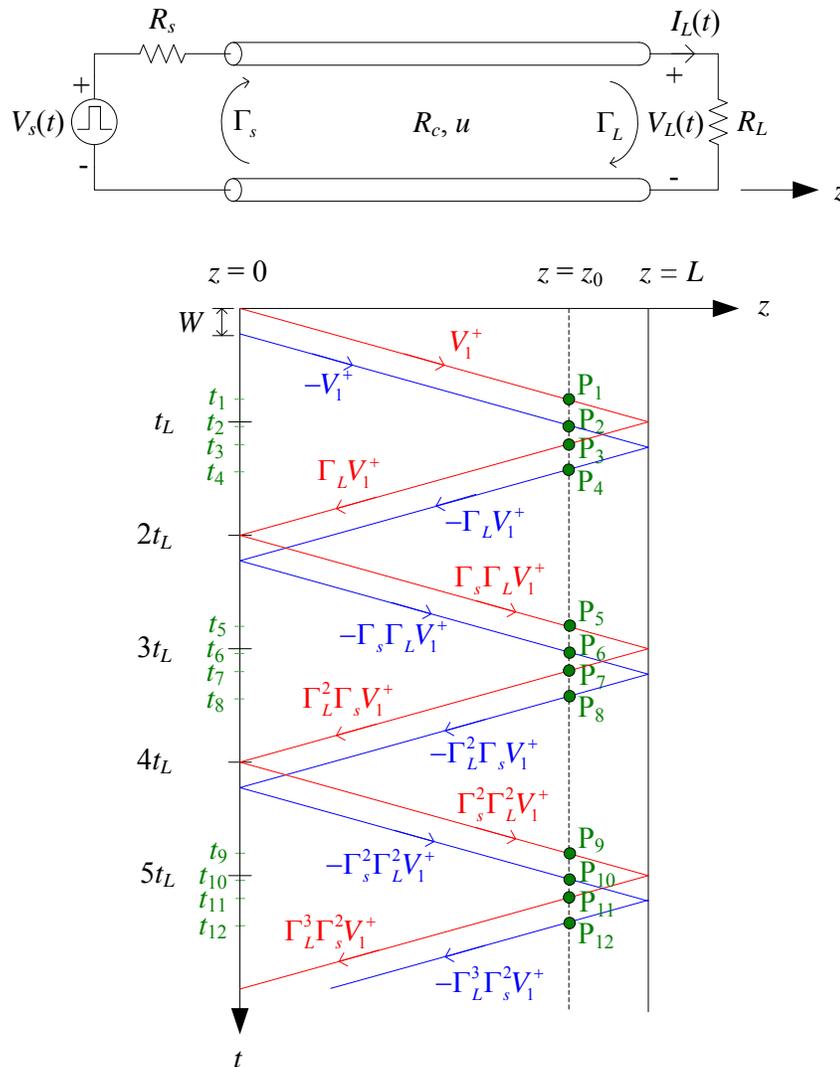


Fig. 2

To use the bounce diagram in this situation:

- (i) **Pick a position z_0** at which to plot $V(z_0, t)$ versus time.

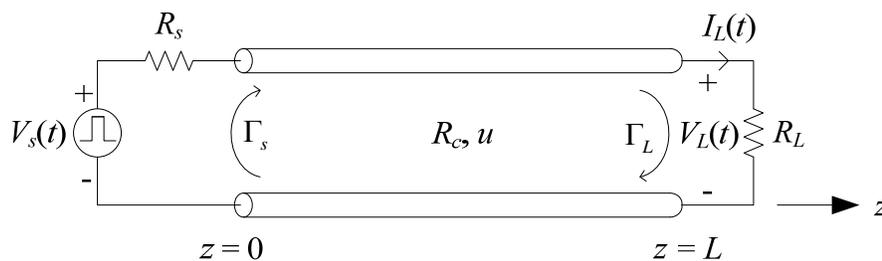
- (ii.) Draw a vertical line at z_0 . It intersects the sloping lines at points P_1 through P_{12} above.
- (iii.) At each of these intersection points, draw horizontal lines and label these times t_1 through t_{12} . These are the times at which new wave fronts arrive and abruptly change the voltage at z_0 .
- (iv.) The voltage at z_0 versus time is then:

<u>Time range</u>	<u>Voltage</u>
$0^+ \rightarrow t_1^-$	0
$t_1^+ \rightarrow t_2^-$	V_1^+
$t_2^+ \rightarrow t_3^-$	$V_1^+ - V_1^+ (= 0)$
$t_3^+ \rightarrow t_4^-$	$V_1^+ - V_1^+ + \Gamma_L V_1^+ (= \Gamma_L V_1^+)$
$t_4^+ \rightarrow t_5^-$	$V_1^+ - V_1^+ + \Gamma_L V_1^+ - \Gamma_L V_1^+ (= 0)$
$t_5^+ \rightarrow t_6^-$	$\Gamma_s \Gamma_L V_1^+$
$t_6^+ \rightarrow t_7^-$	0
$t_7^+ \rightarrow t_8^-$	$\Gamma_L^2 \Gamma_s V_1^+$
$t_8^+ \rightarrow t_9^-$	0
$t_9^+ \rightarrow t_{10}^-$	$\Gamma_s^2 \Gamma_L^2 V_1^+$
$t_{10}^+ \rightarrow t_{11}^-$	0
$t_{11}^+ \rightarrow t_{12}^-$	$\Gamma_L^3 \Gamma_s^2 V_1^+$
etc...	

Depending on the width of the input pulse, these voltages will be different, as we'll see in the next example.

Example N15.1: Imagine that a high speed digital logic gate (modeled by a pulse voltage source of amplitude 1 V, pulse width of 200 ps, and output resistance 900 Ω) drives a load of 25 Ω through a 100- Ω microstrip line ($u = 200$ m/ μ s) that is 8 cm long. Sketch the voltage at the load $V(L, t)$ from 0 to 2.2 ns.

We can model this interconnection problem using a transmission line as shown:



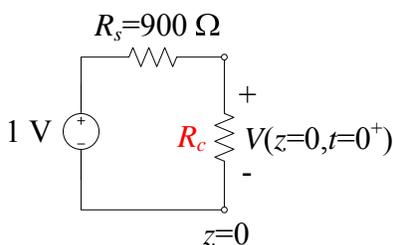
For this TL,

$$t_L = \frac{L}{u} = \frac{8 \text{ cm}}{200 \text{ m}/\mu\text{s}} = 400 \text{ ps}, \quad \Gamma_s = \frac{R_s - R_c}{R_s + R_c} = \frac{900 - 100}{900 + 100} = 0.8,$$

and

$$\Gamma_L = \frac{R_L - R_c}{R_L + R_c} = \frac{25 - 100}{25 + 100} = -0.6$$

At time $t = 0^+$, draw the equivalent lumped-element circuit at the input to the TL:



By voltage division in this circuit

$$V_1^+ = \frac{R_c}{R_c + R_s} \cdot V_s = \frac{100}{100 + 900} \cdot 1 = \frac{1}{10} \text{ V}$$

The pulse width $W = 200$ ps is less than the one-wave transit time t_L , so the bounce diagram in Fig. 2 is applicable, though it's not drawn to precisely represent this specific problem.

We are requested to plot $V(z = L, t)$. For $z_0 = L$, since we're looking at the load end, then referring to the bounce diagram in Fig. 2:

$$\begin{aligned} P_1 &= P_3 \\ P_2 &= P_4 \\ P_5 &= P_7 \\ P_6 &= P_8 \\ &\vdots \end{aligned}$$

- $0^+ \rightarrow t_1^- (= t_L^- = 400 \text{ ps})$:

$$V_L = 0 \text{ V}$$

- $t_1^+ (= 400 \text{ ps}) \rightarrow t_2^- (= t_L^- + W = 600 \text{ ps})$:

$$V_L = V_1^+ + \Gamma_L V_1^+ = \frac{1}{10} - 0.6 \frac{1}{10} = 0.04 \text{ V}$$

- $t_2^+ (= 600 \text{ ps}) \rightarrow t_5^- (= 3t_L^- = 1200 \text{ ps})$:

$$V_L = 0 \text{ V}$$

- $t_5^+ (= 1200 \text{ ps}) \rightarrow t_6^- (= 3t_L^- + W = 1400 \text{ ps})$:

$$\begin{aligned}
 V_L &= \Gamma_s \Gamma_L V_1^+ + \Gamma_L^2 \Gamma_s V_1^+ \\
 &= 0.8(-0.6) \frac{1}{10} + (-0.6)^2 0.8 \frac{1}{10} = -0.0192 \text{ V}
 \end{aligned}$$

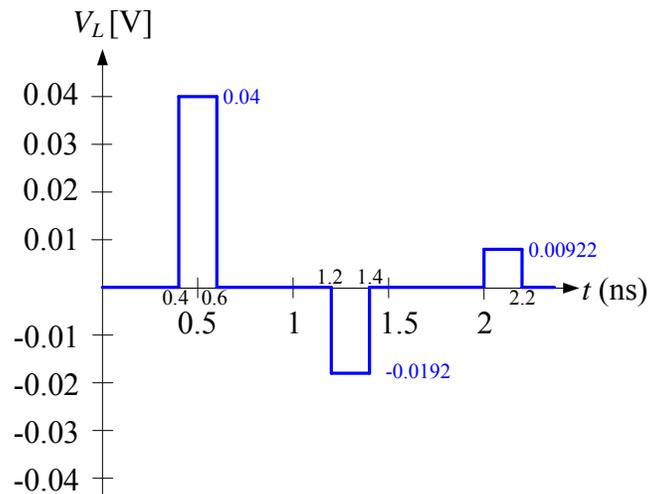
- $t_6^+ (= 1400 \text{ ps}) \rightarrow t_9^- (= 5t_L^- = 2000 \text{ ps})$:

$$V_L = 0 \text{ V}$$

- $t_9^+ (= 2000 \text{ ps}) \rightarrow t_{10}^- (= 5t_L^- + W = 2200 \text{ ps})$:

$$\begin{aligned}
 V_L &= \Gamma_s^2 \Gamma_L^2 V_1^+ + \Gamma_L^3 \Gamma_s^2 V_1^+ \\
 &= 0.8^2 (-0.6)^2 \frac{1}{10} + (-0.6)^3 0.8^2 \frac{1}{10} = 0.00922 \text{ V}
 \end{aligned}$$

Sketch of the output voltage versus time:





Example 7.3



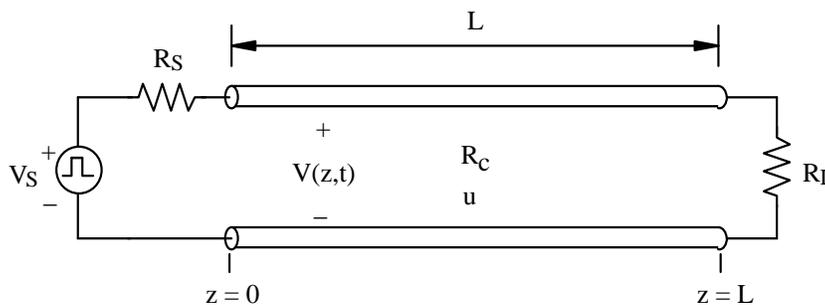
Propagation of Voltage Pulse and Ramp Waveforms on a Transmission Line

Purpose

To compute and visualize the propagation of voltage waves on a transmission line. Two different source voltage waveforms – the pulse and the ramp – will be used in this worksheet. The computation of the voltage on the TL is accomplished using a numerical solution called the finite difference time domain method. (Requires Mathcad 8 Professional, Academic or Explorer.)

Background

A coaxial-cable transmission line is used in this problem as shown in Fig. 7.15a of the text. The transmission line (TL) model of this coax is depicted in the figure below:



Although the load resistance was specified as zero in Example 7.3, in this worksheet we will allow for a nonzero load resistance so that you can investigate other problems of this type that may have a nonzero load resistance.

Two different types of transient voltage excitation are examined in Example 7.3. Both of these will be implemented in this worksheet. The propagation of the transient voltage on this TL will be computed using the FDTD method as described in the [Example 7.2](#) worksheet.

Enter the transmission line, voltage source and FDTD parameters

Choose the parameters for the transmission line (TL):

$$L := 400$$

Length of TL (m).

$$c_{tl} := 100 \cdot 10^{-12}$$

Per-unit-length capacitance of TL (F/m).

$$l_{tl} := 0.25 \cdot 10^{-6} \quad \text{Per-unit-length inductance of TL (H/m).}$$

$$R_L := 0 \quad \text{Load resistance } (\Omega).$$

For this choice of the per-unit-length capacitance and inductance, the propagation velocity, u , and characteristic resistance, R_C , of the TL are:

$$u := \frac{1}{\sqrt{l_{tl} \cdot c_{tl}}} \quad u = 2 \times 10^8 \quad (\text{m/s})$$

$$R_C := \sqrt{\frac{l_{tl}}{c_{tl}}} \quad R_C = 50.000 \quad (\Omega)$$

Now choose the source voltage amplitude and the pulse width (i.e., the length of time for which the source voltage is nonzero):

$$V_0 := 100 \quad \text{Source voltage amplitude (V).}$$

$$R_S := 150 \quad \text{Source resistance } (\Omega).$$

$$W := 6 \cdot 10^{-6} \quad \text{Pulse width of the source voltage (s).}$$

For the numerical **FDTD solution** of the voltage on this transmission line, choose the number of segments in which to subdivide the TL, nz , and the number of time steps to compute the solution, nt :

$$nz := 40 \quad \text{Number of segments to subdivide the TL.}$$

$$nt := 360 \quad \text{Number of time steps to compute the solution.}$$

Compute the uniform spacing of the voltage nodes, Δz , and the time increment Δt :

$$\Delta z := \frac{L}{nz} \quad \Delta t := \frac{\Delta z}{u}$$

The source voltage pulse is assumed to begin at time $t = 0$. For $nt = 360$ time steps, the final time for which a voltage solution will be computed is:

$$t_{\text{end}} := nt \cdot \Delta t \quad t_{\text{end}} = 1.800 \times 10^{-5} \quad (\text{s})$$

Plot the open-circuit source voltage for the pulse and ramp waveforms

We will first plot the pulse and ramp source voltages to better appreciate these two types of open-circuit waveforms that we will use in this worksheet. Here we define the *program* structure $V_S(V_0, t)$ that is used to compute the open-circuit source voltage at any time t :

$$V_S(V_t, t) := \begin{cases} V_0 - \left(V_t - \frac{V_t}{W} \cdot t \right) & \text{if } 0 < t \leq W \\ 0 & \text{otherwise} \end{cases}$$

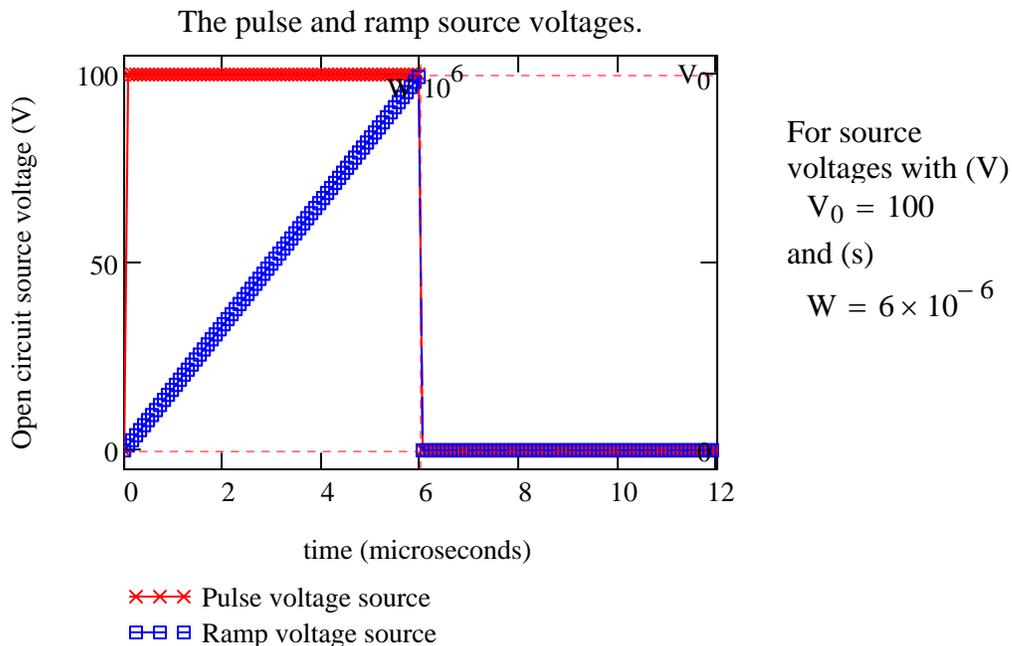
This source voltage definition is capable of computing both the pulse and ramp shapes used in Example 7.3 of the text. V_S is defined here as the difference between two waveforms: a pulse of amplitude V_0 and a linearly *decreasing* voltage (in time) that has amplitude V_t at $t = 0$ and an amplitude of 0 at $t = W$. If the desired voltage source is to have a pulse shape, then V_t should be input as 0. If the desired source is to be an increasing ramp (in time), then V_t should be input as V_0 .

Choose the number of points to plot the source waveforms and a set of times t_p at which to plot $npts := 150$ Number of time points to plot the source voltage.

$t_{p_end} := 2 \cdot W$ Ending time for the plot (s).

$i := 0 .. npts - 1$ $t_{p_i} := i \cdot \frac{t_{p_end}}{npts - 1}$

The pulse and ramp voltage sources are shown as a function of time below:



These two open-circuit source voltages will be identical to those indicated in Figs. 7.15b and 7.15d of the text if $V_0 = 100$ V and $W = 6 \mu s$.

Define the FDTD function to compute the TL voltage

Using the parameters for the TL, the number of segments (nz) and the number of time steps (nt) for the FDTD solution entered earlier in this worksheet, we can now solve for the voltage at each node on the TL at each time step. The program structure FDTD given in the following *referenced worksheet*

☞ Reference:D:\Classes\EE 382 - Applied Electromagnetics\Lecture notes, EE 382\EE 382, Lecture notes, Mathcad wor

will compute the voltage on the TL once a voltage source waveform (either pulse or ramp) has been specified.

The voltage on the TL produced by each of these two sources will now be separately examined in the following four sections of this worksheet.

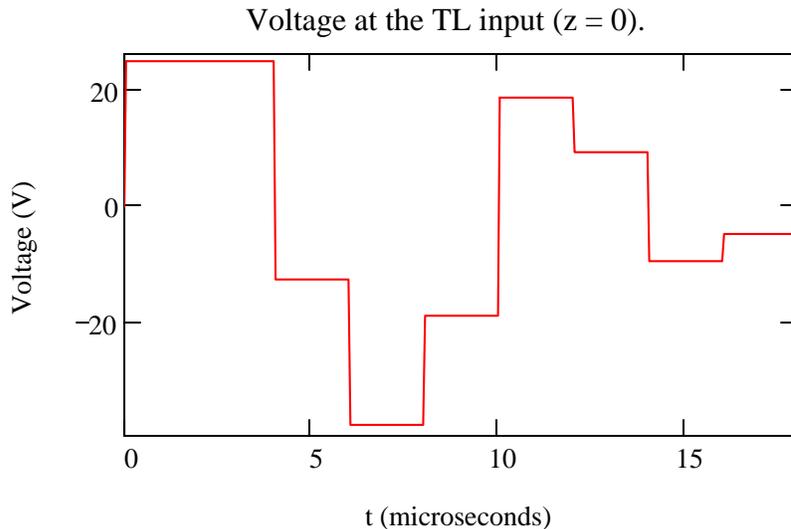
Pulse voltage source

We will first observe the voltage produced on the TL by a pulse source voltage.

Solve for the voltage at all nz+1 nodes on the TL at all times nt with a pulse voltage source excitation:

$$V := \text{FDTD}(0, nz, nt)$$

With the voltages determined at all of the nodes on the TL at the discrete list of times, we will now generate two plots in order to visualize these results. The first plot is the voltage at the input to the TL (z = 0) as a function of time.



For a pulse voltage source.

Such a plot was also constructed in Example 7.3 of the text by other means. Using the same source and TL parameters as given in that example, you will observe that the plot shown above will be identical to that given by the solid line in Fig. 7.15c.

The voltage levels in the above plot can be measured directly from the plot using the Trace command. 

Pulse voltage source: Animated plot of V propagating on the TL

The next plot we will generate is an animation clip for the voltage on the TL as a function of time for this pulse voltage source.

The voltage was computed at $nt = 360$ time steps. Due to the time it takes to construct the animation of this voltage disturbance on the TL, you may *not* wish to have Mathcad construct the animation clip using all of these time steps. Therefore, you can choose the parameter n_{skip} to specify how many time steps to disregard between successive frames when generating the animation clip. For example, if $n_{\text{skip}} = 0$, all time steps are used; if $n_{\text{skip}} = 1$, every second time step is used; if $n_{\text{skip}} = 2$, every third time step is used, etc.

$$n_{\text{skip}} := 2$$

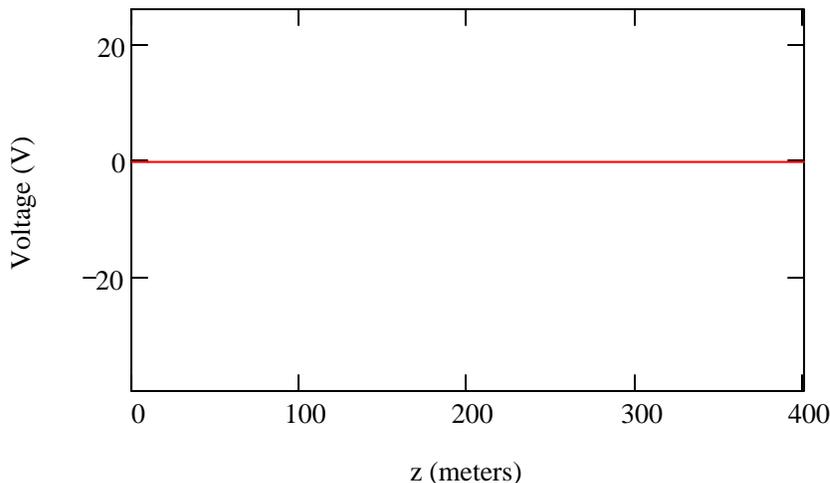
Choose the number of time steps to skip between successive frames in the animation clip.

Compute the z position of all voltage nodes in the FDTD solution and define the parameter time in terms of FRAME:

$$k := 1 .. nz + 1 \quad z_k := (k - 1) \cdot \frac{L}{nz} \quad \text{time} := \frac{\text{FRAME} \cdot (n_{\text{skip}} + 1) \cdot \Delta t}{10^{-6}}$$

Now generate an animation clip of the voltage on this TL. For best results, in the "Animate" dialog box choose $To = 120$.

Voltage on the TL (pulse source).



At time (μs)

time = 0.000

For a TL with

$L = 400$ (m)

$u = 2 \times 10^8$ (m/s)

The source is located at the left-hand edge of this plot and the load is located at the right-hand edge.

One particularly useful application of this animation is to observe the voltage at the input to the TL as a function of time and compare this voltage to that shown in Fig. 7.15(c) in the text (and also in the previous plot). You can use the slider bar in the animation viewer to observe this animation at a desired time instant.

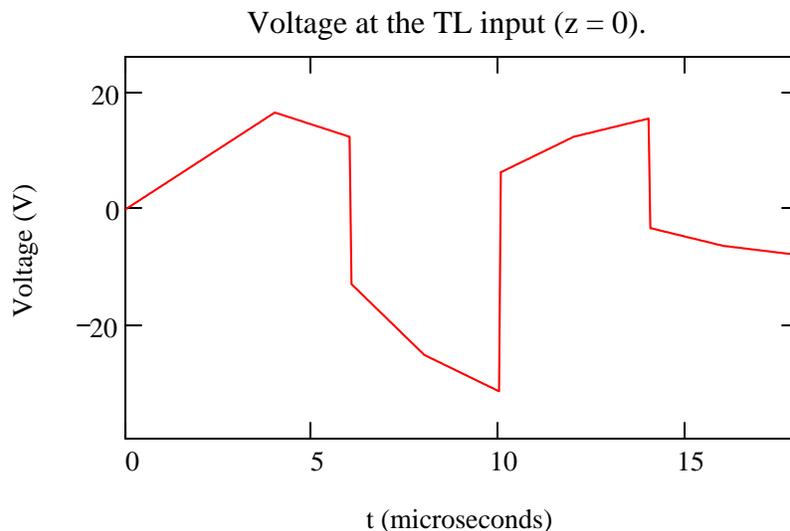
Performing this comparison, in addition to the analysis technique in Example 7.3, is very helpful in appreciating how this input voltage varies as a function of time.

Ramp voltage source

Now we will consider the ramp voltage source excitation as shown earlier in this worksheet. The numerical solution is the same as in the previous pulse example, except now the FDTD program is called with the source parameter V_0 in the first argument below. As discussed earlier, this will cause the voltage source to be the ramp excitation:

$$V := \text{FDTD}(V_0, nz, nt)$$

With the voltages determined at all of the nodes on the TL at the discrete list of times, we will now generate two plots in order to visualize these results. The first plot we will generate is the voltage at the input to the TL ($z = 0$) as a function of time:



For a ramp voltage source.

Using the same source and TL parameters as given in Example 7.3, you will observe that the plot shown above will be identical to that given by the solid line in Fig. 7.15d.

The voltage levels in the above plot can be measured directly from the plot using the Trace command. 

Ramp voltage source: Animated plot of V propagating on the TL

The final plot we will generate is an animation clip of the voltage on the TL as a function of time for this ramp voltage source. First choose the parameter n_{skip} . As described earlier in this worksheet, if $n_{\text{skip}} = 0$, all time steps are used; if $n_{\text{skip}} = 1$, every second time step is used; if $n_{\text{skip}} = 2$, every third time step is used, etc., in the generation of the animation clip:

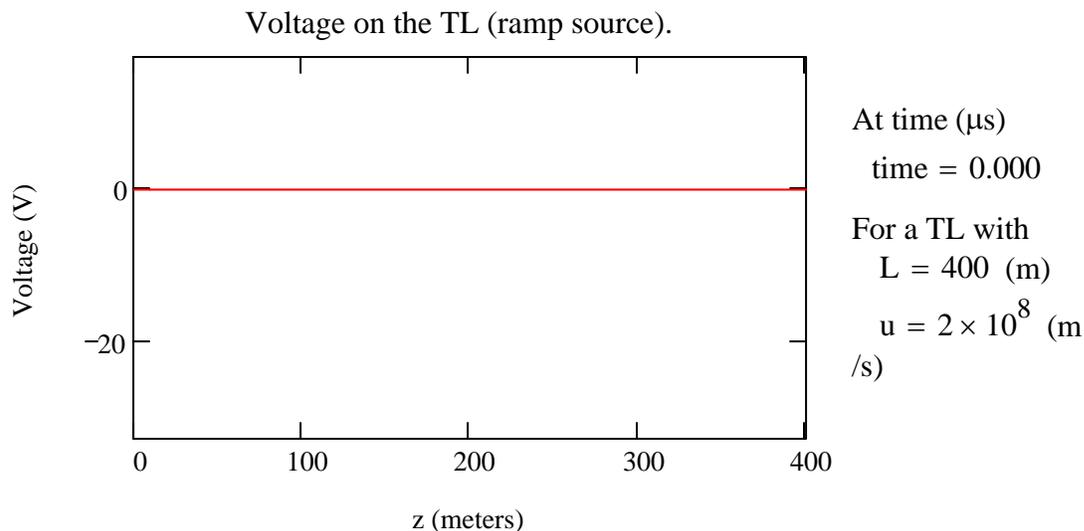
$n_{\text{skip}} := 2$

Choose the number of time steps to skip between successive frames in the animation clip.

Compute the z position of all voltage nodes in the FDTD solution and define the parameter time in terms of FRAME:

$$\text{time} := \frac{\text{FRAME} \cdot (n_{\text{skip}} + 1) \cdot \Delta t}{10^{-6}}$$

Now generate an animation clip of the voltage on this TL. For best results, in the "Animate" dialog box choose $T_0 = 120$.



It is interesting to observe in this animation that the voltage waveform that initially propagates along this TL has the sloped edge pointing towards the right whereas the plot of this ramp waveform shown earlier in this worksheet (and in Fig. 7.15d of the text) has the sloped edge of the voltage on the left. This behavior is due to the fact that the latter plot is the open-circuit source voltage as a function of time, whereas the former plot (i.e., the animation clip above) is the voltage as a function of position (and time) along the TL. In fact, as is discussed in the text following Example 7.3, the voltage waveform we initially observe in this animation clip appears as a "flipped around" version of the open-circuit source voltage. This occurs because the linearly increasing portion of the ramp source is the first to excite the TL and, consequently, the first to begin propagation along the TL. We did not observe this behavior with the pulse input earlier in this worksheet since the leading and trailing edges of this waveform look identical.

You may wish to experiment with other waveforms to more clearly see this behavior. For example, choosing a smaller width for the source, such as $W = 10^{-6}$ (1 μs) will give a ramp (or pulse) waveform that is much smaller than the one-way transient time on the TL using the parameters for the TL given in Example 7.3. When you generate the animation clip for this case, look for the total reflection of the voltage waveform at the load which will cause the incident ramp to undergo an inversion as it propagates back towards the source.



End of worksheet.

