Lecture 14: Bounce Diagrams.

The bounce diagram can be a useful tool for the analysis of unit-step and pulse voltage (or current) responses of TLs.

In the bounce diagram, distance is shown along the horizontal axis and time along the vertical axis. A series of lines are drawn diagonally on this diagram indicating the leading or trailing edges of a voltage (or current) waveform, which are labeled by the amplitude of the voltage (or current) for that time and position on the TL.

It is helpful to present the bounce diagram with an example. Consider the TL shown below with a unit-step excitation.

The sloping lines indicate voltage waves traveling in the $+z$ and $-z$ directions. Each sloping line in this case is labeled with the voltage amplitude of the partial voltage wave traveling in that direction. We can also construct a bounce diagram for the amplitudes of a current wave.
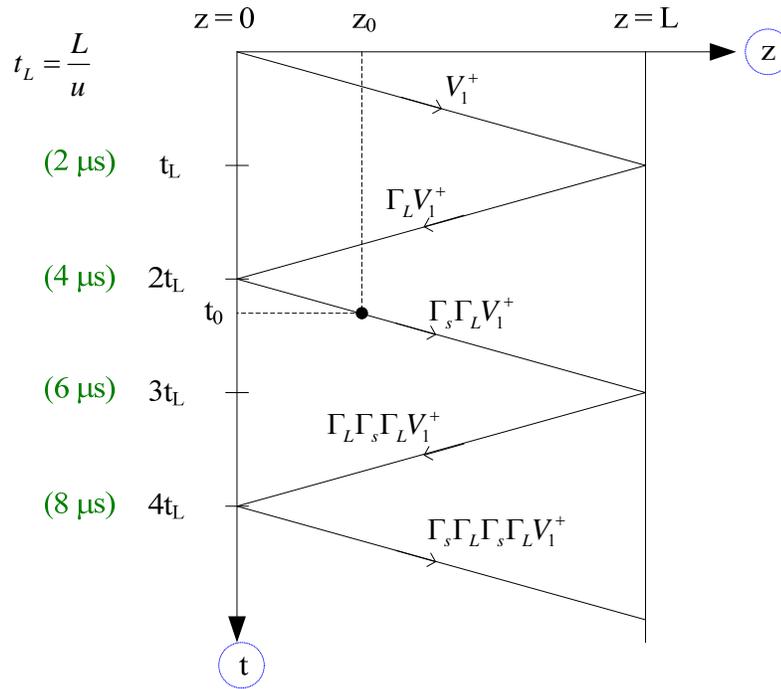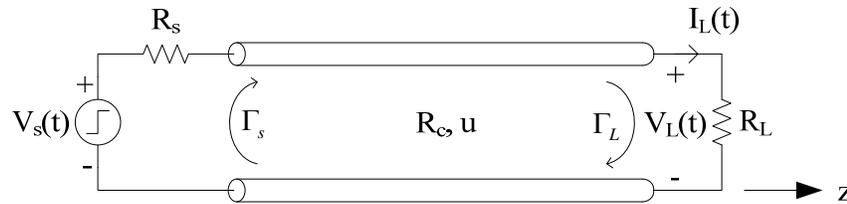
These voltage amplitudes are obtained by multiplying the "incident" voltage by the reflection coefficient at the particular discontinuity (either Γ_L or Γ_s) as shown.

The bounce diagram can be used in two ways: To determine the voltage variation

1. along the TL at a specific time,
2. at a specific z as a function of time.

These two cases will be considered separately below.

Use Bounce Diagram for *V(z,t)* vs. *z* at a Fixed *t*

For this illustration, we'll assume a fixed time t_0 is as shown in the bounce diagram above between $2t_L$ and $3t_L$. To use the bounce diagram in this capacity:

(i.)   Mark t_0 on the vertical *t* axis.

(ii.) Draw a horizontal line at t_0 to intersect with the sloping line of the voltage wave.

(iii.) Draw a vertical line at the intersection. Only times before t_0 are relevant, which is the portion of the graph "above" t_0.

(iv.) The voltage on the TL to the left of z_0 is the sum of all voltages intersecting an imaginary vertical line at z_0^- as:

$$V\left(z,t_0\right)\big|_{z<z_0} = V_1^+ + \Gamma_L V_1^+ + \Gamma_s \Gamma_L V_1^+ \qquad (1)$$

(v.)   The voltage on the TL to the right of z_0 is the sum of all voltages intersecting an imaginary vertical line at z_0^+ as:

$$V\left(z,t_0\right)\big|_{z>z_0} = V_1^+ + \Gamma_L V_1^+ \qquad (2)$$

Example N14.1: Repeat Example N13.1 using a bounce diagram. Sketch the voltage on the TL at the time $t_0 = 4.5$ μs.

For this TL,

$$t_L = \frac{L}{u} = \frac{400 \text{ m}}{200 \text{ m/μs}} = 2 \text{ μs}, \quad \Gamma_L = \frac{1}{3}, \quad \text{and} \quad \Gamma_s = -1.$$

We can use the bounce diagram on page 2 for this example because the t_0 in the bounce diagram is located at 4.5 μs as needed here for this example.

Consequently, from (1):
$$V(z,t_0)\big|_{z<z_0} = V_1^+ + \Gamma_L V_1^+ + \Gamma_s \Gamma_L V_1^+$$

$$= 30 + \frac{1}{3} \cdot 30 + (-1) \cdot \frac{1}{3} \cdot 30 = 30 \text{ V}$$

while from (2):
$$V(z,t_0)\big|_{z>z_0} = V_1^+ + \Gamma_L V_1^+$$

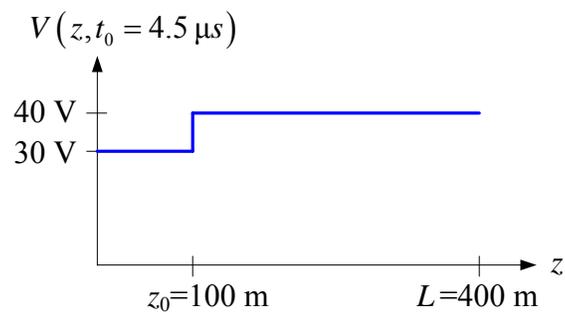$$= 30 + \frac{1}{3} \cdot 30 = 40 \text{ V}$$

To calculate the position z_0 shown in the figure above, we first calculate the <u>total</u> distance, d_{total}, the initial wavefront travels as
$$d_{\text{total}} = u \cdot t_0 = 200 \text{ m/}\mu s \cdot 4.5 \text{ } \mu s = 900 \text{ m}$$
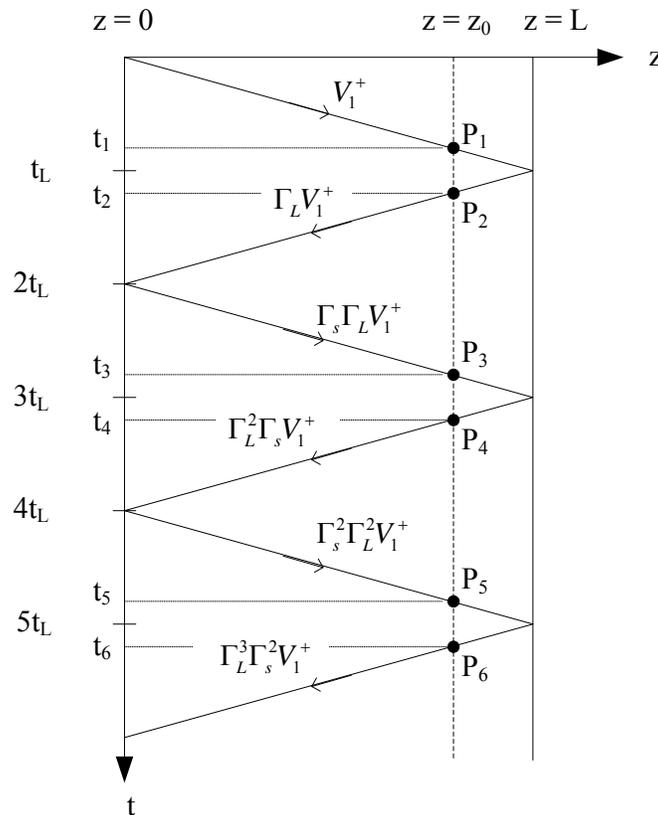Consequenlty,
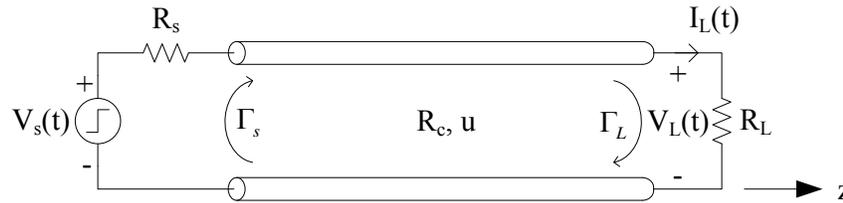$$z_0 = d_{\text{total}} - 2 \cdot L = 100 \text{ m}$$

A sketch of the total voltage everywhere on the TL at time 4.5 μs is then:

$V(z, t_0 = 4.5\,\mu s)$

40 V

30 V

$z_0 = 100$ m $L = 400$ m

z

This is the same voltage as shown in "Fig. 7.12(d)" in Lecture 13.

Use Bounce Diagram for $V(z,t)$ vs. t at a Fixed z


To use the bounce diagram in this situation:

(i.)   Pick a position z_0 at which to plot $V(z_0,t)$ versus time.

(ii.)   Draw a vertical line at z_0. It intersects the sloping lines at points P_1 through P_6 above.

(iii.) At each of these intersection points, draw horizontal lines and label these times t_1 through t_6. These are the times at which the corresponding new wave fronts arrive and abruptly change the voltage at z_0.

(iv.)   The voltage at z_0 versus time is then:

Time range	Voltage
$0^+ \to t_1^-$	0
$t_1^+ \to t_2^-$	V_1^+
$t_2^+ \to t_3^-$	$V_1^+ + \Gamma_L V_1^+$
$t_3^+ \to t_4^-$	$V_1^+ + \Gamma_L V_1^+ + \Gamma_s \Gamma_L V_1^+$
$t_4^+ \to t_5^-$	$V_1^+ + \Gamma_L V_1^+ + \Gamma_s \Gamma_L V_1^+ + \Gamma_L^2 \Gamma_s V_1^+$
$t_5^+ \to t_6^-$	$V_1^+ + \Gamma_L V_1^+ + \Gamma_s \Gamma_L V_1^+ + \Gamma_L^2 \Gamma_s V_1^+ + \Gamma_s^2 \Gamma_L^2 V_1^+$
etc…	

Example N14.2: Consider the coaxial cable shown below. Sketch the voltage at the load $V(L,t)$ for the indicated unit-step input voltage and an open circuit load.
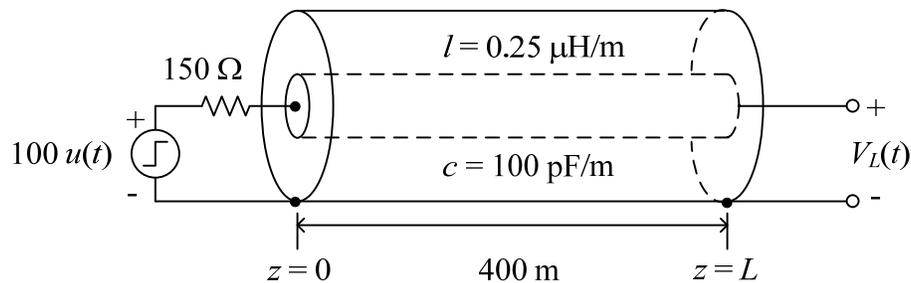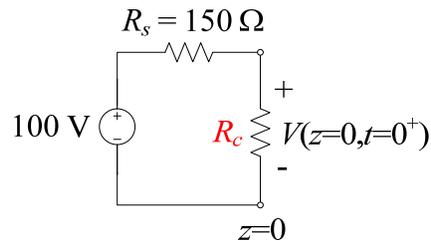

For this TL circuit:

- $R_c = \sqrt{\dfrac{l}{c}} = \sqrt{\dfrac{0.25 \times 10^{-6}}{100 \times 10^{-12}}} = 50 \ \Omega$

- $u = \dfrac{1}{\sqrt{lc}} = 2 \times 10^8 \ \text{m/s} \ (\approx 66.7\%)$

- $t_L = \dfrac{L}{u} = \dfrac{400}{2 \times 10^8} = 2 \ \mu s$

- $\Gamma_L = \dfrac{R_L - R_c}{R_L + R_c} = \dfrac{1 - R_c/R_L}{1 + R_c/R_L} = 1, \quad \Gamma_s = \dfrac{R_s - R_c}{R_s + R_c} = \dfrac{1}{2}$

At time $t = 0^+$, draw the equivalent lumped-element circuit at the input to the TL:

By voltage division in this circuit

$$V_1^+ = \dfrac{R_c}{R_c + R_s} \cdot V_s = \dfrac{50}{50 + 150} \cdot 100 = 25 \ V$$

Draw a vertical line at $z_0 = L^-$ in the bounce diagram. Then:

Time range	V_L	
$0 \to t_L^-$	0	Actually, two events occur at time t_L. Useful to consider looking at a position $z = L^-$:
$t_L^+ \to 3t_L^-$	$V_1^+ + \Gamma_L V_1^+ = 25 + (+1) \cdot 25 = 50 \ V$	
$3t_L^+ \to 5t_L^-$	$50 + \Gamma_s \Gamma_L V_1^+ + \Gamma_L^2 \Gamma_s V_1^+ = 75 \ V$	
$5t_L^+ \to 7t_L^-$	$75 + \Gamma_s^2 \Gamma_L^2 V_1^+ + \Gamma_L^3 \Gamma_s^2 V_1^+ = 87.5 \ V$	

Sketch of voltage at the load:
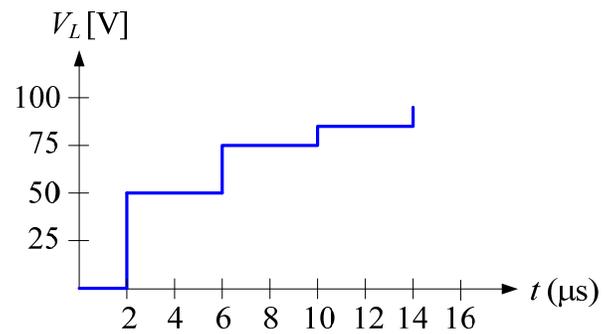
Compare this result with that from the *VisualEM* "Section 7.2/Problem 7.2.4" worksheet.
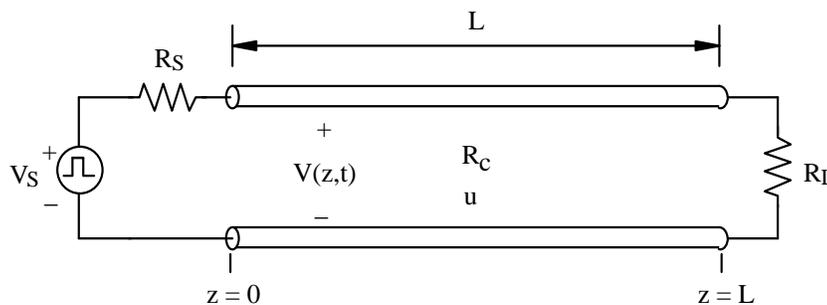
Section 7.2 and Problem 7.2.4

Transient Voltage and Current at the Terminals of a Transmission Line

Purpose

To compute the voltage and current at the input and load terminals of a transmission line that is excited by a unit-step voltage source. The steady-state behavior of the voltage and current at the input and load terminals is examined. The computation of the voltage and current on the TL is accomplished using the finite difference time domain method. (Requires Mathcad 8 Professional, Academic or Explorer.)

Enter TL and voltage source parameters

In this worksheet we will compute and plot the voltage and current at both the input ($z = 0$) and output ($z = L$) terminals of the transmission line (TL) shown in the figure below:

The computation of the voltage and current at the input and load terminals of the TL will be accomplished using the finite difference time domain (FDTD) numerical method. The FDTD method was described previously in the **Example 7.2** worksheet and was also used in the **Example 7.3** worksheet.

Choose the parameters for the transmission line:

$L := 400$ Length of TL (m).

$u := 200 \cdot 10^6$ Propagation velocity of the TL (m/s).

$R_C := 50$ Characteristic resistance of the TL (Ω).

$R_L := 10^6$ Load resistance (Ω).

Note: To model a *short-circuit* load, use a small value for R_L (such as 10^{-6}) rather than zero. This approximation for the short-circuit load will avoid numerical errors in Mathcad when the current through the load is computed. Conversely, for an *open-circuit* load use a large value for R_L such as 10^6. These approximations for R_L are similar to what is used for SPICE as discussed in Section 7.2.1 of the text.

For the choice of the the propagation velocity, u, and characteristic resistance, R_C, the per-unit-length parameters for this TL are:

$$c_{tl} := \frac{1}{u \cdot R_C} \qquad c_{tl} = 1.000 \times 10^{-10} \qquad \text{Per-unit-length capacitance of TL (F/m).}$$

$$l_{tl} := c_{tl} \cdot R_C^{\,2} \qquad l_{tl} = 2.500 \times 10^{-7} \qquad \text{Per-unit-length inductance of TL (H/m).}$$

Now choose the source voltage amplitude and width.

$V_0 := 100$ Source voltage amplitude (V).

$R_S := 150$ Source resistance (Ω).

Note: To model a source with zero internal resistance, use a small value for R_S (such as 10^{-6}) rather than zero. This approximation for the internal source resistance will avoid numerical errors in Mathcad when the current at the input terminals of the TL is computed. This approximation for R_S is similar to what is used for SPICE as discussed in Section 7.2.1 of the text.

For the numerical **FDTD solution** for the voltage on this transmission line, choose the number of segments in which to subdivide the TL, nz, and the number of time steps to compute the solution, nt:

$nz := 40$ Number of segments to subdivide the TL.

$nt := 400$ Number of time steps to compute the solution.

Compute the uniform spacing of the voltage nodes, Δz, and the time increment Δt:

$$\Delta z := \frac{L}{nz} \qquad \Delta t := \frac{\Delta z}{u}$$

The source voltage pulse is assumed to begin at time $t = 0$. For $nt = 400$ time steps, the final time for which a voltage solution will be computed is:

$$t_{end} := nt \cdot \Delta t \qquad t_{end} = 2.000 \times 10^{-5} \qquad \text{(s)}$$

Solve for the voltage on the TL

To model the *unit-step* source voltage, we will use a pulse voltage source with a pulse width W that is larger than the maximum time we will observe the voltage and the current on the TL:

$$W := 2 \cdot t_{end}$$ Width of the source voltage pulse (s).

Using these parameters for the TL, the voltage source and the number of segments (nz) and the number of time steps (nt) for the FDTD solution, we can now solve for the voltage at each node on the TL at each time step. This FDTD solution is performed in the following *referenced worksheet*.

⬛ Reference:D:\Classes\EE 382 - Applied Electromagnetics\Lecture notes, EE 382\EE 382, Lecture notes, Mathcad wo

The final result from this FDTD solution is the list $V_{k,n}$ which contains the values of the voltage at the kth position in z along the TL (beginning at z = 0) at the nth time instant (beginning at t = 0).

Plot the voltage at the input and load terminals of the TL

With the voltage computed at all of the nodes on the TL at the discrete list of times, we are now able to generate a number of plots to visualize these results. The first plots we will generate are the voltages at the input (z = 0) and the load (z = L) terminals as a function of time.

For a TL with L = 400 (m), $u = 2 \times 10^8$ (m/s), $R_C = 50$ (Ω) and a unit-step source with $V_0 = 100$ (V) and $R_S = 150$ (Ω):

Voltage at the input to the TL.

Input voltage (V)

t (microseconds)
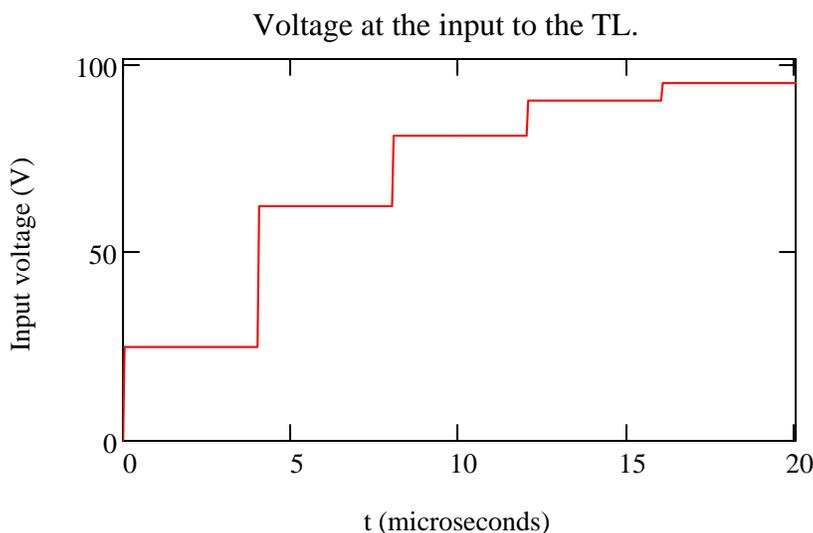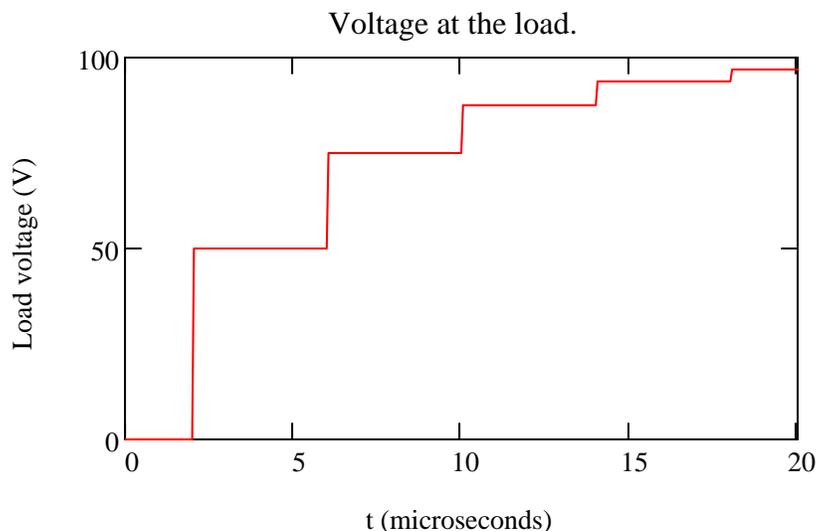
Voltage at the load.

Recall that the voltage source is a unit-step function that transitions from 0 to V_0 Volts at t = 0.

We can observe from these two plots (for the TL parameters given in Prob. 7.2.4) that after approximately 6-7 μs, the voltage at both the input and load terminals of the TL have *nearly* reached the same value. That is, a steady-state response has nearly been achieved.

The voltage levels in the above plot can be measured directly from the plot using the Trace command.

Also, the fine variation of the voltage in the steady-state region of the two plots can be more clearly seen using the Zoom command.

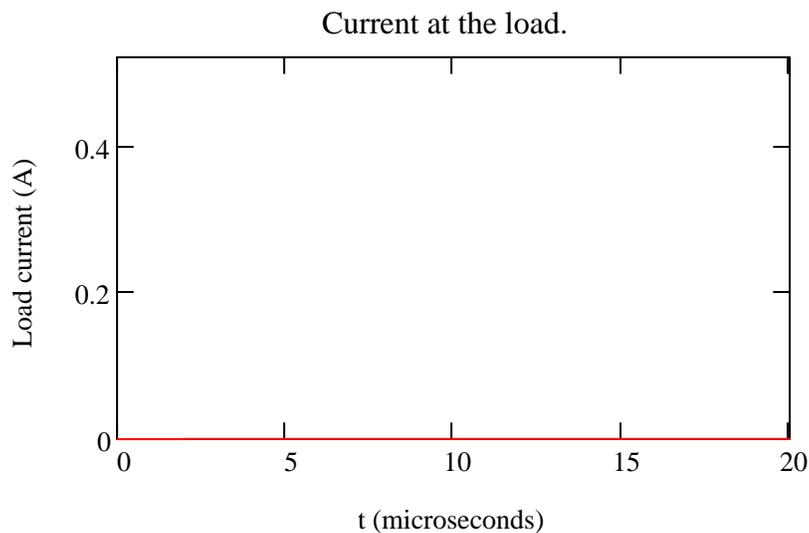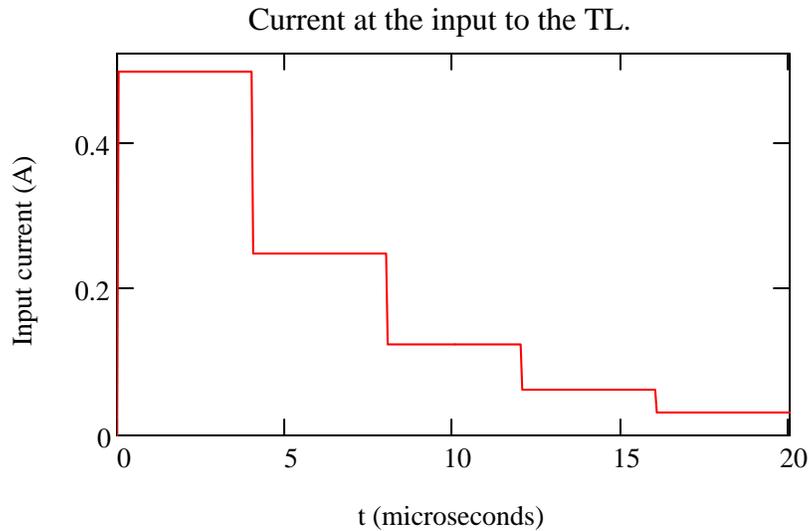### *Plot the current at the input and the load terminals of the TL*

Next, we will plot the current at the input and load terminals of the TL. The input current is computed by dividing the difference in voltage between the source and the first node in the FDTD model of the TL by the source resistance:

$$I_{in_n} := \frac{V_S(n \cdot \Delta t) - V_{1,n}}{R_S}$$

(See the **Example 7.2** worksheet which contains a sketch of the voltage node locations on the TL.) Similarly, the current at the load end of the TL is computed by dividing the voltage at the last node in the FDTD model by the load resistance:

$$I_{load_n} := \frac{V_{nz+1,n}}{R_L}$$

Now plot the current at the input of the TL (z = 0) and at the load (z = L). For a TL with L = 400 (m), u = 2×10^8 (m/s), R_C = 50 (Ω) and a unit-step source with V_0 = 100 (V) and R_S = 150 (Ω):

Current at the input to the TL.

Current at the load.

We can see from these two plots (for the TL parameters given in Prob. 7.2.4) that the currents at the input and load terminals have nearly the same value after approximately 6-7 µs. The current has reached its steady-state response after this amount of time, which we have already seen for the voltage at these two terminals in the earlier plots.

Animated plot of the voltage on the TL

The final plot we will generate is an animation clip of the voltage everywhere on the TL as a function of time for this unit-step voltage source.

The voltage was computed at $nt = 400$ time steps. Due to the time it takes to construct the animation of this voltage disturbance on the TL, you may *not* wish to have Mathcad construct the animation clip using all of these time steps. Therefore, you can set the parameter n_{skip} to specify how many time steps to disregard between successive frames when generating the animation clip. For example, if $n_{skip} = 0$, all time steps are used; if $n_{skip} = 1$, every second time step is used; if $n_{skip} = 2$, every third time is used, etc.

$n_{skip} := 3$ 　　　　Choose the number of time steps to skip between successive frames in the animation clip.

Compute the z position of all voltage nodes in the FDTD solution and define the parameter time in terms of FRAME:

$$k := 1 .. nz + 1 \quad z_k := (k - 1) \cdot \frac{L}{nz} \qquad time := \frac{FRAME \cdot (n_{skip} + 1) \cdot \Delta t}{10^{-6}}$$

Now generate an animation clip of the voltage on this TL. For best results, in the "Animate" dialog box choose $To = 100$.

Voltage on the TL.

At time (μs)
　time = 0.000

For a TL with
　$L = 400$ (m)
　$u = 2 \times 10^8$ (m/s)
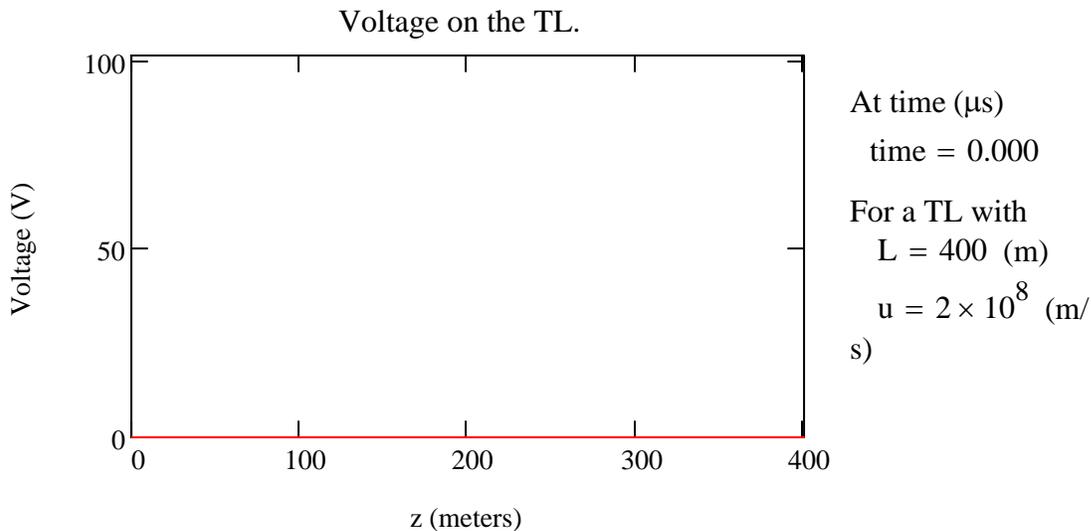
As you can see in the animation clip, after approximately 6 μs (for the TL parameters given in Prob. 7.2.4) that the voltage on the TL does not vary with time. In other words, after this length of time the voltage is constant everywhere on the TL – within the displayed precision of the line plot – and the steady state has been achieved. This result is consistent with the conclusions concerning the time for steady-state to be reached for the voltage at the input and load terminals of the TL as we saw earlier in this worksheet.

For the TL parameters given in Prob. 7.2.4, does this voltage on the TL in the animation clip after approximately 6 μs agree with the expected steady-state voltage? (Hint: you can use simple electrical circuit analysis – in particular, voltage division – to compute the steady-state voltage. Note that, interestingly enough, the characteristic resistance of the TL plays no role in the steady-state value of this voltage.)

End of worksheet.