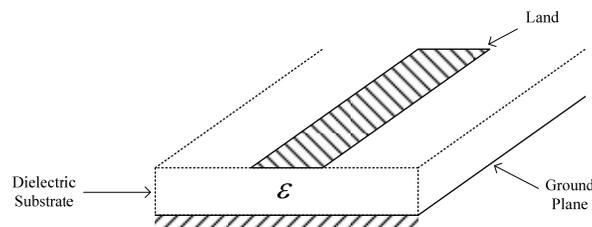# Lecture 11: Transmission Lines and Distributed l and c.

Printed circuit boards (PCBs) are a very common method to connect electrical elements together such as surface mounted resistors, inductors, capacitors, and integrated circuits, for example.

Lands connect the circuit elements together. These are the "printed wires" on the PCB:


While this is a common method for interconnecting components, there is a big difference between such lands for low and high frequency circuits.

As you'll see in this course (and in more detail in EE 481/581 *Microwave Engineering*), the width and thickness of the land, the thickness of the substrate, and the dielectric constant of the substrate all effect the performance of the land. For high frequency engineering, a land is much more than just a printed-wire interconnection!

Signal Propagation Delay

Imagine that two ICs are connected together as shown:
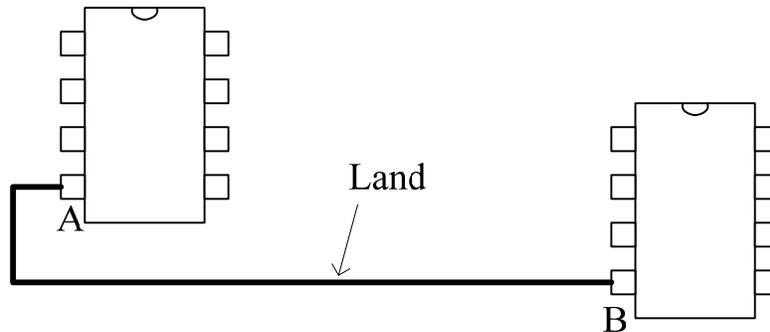
When the voltage at A changes state, does that new voltage value appear instantly at B? No, of course not.

Think of one person speaking to another. If they are close together, the sound seems to arrive at the observer instantly. If they are separated by a large distance, there is a propagation delay time as the sound wave travels to the observer. Not an instantaneous effect.

A similar phenomenon occurs with electrical signals. With high-speed digital circuits, even distances as small as six inches may be "far" and the propagation delay time for a voltage to appear at another IC may be significant.

So how does this phenomenon work? In the words of Dr. Albert Einstein:

"You see, wire telegraph is a kind of a very, very long cat. You pull his tail in New York and his head is meowing in

Los Angeles. Do you understand this? And radio operates exactly the same way: you send signals here, they receive them there. The only difference is that there is no cat."

Transmission Lines

This propagation of voltage signals is modeled as a so-called **transmission line (TL)**. We will see that voltage and current can propagate along a TL as **waves**! Fantastic.

The transmission line model can be used to solve many, many types of high frequency problems, either exactly or approximately. TL geometries include those in text Fig. 11.1:
  • Two-wire (parallel)
  • Wire above a ground plane, and
  • Coaxial cable.

All true TLs share one common characteristic: the $\bar{E}$ and $\bar{H}$ fields are all perpendicular to the direction of propagation, which is the long axis of the geometry. These are called TEM fields for transverse electric and magnetic fields.

An excellent example of a TL is a coaxial cable. On a TL, the voltage and current vary along the structure in time t and spatially in the z direction, as indicated in the figure below. There are no instantaneous effects.
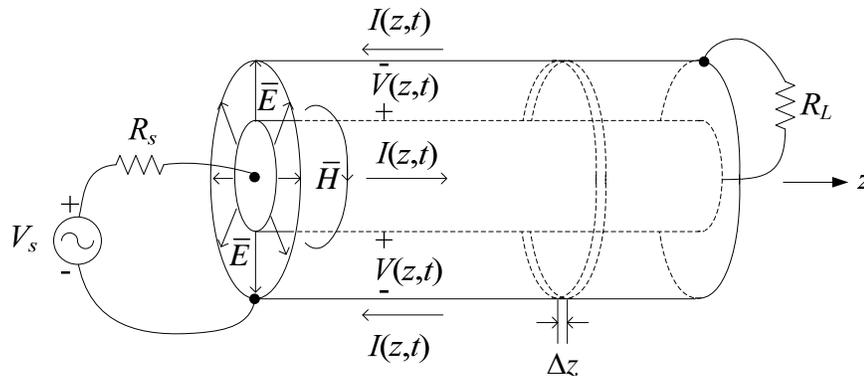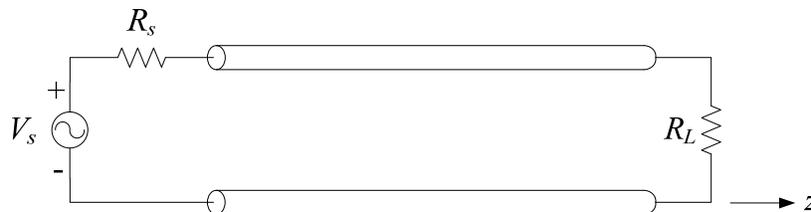
Fig. 1

A common circuit symbol for a TL is the two-wire (parallel) symbol to indicate any transmission line. For example, the equivalent circuit for the coaxial structure shown above is:

Per-Unit-Length Parameters for Transmission Lines

How do we solve for $V(z,t)$ and $I(z,t)$? We first need to develop the governing equations for the voltage and current, then solve these equations.

Notice in Fig. 1 above that there is conduction current in the center conductor and outer shield of the coaxial cable and a displacement current between these two conductors where the electric field $\bar{E}$ is varying with time. **Each of these currents has an associated impedance**:

- Conduction current impedance effects:
  - o Inductance, *L*, due to the current in the conductors and the magnetic flux linking the current path. (Faraday's law.)
  - o Resistance, *R*, due to losses in the conductors,
- Displacement current impedance effects:
  - o Capacitance, *C*, due to the time varying electric field between the two conductors. (Displacement current, Maxwell's law.)
  - o Conductance, *G*, due to losses in the dielectric between the conductors,

We will ignore all losses ($R=0$ and $G=\infty$) for now, but will add them in later in this course.

To develop the governing equations for $V(z,t)$ and $I(z,t)$, we will consider only a small section Δz of the TL. This Δz is so small that the electrical effects are occurring instantaneously and we can simply use circuit theory to draw the relationships between the conduction and displacement currents. This equivalent circuit is shown below (text Fig. 11.5 without losses):
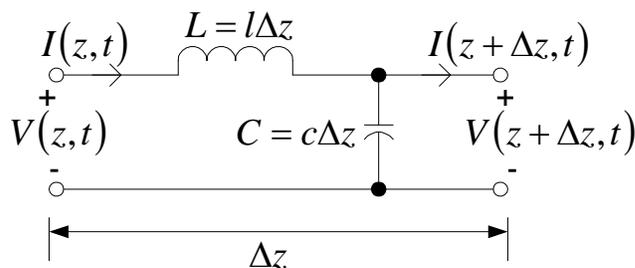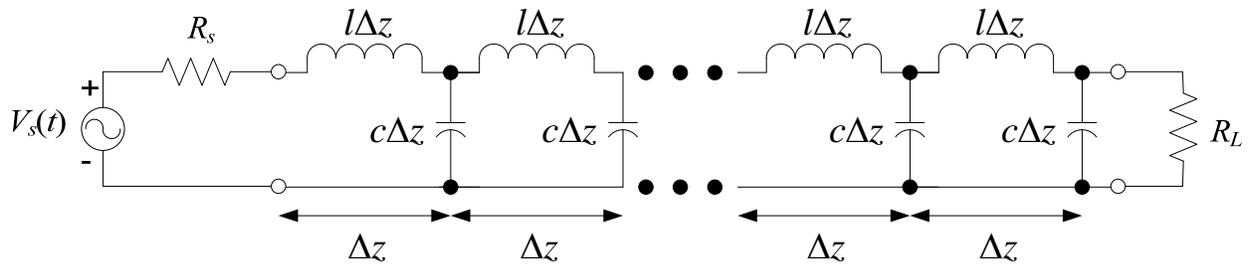

Fig. 2

The variables l and c are per-unit-length (PUL) parameters with units of H/m and F/m, respectively.

A TL can be constructed by cascading many, many of these subsections along the total length of the TL:

This is a general model. In other words, it applies to any TL supporting a TEM field. However, the specific l and c values change depending on the specific geometry (whether it is a two-wire, coax, or other geometry).

Transmission Line Equations

To develop the governing equations for $V(z,t)$, we apply KVL in Fig. 2 above

$$V(z,t) = \underbrace{l\Delta z}_{=L}\frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t) \tag{1}$$

Similarly, for the current $I(z,t)$ apply KCL at the node

$$I(z,t) = \underbrace{c\Delta z}_{C}\frac{\partial V(z,t)}{\partial t} + I(z+\Delta z,t) \tag{2}$$

Then:

1. Divide (1) by Δz:
$$\frac{V(z+\Delta z,t)-V(z,t)}{\Delta z}=-l\frac{\partial I(z,t)}{\partial t} \qquad (3)$$
   In the limit as $\Delta z \to 0$, the term on the LHS in (3) is the forward difference definition of the derivative. Hence,
$$\boxed{\frac{\partial V(z,t)}{\partial z}=-l\frac{\partial I(z,t)}{\partial t}} \qquad (4)$$

2. Divide (2) by Δz:
$$\frac{I(z+\Delta z,t)-I(z,t)}{\Delta z}=-c\frac{\partial V\left(z+\Delta z^{0},t\right)}{\partial t} \qquad (5)$$
   Again, in the limit as $\Delta z \to 0$ the term on the LHS is the forward difference definition of the derivative. Hence,
$$\boxed{\frac{\partial I(z,t)}{\partial z}=-c\frac{\partial V(z,t)}{\partial t}} \qquad (6)$$

Equations (4) and (6) are a pair of coupled, first order partial differential equations for $V(z,t)$ and $I(z,t)$. These two equations are called the **transmission line equations**, though sometimes they're called the telegrapher's equations.

Recap: We expect that *V* and *I* are not constant along high-speed digital circuit interconnects and that changes to *V* and *I* do not

occur instantaneously everywhere along the TL. Rather, (4) and (6) dictate how *V* and *I* vary along the TL at all times.

TL Wave Equations

We will now combine (4) and (6) in a special way to form two equations, each a function of *V* or *I* only.

To do this, take $\dfrac{\partial}{\partial z}$ of (4) and $\dfrac{\partial}{\partial t}$ of (6):

• $\dfrac{\partial}{\partial z}(4):$

$$\frac{\partial^2 V(z,t)}{\partial z^2} = -l\frac{\partial^2 I(z,t)}{\partial z\partial t} \tag{7}$$

• $\dfrac{\partial}{\partial t}(6):$

$$\frac{\partial^2 I(z,t)}{\partial t\partial z} = -c\frac{\partial^2 V(z,t)}{\partial t^2} \tag{8}$$

Substituting (8) into (7) gives:

$$\boxed{\frac{\partial^2 V(z,t)}{\partial z^2} = lc\frac{\partial^2 V(z,t)}{\partial t^2}} \tag{9}$$

Similarly,

$$\boxed{\frac{\partial^2 I(z,t)}{\partial z^2} = lc\frac{\partial^2 I(z,t)}{\partial t^2}} \tag{10}$$

Equations (9) and (10) are the governing equations for the *z* and *t* dependence of *V* and *I*. These are very special equations. In fact, they are **wave equations** for *V* and *I*!

Voltage and Current Waves

You probably don't have any direct experimental evidence that this propagation of voltage and current waves is actually real. You'll get a chance for this in the special laboratory project for this course.

But here's a description of a phenomenon that occurs because of the wave nature of voltages and currents. Imagine a two-wire TL with a short circuit connected at the load end:
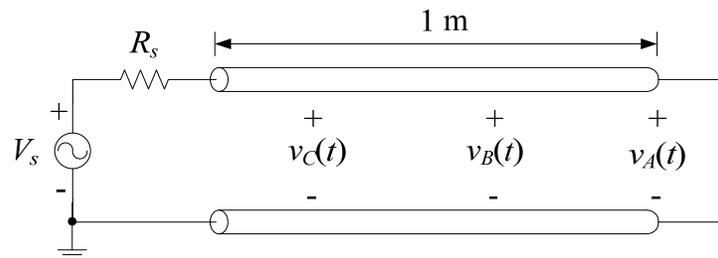
The measured voltage $v_A(t) = 0$ for all time. One may expect the voltages $v_B(t)$ and $v_C(t)$ to be zero for all time as well. That is indeed the case when the frequency of operation is small enough. For example, for the 1-m length TL shown above, this is the case when $f \lesssim 2$ MHz.

But when the frequency becomes high enough, say $f \gtrsim 20$ MHz the voltage and current will not be zero! Why? Because the voltage and current waves propagate along this TL, reflect from the short-circuit load, and add constructively in some places

along the TL and add destructively in others. In the case of the above TL operating near 200 MHz:
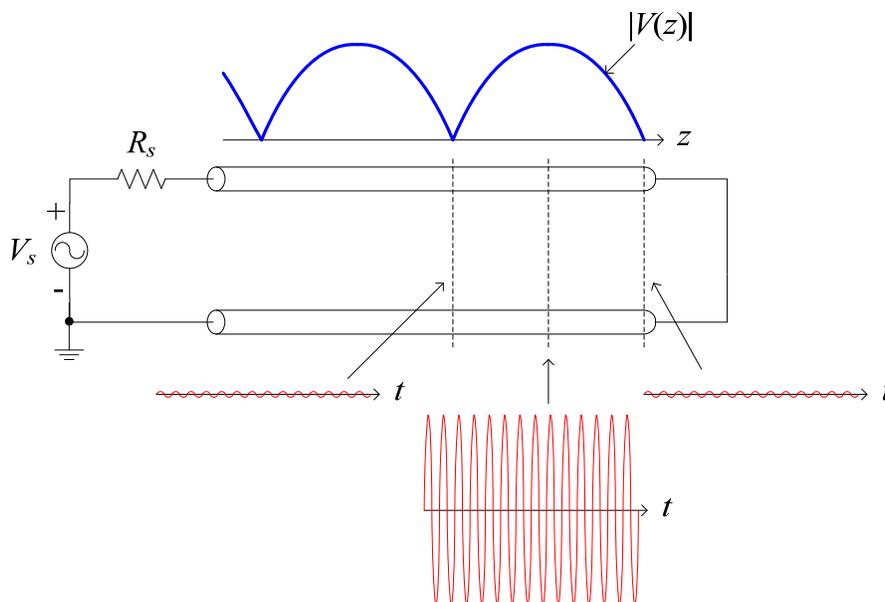
This is **a new phenomenon** and one not described by circuit theory in your earlier courses.