Lecture 10: Non-ideal Behavior of Physical Circuit Elements. Skin Effect.

At "low" frequencies (say $\lesssim$ 1 MHz), physical resistors, capacitors, and inductors usually have terminal characteristics nearly identical to ideal theoretical prediction.

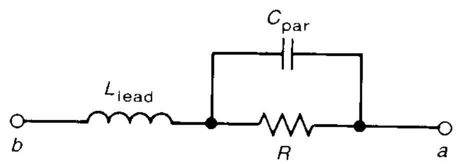
However, at "high" frequencies (say $\gtrsim$ 100 MHz), resistor, capacitor, and inductor circuit elements can behave very differently than expected.

We will consider in this lecture the frequency behavior of these three circuit elements as well as the current density distribution across a round wire at high frequencies.

Non-Ideal Resistor

The leads of the resistor create an inductance L_{lead} and a capacitance C_{par} (for "parasitic"). Additional capacitance from the resistor body can also contribute.

An equivalent circuit representation for a physical resistor is:



Measured frequency response of a 1-kΩ carbon resistor with ¼" lead lengths:

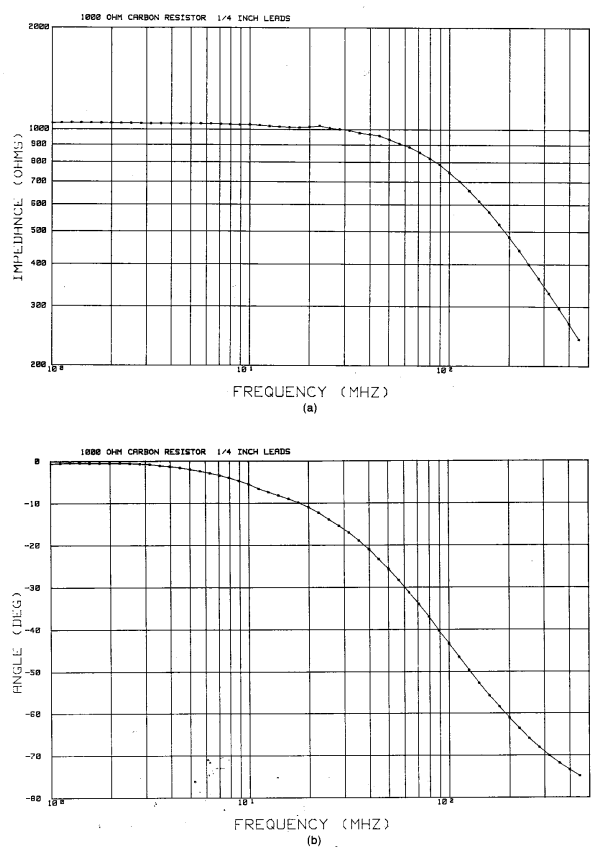


FIGURE 6.19 Measured impedance of a 1000 Ω, carbon resistor having $\frac{1}{4}$ inch lead lengths: (a) magnitude; (b) phase.

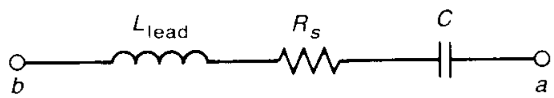
(C. R. Paul, *Introduction to Electromagnetic Compatibility*. New York: John Wiley & Sons, 1992.)

Non-Ideal Capacitor

The leads of the capacitor create an inductance L_{lead} and a capacitance C_{par}. It may often be the case that the C of the capacitor is so large that C_{par} can be ignored.

The resistance of the leads and the conduction current **through** the dielectric of the capacitor can be modeled by a resistance R_s (which should be very large).

An equivalent circuit representation for a physical capacitor:



Measured frequency response of a 470-pF ceramic capacitor with essentially no lead lengths:

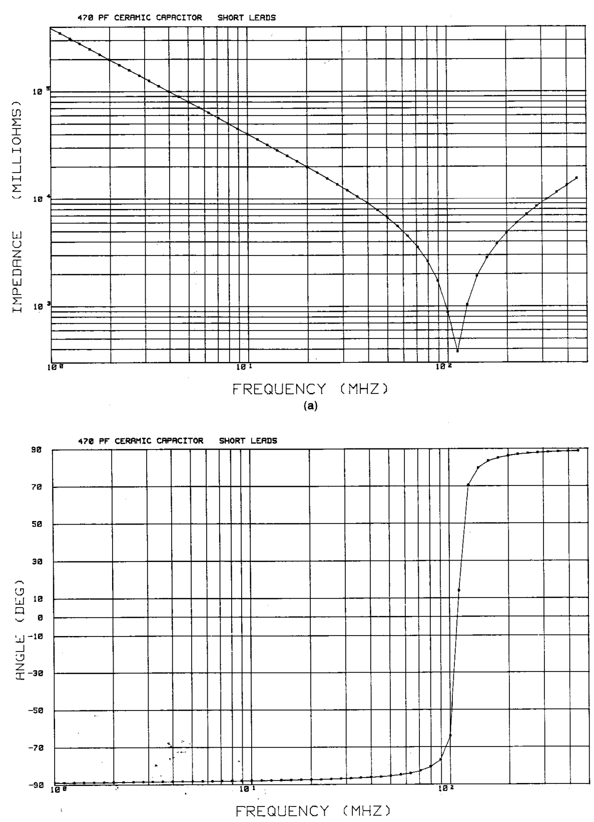


FIGURE 6.25 Measured impedance of a 470 pF ceramic capacitor with short lead lengths: (a) magnitude; (b) phase.

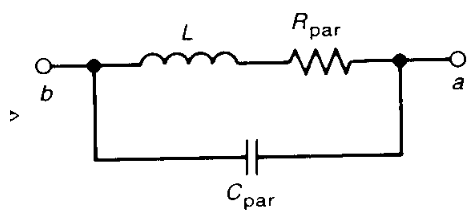
(C. R. Paul, *Introduction to Electromagnetic Compatibility*. New York: John Wiley & Sons, 1992.)

Non-Ideal Inductor

The leads of the inductor create a resistance as well as an inductance. However, both of these are usually much smaller than the resistance of the windings R_{par} and the inductance L of the inductor.

Additionally, a parasitic capacitance C_{par} between turns of wire in the windings is also present.

An equivalent circuit representation for a physical inductor:



Measured frequency response of a 1.2-μH inductor with "very short" lead lengths:

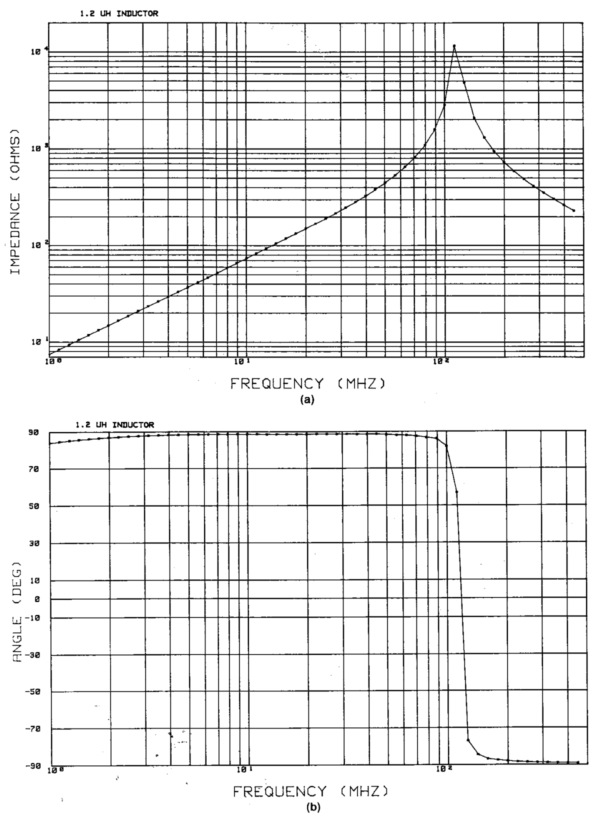


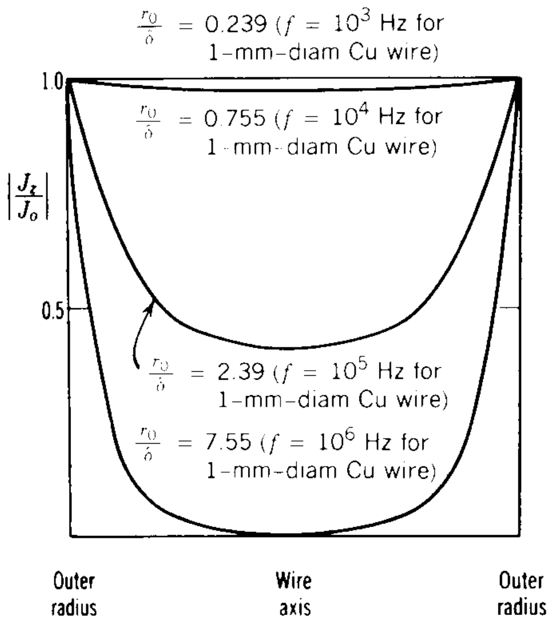
FIGURE 6.32 Measured impedance of a 1.2 μH inductor having very short lead lengths: (a) magnitude; (b) phase.

(C. R. Paul, *Introduction to Electromagnetic Compatibility*. New York: John Wiley & Sons, 1992.)

Skin Effect

On a related topic, you've likely seen in EE 381 that the current density is uniform over the cross section of a wire that is supporting a direct current.

It was likely mentioned then that this behavior is not present at higher frequencies. Shown in the figure below is the behavior of the current density in a 1-mm diameter copper wire at four frequencies (1 kHz, 10 kHz, 100 kHz, and 1 MHz.)



(S. Ramo, J. R. Whinnery and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: John Wiley & Sons, third ed.,1994.)

As the frequency increases, the current density becomes concentrated near the outer surface of the round wire. This behavior of the current density is called the "**skin effect**".