

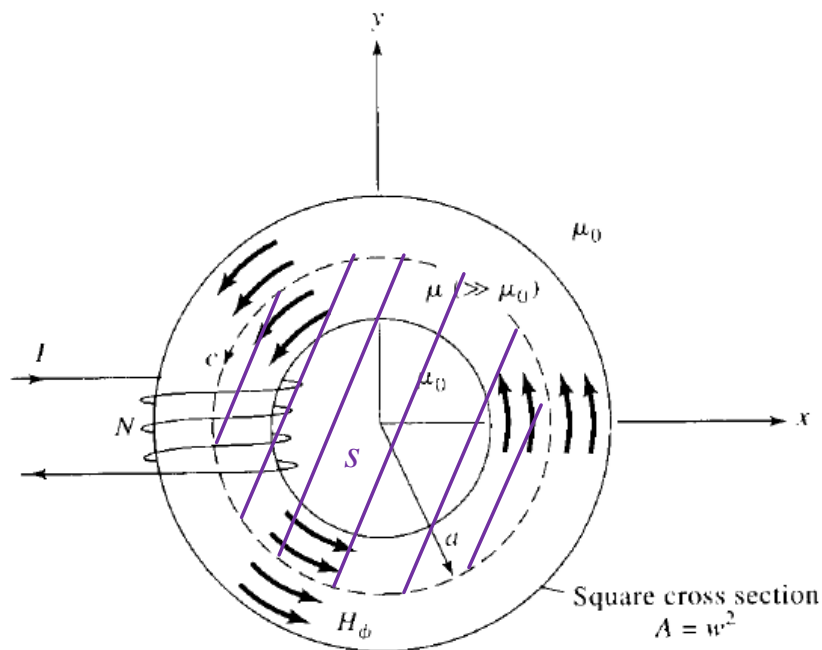
# Lecture 1: Magnetic Circuits.

The first topic of this course is actually a continuation of magnetostatics from EE 381 last semester. This topic is **magnetic circuit analysis** and it's a lumped-element method for solving certain types of magnetic field problems.

Magnetic circuit analysis is **not always applicable**, but when it is it can greatly simplify the solution to magnetics problems.

This process is similar to the simplification of electric circuits using  $R$ ,  $L$ , and  $C$  lumped elements to represent the effects of actual devices with physical dimensions.

To illustrate magnetic circuit analysis, consider the toroid:



This problem is similar to toroid problems you have encountered earlier in EE 381. There is one difference though; the turns of wire exist only over a **portion of the toroid**.

However, with  $\mu \gg \mu_0$  it can be shown through other means that the  $\bar{B}$  field will largely stay within the toroid. Here we will ignore all such **flux leakage** effects. Then, by Ampère's law

$$\oint_{c(s)} \bar{H} \cdot d\bar{l} = NI$$

Employing symmetry, and using  $d\bar{l} = \hat{a}_\phi r d\phi$

$$B_\phi = \frac{\mu NI}{2\pi r} \left[ \frac{\text{Wb}}{\text{m}^2} \right] \quad a - \frac{w}{2} < r < a + \frac{w}{2} \quad (1)$$

We will assume that this toroid has a small cross section such that  $B_\phi$  is **nearly uniform** over the cross section. Therefore,

$$B_\phi \approx \frac{\mu NI}{2\pi a} \equiv B \quad (2)$$

inside the toroid, and zero outside.

Now, the magnetic flux is

$$\psi_m = \int_s \bar{B} \cdot d\bar{s} \approx BA \quad [\text{Wb}] \quad (3)$$

where  $A = w^2$  is the cross-sectional area. Therefore, with (2) in (3)

$$\psi_m = \frac{\mu NI}{2\pi a} w^2 \quad [\text{Wb}] \quad (4)$$

## Equivalent Magnetic Circuit

We now develop the **equivalent magnetic circuit** for this toroid using this last equation by defining

$$\psi_m \equiv \frac{V_m}{\mathfrak{R}} \quad (5)$$

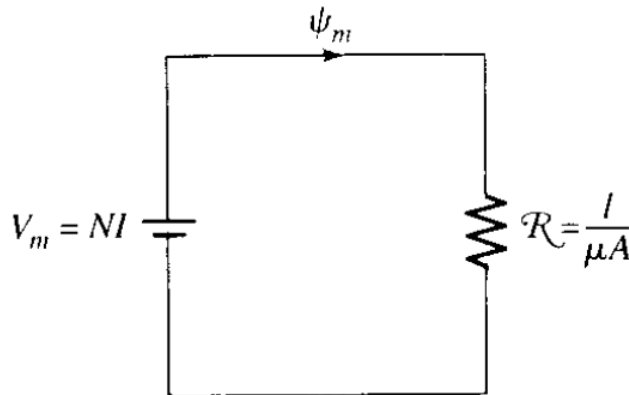
where

- $V_m = NI (= \mathfrak{F})$  [A·t] is the source. Called an **mmf** or “magnetomotive force” with units of Ampère-turns,
- The **reluctance** of the core (units of H<sup>-1</sup>).

$$\mathfrak{R} \equiv \frac{l}{\mu A} \quad [\text{H}^{-1}] \quad (6)$$

where  $l = 2\pi a$  is the mean length (circumference).

The **equivalent magnetic circuit** for the toroid can be drawn as

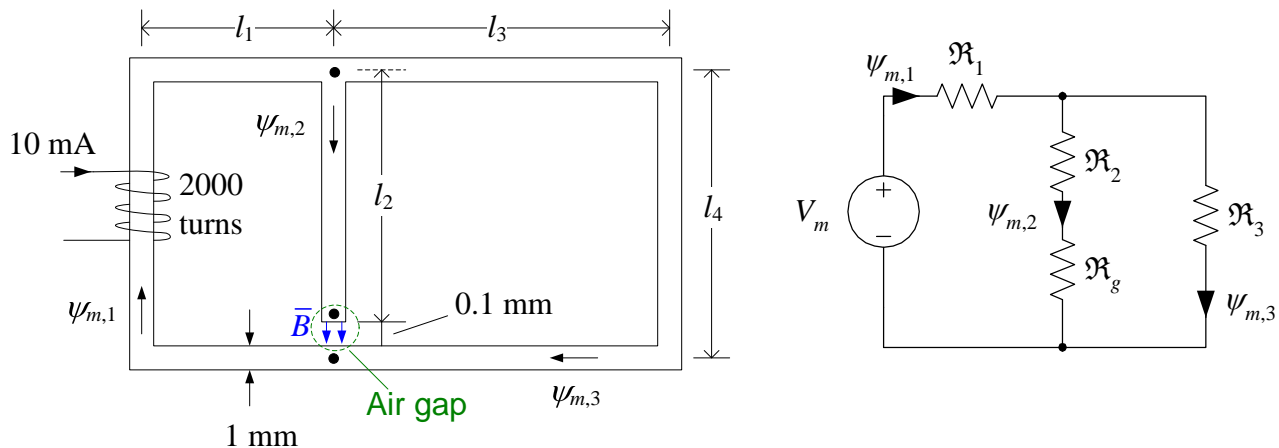


There is a direct analogy between electrical and magnetic circuits.

## Analogous quantities in electrical and magnetic circuit analysis.

Electrical Circuits		Magnetic Circuits	
conductivity (S/m)	$\sigma$	permeability (H/m)	$\mu$
emf (V)	$V$	mmf (A-t)	$V_m$
current (A)	$I$	magnetic flux (Wb)	$\psi_m$
resistance ( $\Omega$ )	$R = \frac{l}{\sigma A}$	reluctance ( $H^{-1}$ )	$\mathfrak{R} = \frac{l}{\mu A}$
Ohm's law	$V = IR$	—	$V_m = \psi_m \mathfrak{R}$
KVL (around a loop)	$\sum_i V_i = \sum_j R_j I_j$	—	$\sum_i V_{m,i} = \sum_j \mathfrak{R}_j \psi_{m,j}$
KCL (at a junction)	$\sum_j I_j = 0$	—	$\sum_j \psi_{m,j} = 0$

**Example N1.1:** Determine the magnetic flux through the air gap in the geometry shown below. The structure is assumed to have a square cross section of area  $10^{-6} \text{ m}^2$ , a core with  $\mu_r = 1,000$ , and dimensions  $l_1 = 1 \text{ cm}$ ,  $l_3 = 3 \text{ cm}$ , and  $l_4 = 2 \text{ cm}$ .



Notice that we can subdivide this circuit so there are only three nodes in this problem. These nodes are indicated by the dots in the left-hand figure. We, of course, could create other equivalent

circuits with more nodes, but that would just increase the complexity of the analysis without adding any other usefulness.

So how do we solve such a problem? Can we use Ampère's law

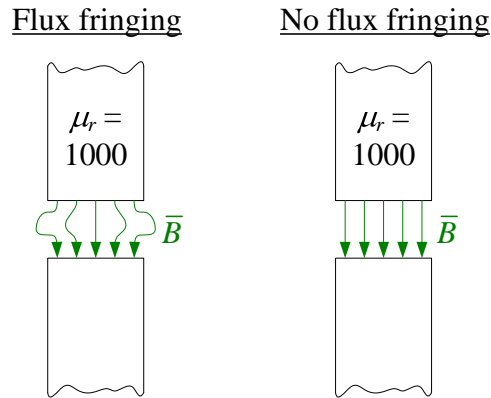
$$\oint_c \vec{H} \cdot d\vec{l} = NI$$

No, we cannot because there isn't sufficient geometrical symmetry for us to **solve** for  $\vec{H}$  using Ampère's law.

The assumptions inherent in magnetic circuits allow us to find an **approximate** solution, however. A drawback to magnetic circuit analysis is that generally we can't check our solutions with a simple analytical formula because there isn't one. Usually our only recourse to check the accuracy of our magnetic circuit solutions is to use a **computational magnetics tool**, which can compute  $\vec{H}$  everywhere in space.

A distinguishing characteristic of this particular example problem is the **air gap**. We'll assume

- The length of the gap is small with respect to the cross section, thus
- No "flux fringing" effects:



Summary of assumptions:

1. **No “flux leakage”**:  $\bar{B}$  remains entirely within the magnetic material and the small air gap.
2. **No “flux fringing”**:  $\bar{B}$  remains vertical in the air gap in this problem.

With no “flux fringing,” then the air gap can be modeled as another reluctance in series with  $\mathfrak{R}_2$  and shown in the equivalent magnetic circuit above.

Compute the four reluctances using (6):

- $\mathfrak{R}_1 = \frac{2l_1 + l_4}{1000\mu_0 A} = 31.83 \times 10^6 \text{ H}^{-1}$
- $\mathfrak{R}_2 = \frac{l_4 - l_g - 0.001/2}{1000\mu_0 A} = 15.44 \times 10^6 \text{ H}^{-1}$
- $\mathfrak{R}_3 = \frac{2l_3 + l_4}{1000\mu_0 A} = 63.66 \times 10^6 \text{ H}^{-1}$
- $\mathfrak{R}_g = \frac{l_g}{\mu_0 A} = 79.58 \times 10^6 \text{ H}^{-1}$

The magnetic flux through the source coil is the mmf divided by the total reluctance seen by the source:

$$\psi_{m,1} = \frac{V_m}{\mathcal{R}_{\text{total}}} = \frac{NI}{\mathcal{R}_1 + (\mathcal{R}_2 + \mathcal{R}_g) \parallel \mathcal{R}_3} = 0.286 \times 10^{-6} \text{ Wb}$$

Using “flux division” (analogous to current division), then

$$\psi_{m,2} = \frac{\mathcal{R}_3}{\mathcal{R}_3 + \mathcal{R}_2 + \mathcal{R}_g} \psi_{m,1} = 0.115 \times 10^{-6} \text{ Wb}$$

A practical application of this problem could be to find  $\bar{B}$  in the air gap, for example. This can be determined from  $\psi_{m,2}$  and the no flux fringing and no flux leakage assumptions.