

# Lecture 5: RC Filters. Series Resonance and Quality Factor. Matching. Soldering.

Review the following sections in your text:

1. Section 3.1 – Complex Numbers.
2. Section 3.2 – Exponential Function.
3. Section 3.3 – Phasors.
4. Section 3.4 – Impedance.

---

## RC Filters

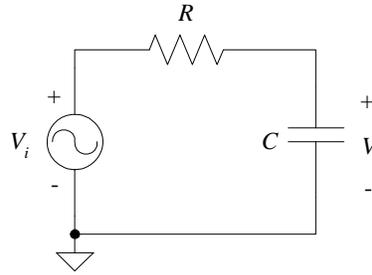
The **sinusoidal steady state** (aka time harmonic) is an extremely useful domain to work in. This is especially true for linear, analog electronics and narrowband communication circuits.

Advantages of sinusoidal steady state:

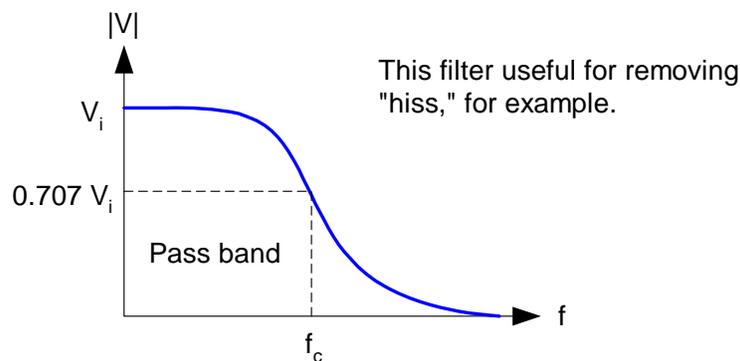
- Impedance can be used,
- Circuit analysis is simpler,
- Characterize filters, etc.

Let's consider two examples. Both involve series connections of a resistor and a capacitor.

**(1) Low pass filter** (see Fig. 3.4)



As the frequency of the input sinusoidal voltage changes, the magnitude (and phase) of the output voltage changes.



A low pass filter is used in the NorCal 40A near the transmitter output (which is also the receiver input). This low pass filter is called the Harmonic Filter, but it is not constructed from  $R$  and  $C$  elements. It uses  $L$  and  $C$  elements instead (see Ch. 5).

From the above circuit

$$V = \frac{Z_c}{Z_c + Z_R} V_i = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i = \frac{1}{1 + j\omega CR} V_i$$

With  $\tau = RC$ , then

$$V = \frac{1}{1 + j\omega\tau} V_i \quad (3.74),(1)$$

Now, the **cutoff frequency**  $\omega_c$  of a low- (or high-) pass filter occurs when

$$|V| = \frac{V_i}{\sqrt{2}}$$

This is also called the half-power frequency, for obvious reasons. From the denominator of (3.74), we see that  $\omega_c$  is

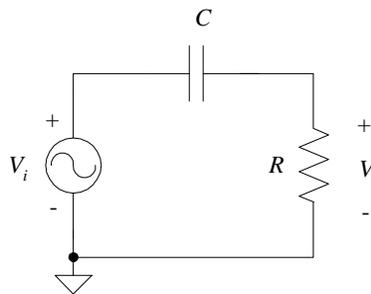
$$\omega_c \tau = 1 \quad \text{or} \quad \omega_c = \frac{1}{\tau} \quad [\text{rad/s}] \quad (3.76), (2)$$

Note these two special regions of operation for the low-pass filter:

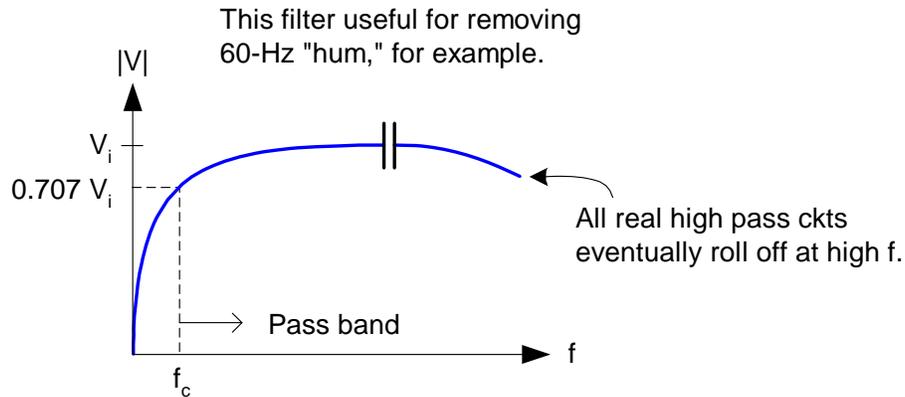
(a) Low  $f$ , where  $f \ll f_c$ . From (3.74),  $V \approx V_i$ ,

(b) High  $f$ , where  $f \gg f_c$ . From (3.74),  $V \approx \frac{V_i}{j\omega\tau}$ . (3.77), (3)

**(2) High pass filter** (see Fig. 3.5). Just interchange  $R$  and  $C$ .



The frequency response of this filter is shown below.



There are no high pass filters in the NorCal 40A, to the best of my knowledge.

From the above circuit, we can easily determine that

$$V = \frac{V_i}{1 + \frac{1}{j\omega\tau}}$$

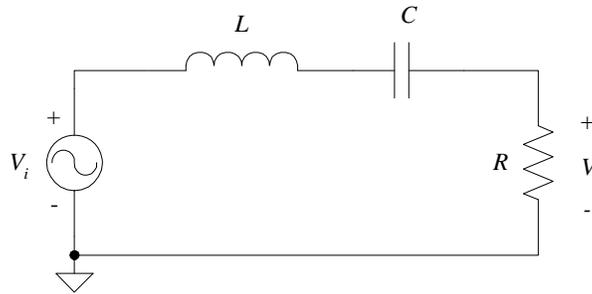
In the two special regions of operation for the high-pass filter:

- (a) Low  $f$ , where  $f \ll f_c$ ,  $V \approx j\omega\tau V_i$ ,
- (b) High  $f$ , where  $f \gg f_c$ ,  $V \approx V_i$ .

## Series Resonant Circuits

In addition to the low pass filter, another very common type of filter in communications electronics is the **bandpass** filter. (The opposite is the bandstop, or “notch,” filter.)

The series RLC circuit in Fig. 3.6 is one example of a bandpass filter.



In the frequency domain (sinusoidal steady state)

$$V = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} V_i = \frac{R}{R + j \underbrace{\left( \omega L - \frac{1}{\omega C} \right)}_{\equiv X}} V_i \quad (1)(4)$$

In the denominator  $X = \omega L - 1/\omega C$  is the sum of the (signed) reactances in the circuit. Notice that at

- (i) **low  $f$** ,  $|X|$  is large due to the capacitive reactance,
- (ii) **high  $f$** ,  $|X|$  is again large, but now due to the inductive reactance.

In either case,  $V$  in (1) will be small due to this large impedance.

However, in-between these extremes  $|X|$  can be small. In fact,  $X$  becomes zero at the special frequency  $\omega_0 = 2\pi f_0$ , which we can find from

$$X = 0 = \omega_0 L - \frac{1}{\omega_0 C} \quad \Rightarrow$$

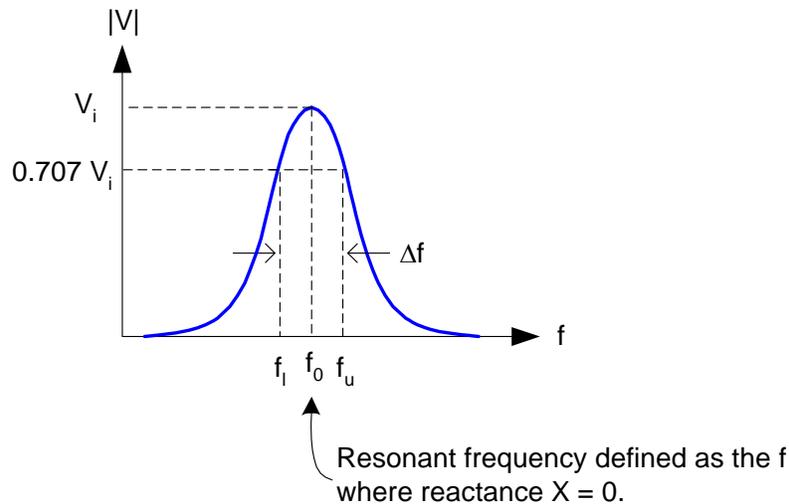
as  $\omega_0 = \frac{1}{\sqrt{LC}}$  [rad/s] and  $f_0 = \frac{1}{2\pi\sqrt{LC}}$  [Hz] (5)

This frequency  $f_0$  is called the **resonant frequency**.

At the resonant frequency  $f_0$ , from (1) we find that

$$V = V_i$$

Note that this is true regardless of  $R$ . (Not really that interesting, though, because we've ignored any Thévenin resistance in the source.)



Away from  $f_0$ ,  $|X|$  increases and  $|V|$  drops. There are two frequencies on either side of  $f_0$  where the **reactance magnitude  $|X|$  equals the resistance  $R$**  in the circuit. These frequencies  $f_u$  and  $f_l$  are the upper and lower **half-power frequencies**, as indicated in the figure above.

The ratio of the reactance of  $L$  (**or**  $C$ ) at the resonance frequency to the resistance in a series resonant circuit is called the **Quality Factor  $Q_s$**  (see p. 60 of the text):

$$Q_s = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} \quad (3.90),(6)$$

Also, as we've discussed in a previous lecture,

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$$

That is,  $Q$  is the ratio of the resonant frequency divided by the 3-dB frequency bandwidth.

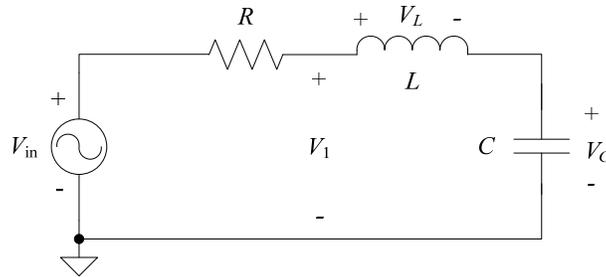
A series RLC circuit is used as the **RF Filter** in the NorCal 40A. (See Fig. 1.13 and the circuit on the front flap.) You'll start the construction of this filter in Prob. 8. Recall that in a superhet receiver, the  $Q$  of this filter doesn't need to be all that large to reject the VFO image (see Figs. 1.11 and 1.12).

The  $Q$  of discrete RLC circuits is typically less than 100. Remarkably, the  $Q$  of quartz crystals is on the order of 100,000. This is the "unloaded"  $Q$ , meaning the crystal is not connected to other circuit components.

---

## Reactive-Element Voltages in Resonant Circuits

It is very interesting to examine the **voltages across reactive elements** in a resonant circuit near the resonance frequency. For the sake of this investigation, we'll again look at a series resonant circuit:



The voltage across the capacitor is

$$V_C = \frac{1/(j\omega C)}{R + j\omega L + 1/(j\omega C)} V_{in} \quad (7)$$

At the resonance frequency  $\omega = \omega_0$  the sum of the  $L$  and  $C$  impedances are

$$j\omega_0 L + 1/(j\omega_0 C) = 0$$

Substituting this into (7) at the frequency  $\omega = \omega_0$  gives

$$V_C|_{\omega_0} = \frac{1}{j\omega_0 CR} V_{in} \quad (8)$$

Using the definition of  $Q_s$  in (6) for this series resonant circuit, (8) can be expressed as

$$V_C|_{\omega_0} = -jQ_s V_{in} \quad (9)$$

In a resonant circuit with  $Q_s > 1$ , we can see from (9) that the voltage magnitude across the capacitor will be **larger** than the input voltage magnitude! (Note that there are no active “amplifiers” in this circuit. The circuit is completely passive.)

In a high- $Q$  circuit, this voltage can become very large. Consequently, the reactive components in high- $Q$  circuits must be selected to withstand this voltage without damage.

We can use (9) as an **alternative definition of  $Q_s$** . That is,  $Q_s$  is the ratio of the capacitor and source voltage magnitudes at the resonant frequency. Further, since

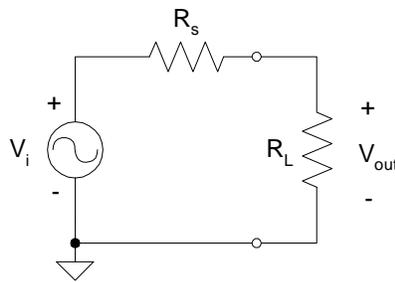
$$V_L|_{\omega_0} = -V_C|_{\omega_0} \quad (10)$$

then  $Q_s$  is also the ratio of the inductor and source voltage magnitudes at the resonant frequency.

Lastly, it is curious to note that because of (10),  $V_1$  in the above circuit equals **zero** at the resonant frequency.

## Matching Networks

As mentioned in Problem 7 (p. 66 of the text), it is useful to design amplifiers with **low output impedance**. Why? Consider this simple circuit



$$\text{With } V_{out} = \frac{R_L}{R_s + R_L} V_i \quad \text{then} \quad P_{out} = \frac{V_{out}^2}{R_L} = \left( \frac{1}{1 + \frac{R_s}{R_L}} \right)^2 \frac{V_i^2}{R_L}.$$

Now, note that if  $R_s \ll R_L$ , then

$$P_{\text{out}} \approx \frac{V_i^2}{R_L} \quad [\text{W}]$$

Very interesting. We see here that if the “output” resistance (the “look back” resistance) is small wrt the load, then the **output power is inversely proportional to  $R_L$** .

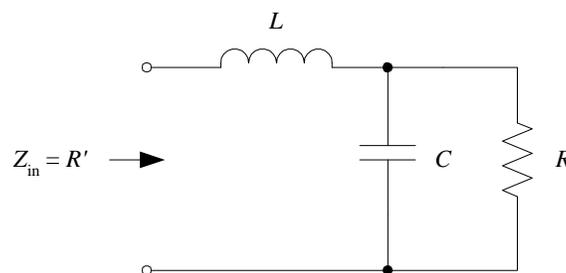
What’s the consequence of this? If  $P_{\text{out}} = 1 \text{ W}$  with  $R_L = 50 \ \Omega$ , for example, then we would expect to increase  $P_{\text{out}}$  to 10 W by lowering  $R_L$  to 5  $\Omega$ . This would only be true if  $R_s \ll 5 \ \Omega$ .

$$\text{Tinker toys} \Leftrightarrow \begin{cases} \text{high input impedance} \\ \text{low output impedance} \end{cases}$$

A **matching network** is used to transform an impedance from one value to another. One use of such a network would be to lower the impedance seen by an amplifier for increased output power.

Matching networks have many applications, many of which revolve around **impedance matching**.

A simple **L-matching network** is shown in Fig. 3.8 of Problem 7:



This network is used in Problem 14 (L4 and C14) for impedance matching between the IF Filter and Product Detector.

A disadvantage to this type of matching network is that is **narrow banded**. That is, it works well only within a small frequency band centered at the design frequency of the matching network.

---

In Problem 7, you will develop important equations that allow you to transform a two-element parallel circuit to an equivalent series one. You will use this later in the course.

As discussed in Problem 7, you can use “ $Q$ ” as a tool in this derivation. (Actually, I did not approach the problem this way, though the “ $Q$ ’s” will end up being equal.) Note that this is not a resonant circuit in this problem.

---

## **Soldering and Desoldering Tips**

You’ll begin soldering in this course starting with Problem 8 – Series Resonance. Here are some useful tips for soldering and the inevitable desoldering:

- ✓ Use the Weller soldering stations located in EEP 127.
- ✓ Wet the sponge.

- ✓ Adjust the soldering station to 700 °F. Wait for a flashing red LED before soldering.
- ✓ The iron is very hot! Be careful.
- ✓ Place the PCB in the Panavise. Solder on the bottom of the PCB.
- ✓ You can use the powered filters in the lab to help remove any lead vapors as you're soldering and desoldering.
- ✓ To solder:
  - Touch the tip of the soldering iron simultaneously to the pad on the PCB and the component lead. Wait a second or two, then touch a solder strand to the connection. The solder is thin, so you may need to melt a ¼ inch section or so.
  - Remove the solder strand.
  - Leave the iron to heat the joint for a moment or two.
  - Remove the iron.
  - The solder joint should be relatively shiny.
- ✓ Run the soldering tip across the wet sponge to clean it up.
- ✓ Don't leave the iron on for more than a few minutes. It will warm up quickly when you turn it back on.

- ✓ To desolder with braid, turn the board over to view the bottom. Place the braid over the joint and heat with the iron. The solder will be “soaked” up by the braid. Use needle nose pliers on the other side of the board to loosen the component. This takes practice. Trim off used braid with side cutters. (You can also use a desoldering pump.)
- ✓ Wash your hands when finished to remove any lead that may have accumulated on your hands.