

Lecture 35: Receiver Intermodulation and Dynamic Range.

On the opposite end from small signals (i.e., those near the noise floor), *strongly* received signals can also limit the performance of receivers.

Isn't this statement paradoxical? On the contrary: as we'll see in this lecture, two strong signals that are near to each other in frequency can create spurious receiver outputs.

These new spurs are produced when unintentional mixing occurs in a **receiver amplifier or mixer**. This unintentional mixing happens in the receiver "front end" (e.g., RF pre-amplifier and/or RF mixer) when:

1. Two or more signals are received that are close together in frequency, and
2. One or more of these signals is so strong that circuit components in the receiver front end behave nonlinearly in an unintended fashion.

Examples of this latter situation include large input signals that drive the semiconductor devices in a mixer into nonlinear behavior leading to unintended mixing, or a small-signal amplifier driven into nonlinear behavior so that it is no longer a linear amplifier.

As discussed next, it is possible in such a circumstance that unintended audio output signals will be produced, in addition to the intended signal. This is, by definition, a spurious output, or spur. These particular spurs are called **intermodulation products**.

It is important to realize that these spurs are **different** from those considered earlier in this course (i.e., image frequencies and mixer products).

Mathematics of Intermodulation Products

To understand the origin of **intermodulation products (IPs)**, consider the **Taylor series** expansion of a “weakly nonlinear” output voltage from the radio produced by the input voltage signal V_i

$$V = \underbrace{G_v V_i}_{\text{desired output}} + \underbrace{G_2 V_i^2 + G_3 V_i^3 + G_4 V_i^4 + G_5 V_i^5 + \dots}_{\text{IPs}} \quad (14.41)$$

The voltage gain coefficients G_v , G_2 , etc., are overall voltage gain coefficients for the entire radio receiver chain.

The IPs can occur when the input voltage, V_i , is the **sum of two** (or more) **strong signals** that are close together in frequency:

$$V_i = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t) \quad (14.42)$$

Let's choose $V_1 = V_2 = V$ and $\omega_1 < \omega_2$, as in the text.

We will now substitute (14.42) into (14.41) and expand each of the H.O. terms. We will assume that V_1 and V_2 are large enough that the H.O. terms in (14.41) are appreciable in size to $G_v V_i$.

- **Second-order products:**

$$V_i^2 = \left[V \cos(\omega_1 t) + V \cos(\omega_2 t) \right]^2 \quad (1)$$

Using *Mathematica*, we can symbolically expand and then simplify this expression:

```

Vi = V * Cos[ω1 * t] + V * Cos[ω2 * t] ;
Factor[TrigReduce[Vi]]

V (Cos[t ω1] + Cos[t ω2])

Factor[TrigReduce[Vi * Vi]]

1/2 V^2 (2 + Cos[2 t ω1] + Cos[2 t ω2] + 2 Cos[t ω1 - t ω2] + 2 Cos[t ω1 + t ω2])

```

From this result, we see that

$$V_i^2 = \frac{V^2}{2} \left[2 + \cos(2\omega_1 t) + \cos(2\omega_2 t) + 2\cos(\omega_1 t - \omega_2 t) + 2\cos(\omega_1 t + \omega_2 t) \right]$$

Now, let's imagine that we have two **strong input** signals at

$$f_1 = 7.030 \text{ MHz} \quad \text{and} \quad f_2 = 7.040 \text{ MHz}$$

as in Prob. 35. (These signals are within the intended bandwidth of the NorCal40A, but may not be precisely the intended input signal.) **IP spurs** should then be located at the RF Mixer output frequencies of

$$2f_1 = 14.060 \text{ MHz}$$

$$2f_2 = 14.080 \text{ MHz},$$

$$|f_1 - f_2| = 0.010 \text{ MHz}$$

$$f_1 + f_2 = 14.070 \text{ MHz}$$

All of these IP spurs are located far from the intended RF output of the mixer signal and would be well attenuated by the IF Filter.

- **Third-order products:**

$$V_i^3 = \left[V \cos(\omega_1 t) + V \cos(\omega_2 t) \right]^3 \quad (2)$$

Again using *Mathematica*:

```
Factor[TrigReduce[Vi * Vi * Vi]]
1/4 V^3 (9 Cos[t ω1] + Cos[3 t ω1] + 9 Cos[t ω2] + Cos[3 t ω2] +
3 Cos[t ω1 - 2 t ω2] + 3 Cos[2 t ω1 - t ω2] + 3 Cos[2 t ω1 + t ω2] + 3 Cos[t ω1 + 2 t ω2])
```

Using f_1 and f_2 defined above, then the **IP spurs** are located at the RF Mixer output frequencies of:

$$f_1 = 7.030 \text{ MHz (actually one fundamental)}$$

$$f_2 = 7.040 \text{ MHz (the other fundamental)}$$

$$3f_1 = 21.090 \text{ MHz}$$

$$3f_2 = 21.120 \text{ MHz}$$

$$2f_2 - f_1 = 7.050 \text{ MHz (very near input } f)$$

$$2f_1 - f_2 = 7.020 \text{ MHz (again, very near input } f)$$

$$2f_2 + f_1 = 21.110 \text{ MHz}$$

$$2f_1 + f_2 = 21.100 \text{ MHz}$$

The two spurs near the intended operational frequencies of the receiver are defined in the text as

$$f_{\underline{3}} = 2f_1 - f_2 \quad (14.37)$$

and

$$f_{\bar{3}} = 2f_2 - f_1 \quad (14.38)$$

These two particular third-order IPs are often troublesome because they can be close in value to the signal frequencies that created them (when $f_1 - f_2$ is small). If f_1 and f_2 lie near or within the passband of the RF Filter, then it is conceivable offending IPs at $f_{\underline{3}}$ and/or $f_{\bar{3}}$ **could pass through the IF Filter** if the receiver were tuned to these frequencies and spurious outputs will occur.

- **Fourth-order products:**

$$V_i^4 = \left[V \cos(\omega_1 t) + V \cos(\omega_2 t) \right]^4 \quad (3)$$

Again using *Mathematica*:

```
Factor[TrigReduce[Vi * Vi * Vi * Vi]]
1/8 V^4 (18 + 16 Cos[2 t ω1] + Cos[4 t ω1] + 16 Cos[2 t ω2] + Cos[4 t ω2] +
4 Cos[t ω1 - 3 t ω2] + 6 Cos[2 t ω1 - 2 t ω2] + 24 Cos[t ω1 - t ω2] + 4 Cos[3 t ω1 - t ω2] +
24 Cos[t ω1 + t ω2] + 4 Cos[3 t ω1 + t ω2] + 6 Cos[2 t ω1 + 2 t ω2] + 4 Cos[t ω1 + 3 t ω2])
```

Using f_1 and f_2 defined above, these IP spurs are located at:

$$2f_1 = 14.060 \text{ MHz}$$

$$2f_2 = 14.080 \text{ MHz}$$

$$4f_1 = 28.120 \text{ MHz}$$

$$4f_2 = 28.160 \text{ MHz}$$

$$3f_2 - f_1 = 14.090 \text{ MHz}$$

$$3f_1 - f_2 = 14.050 \text{ MHz}$$

$$3f_2 + f_1 = 28.150 \text{ MHz}$$

$$3f_1 + f_2 = 28.130 \text{ MHz}$$

$$|2f_1 - 2f_2| = 0.020 \text{ MHz}$$

$$2f_1 + f_2 = 21.110 \text{ MHz}$$

$$|f_1 - f_2| = 0.010 \text{ MHz}$$

$$f_1 + f_2 = 14.070 \text{ MHz}$$

None of these IPs are close to the receiver's input bandwidth, so they couldn't serve as an inadvertent input signal producing a spurious output.

- **Fifth-order products:**

$$V_i^5 = [V \cos(\omega_1 t) + V \cos(\omega_2 t)]^5 \quad (4)$$

You will determine these IP frequencies in Prob. 35. As stated in the text, the two fifth-order IP spurs that can cause trouble in the NorCal 40A are

$$f_{\underline{5}} = 3f_1 - 2f_2 \quad (14.39)$$

and

$$f_{\bar{5}} = 3f_2 - 2f_1 \quad (14.40)$$

In the example here using the f_1 and f_2 specified above, then

$$f_{\underline{5}} = 7.010 \text{ MHz} \quad \text{and} \quad f_{\bar{5}} = 7.060 \text{ MHz}$$

These two spurs are within the passband of the receiver. Together with $f_{\underline{3}}$ and $f_{\bar{3}}$, these signals **could pass through the IF Filter** if the receiver were tuned to any of them, producing spurious outputs. This is bad news!

- **Higher-order products:**

No other IP spurs are close to the input frequency, or generally do not have appreciable signal level.

Lastly, IP spurs are *always* present in a receiver. However, only when the input signals are sufficiently strong do the IPs rise above the noise floor and, perhaps, become large enough to cause audio output.

Are IP Spurs *Really* a Problem?

Intermodulation product spurs can be a real problem in a receiver. This is **probably truer today than it was 40 years ago**.

There are two primary reasons:

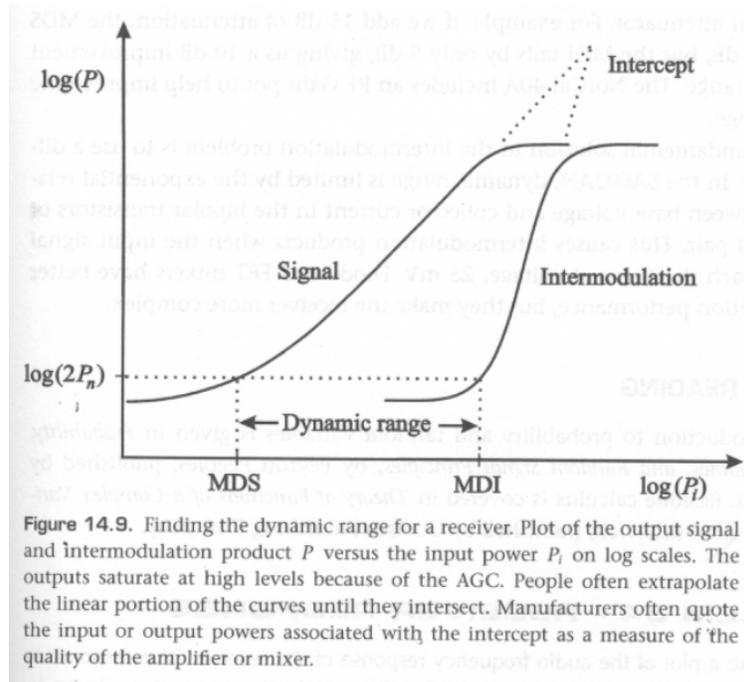
1. **Solid-state amplifiers** are easier to drive into nonlinear behavior than tube amplifiers. From (14.41), we see that IP spurs are due to nonlinear behavior.

- There are **more RF signals** today such as wireless PCS, radio stations, microwave datalinks, etc. If you are too close to a transmitter, your receiver “front end” may be driven to nonlinearity.

If you were concerned with jamming an adversary’s radio or radar, perhaps you could take advantage of IP spurs. How?

Effects of Intermodulation Products

The effects of intermodulation products are illustrated below in Fig. 14.9. It turns out that the slope of the “signal” and “intermodulation” plots are approximately equal to the order of the dominant IP. Interesting!



To understand this important and interesting last statement, first note that the time average signal power is expressed as

$$P = \frac{V_{rms}^2}{R} \quad (5)$$

Next, from the Taylor's series expansion used earlier in this lecture

$$V_{out} = \underbrace{G_v V_i}_{\text{desired output}} + \underbrace{G_2 V_i^2 + G_3 V_i^3 + G_4 V_i^4 + G_5 V_i^5 + \dots}_{\text{IPs}} \quad (14.41)$$

and using (14.42) we can deduce that for a particular IP of order n :

$$V_{n,rms} \propto V_{i,rms}^n \quad (6)$$

More specifically, this important fact can be verified from (1) above where $V \propto V^2$, from (2) above where $V \propto V^3$, etc.

Consequently, substituting (6) into (5) we find

$$P \propto \frac{(V_{i,rms}^n)^2}{R} = \frac{(V_{i,rms}^2)^n}{R} \quad (7)$$

For a log-log plot of this output power versus P_i , we need to rearrange (7) so that P_i is explicitly present:

$$\log(P) = \log \left[\frac{(V_{i,rms}^2)^n}{R} \frac{R^n}{R^n} \right] = \log \left[\left(\frac{V_{i,rms}^2}{R} \right)^n \right] + \log(R^{n-1})$$

Simplifying gives

$$\log(P) = n \log P_i + \log(R^{n-1}) \quad (8)$$

This is, of course, an equation for a straight line. The slope of this P versus P_i curve equals n , which is the order of the IP. Hence, we have proven this interesting conjecture that the slope of the signal and intermodulation curves in Fig. 14.9 are equal to the order of the intermodulation product ($n = 1$ for signal, and $n = 3$ for the NorCal 40A).

Dynamic Range

The **minimum detectible intermodulation input (MDI)** is the input signal that produces a total output signal $= 2P_n$ (signal + noise) for the *dominant* IP.

Then, by definition

$$\boxed{\text{Dynamic range} = \text{MDI} - \text{MDS} \text{ [W]}} \quad (14.47)$$

and is illustrated above in Fig. 14.9.

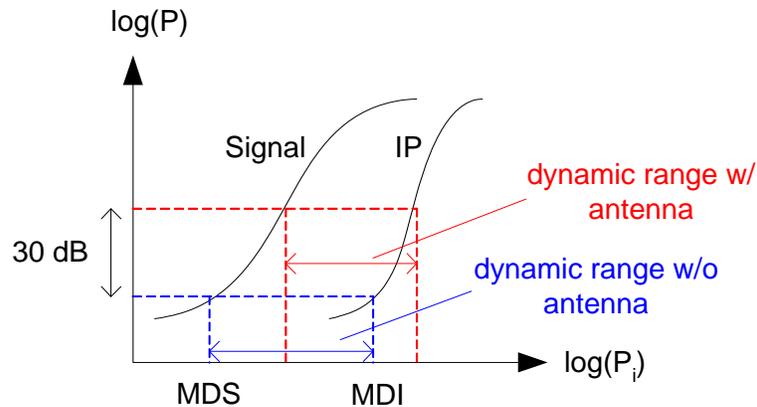
Dynamic range is:

- The range of useful input signal power levels for a receiver.
- Limited by noise for small signals and by receiver front-end nonlinearities for large signals.

Good receivers have a dynamic range ≈ 100 dB or so.

Effects of Antenna Noise on Dynamic Range

Antenna noise can have a marked effect on dynamic range. In the NorCal 40A, the antenna noise is approximately 30 dB above the receiver noise:



From this plot, with:

- the slope of the signal curve equal to 1:1 (linear power amplification), and
- the slope of the IP curve equal to 3:1 (dominant third-order IP),

then increasing the noise floor by 30 dB due to the antenna noise causes the:

1. MDS to increase by $30 \text{ dB} \cdot \frac{1}{1} = 30 \text{ dB}$
2. MDI to increase by $30 \text{ dB} \cdot \frac{1}{3} = 10 \text{ dB}$

Therefore, with the antenna attached to the receiver, the **dynamic range decreases by 20 dB!**

Interestingly, we can **retrieve some of the dynamic range** by introducing attenuation at the front end, though you will sacrifice receiver sensitivity and you may decrease the loudness of the signal.

For example, if we add 15 dB of attenuation at the front end of the receiver, then

1. MDS decreases by 15 dB ($= 15 \text{ dB} \cdot \frac{1}{1}$),
2. MDI decreases by 5 dB ($= 15 \text{ dB} \cdot \frac{1}{3}$).

Consequently, the dynamic range increases by $15 - 5 = 10$ dB. In practice, you can use your RF Gain pot to **improve dynamic range** if you need to.

Of course, the best way to increase dynamic range is with a better mixer design (lower noise, less susceptibility to IP). This would involve a more complicated design and likely more expense.