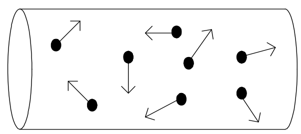
Lecture 34: Nyquist Noise Formula. Cascading Noisy Components. Noise Figure.

Due to thermal agitation of charges in resistors, attenuators, mixers, etc., such devices produce noise voltages and currents.

For example, in a resistor the charges move randomly due to thermal agitation:



As you know, applying a voltage across a resistor makes it warm. Conversely, heat in a resistor *produces* voltage and current in the resistor. We'll call these two quantities "noise voltage" and "noise current," respectively.

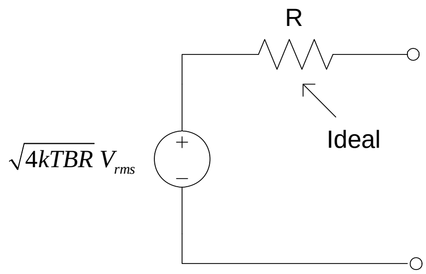
By this reasoning, we wouldn't expect much electrical noise to be generated in an inductor or capacitor.

The famous Nyquist noise formula states that the RMS thermal noise voltage generated by a "noisy" resistor is

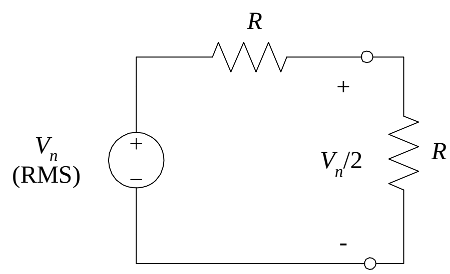
$$\boxed{V_n = \sqrt{4kTBR} \quad [\mathrm{V_{rms}}]} \qquad (14.20)$$

where k = Boltzman's constant = 1.38×10^{-23} J/K,
T = temperature in K,
B = bandwidth (taken has 1 Hz in the text), and
R = resistance (Ω).

From this formula, we can produce an equivalent circuit model for a "noisy resistor":



We can now ask: What is the maximum available noise power from this noisy resistor? To determine this, we'll attach a perfect (i.e., noiseless) resistor R to this circuit [Fig. 14.1(b)]:



The available noise power P_n may now be computed as

$$P_n = \frac{(V_n/2)^2}{R} \underbrace{=}_{(14.20)} \frac{\left(\sqrt{4kTBR}\right)^2}{4R}$$

or

$$\boxed{P_n = kTB \quad [\text{W}]} \tag{1}$$

Again, this P_n is the (maximum) available noise power from a noisy resistor.

From the definition of noise power density $P_n = NB$ in (14.5) and (1), we find

$$\boxed{N = \frac{P_n}{B} = kT \quad [\text{W/Hz}]} \tag{14.21}$$

which is the (maximum) available noise power density from a noisy resistor.

Note that this available noise power density in (14.21) is **NOT** dependent on the value of *R*! However, after careful thought this is perhaps not too surprising since we're dealing with the *maximum* power that is available.

Noise Temperature

This last formula (14.21) is so simple that it is often convenient to use *temperature* as a measure of noise power density as

$$T_e = \frac{N}{k} \quad [\mathrm{K}] \tag{14.22}$$

where T_e is the effective noise temperature. This is commonly done, even if the noise is not thermal in origin!

In the case of receivers, amplifiers, mixers, and attenuators, the noise temperature of the device is defined by dividing the equivalent <u>input</u> noise power density N_{input} by k as

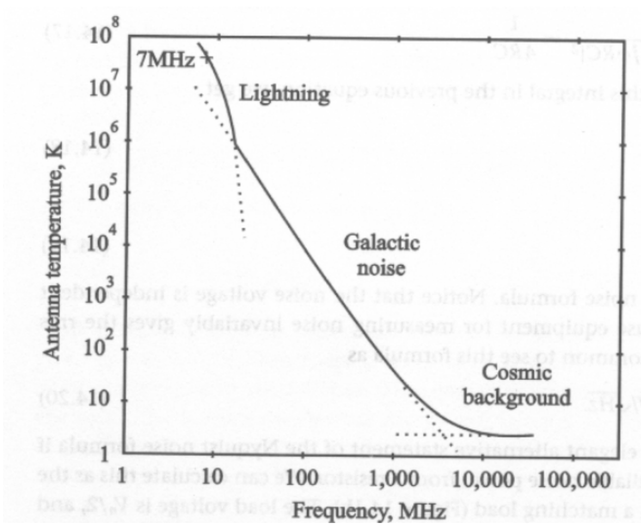
$$T_n = \frac{N_{\text{input}}}{k} = \frac{N}{kG} \tag{14.23}$$

But, with NEP = N/G then

$$\boxed{T_n = \frac{\text{NEP}}{k} \quad [\mathrm{K}]} \tag{14.23}$$

Note that if we are considering anything but a resistor, T_n is an **effective** temperature and has nothing to do with the physical environment.

It is also common to define an equivalent noise temperature for an antenna. Antennas actually produce very little noise themselves. Instead, they receive noise signals from natural and manmade sources (Fig. 14.2):



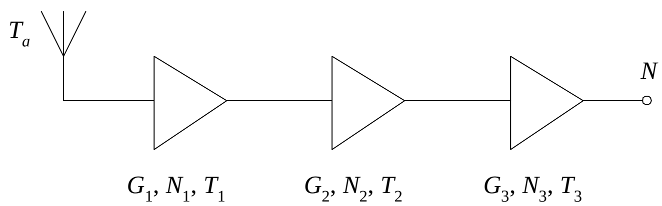
Cascading Noisy Components

When we connect parts of a receiver together, it's important to know the overall output noise power density as well as which subsections contribute most to this noise. Then we can design those portions of the circuit to reduce the output noise power.

If sources of noise in a receiver are "uncorrelated," then noise power from one section can simply be added to the next.

- Uncorrelated signals: random thermal variation is an example.
- Correlated signals: power supply fluctuations that simultaneously affect many subsystems is an example.

Fig. 14.4 shows an example of cascading noisy components:



This sample receiver consists of four subsystems: an antenna and three cascaded amplifiers.

With uncorrelated signals, the output noise power can found by adding the amplified noise powers from each stage:

$$P_{n,\text{out}} = P_{n,3} + P_{n,2} \cdot G_3 + P_{n,1} \cdot G_2 G_3 + P_{n,a} \cdot G_1 G_2 G_3$$

Dividing by the bandwidth of the system, we find

$$\frac{P_{n,\text{out}}}{B} = \frac{P_{n,3}}{B} + \frac{P_{n,2}}{B} \cdot G_3 + \frac{P_{n,1}}{B} \cdot G_2 G_3 + \frac{P_{n,a}}{B} \cdot G_1 G_2 G_3$$

Consequently, from this last expression and using the definition (14.5) that $P_n = N \cdot B$, we find

$$N = N_3 + N_2 G_3 + N_1 G_2 G_3 + \underbrace{kT_a}_{N_a} G_1 G_2 G_3 \qquad (14.28)$$

where N_a is the noise power density from the antenna. We can deduce from this expression that the output noise power density N is the sum of the amplified noise power densities (a sum since the noise contributions are uncorrelated).

Notice that the noise power density from the last stage (N_3) appears directly at the output. However, the noise power densities of all other stages are multiplied by the gain of succeeding stages.

In terms of an effective receiver noise temperature T_r, we can begin with:

$$T_n = \frac{N}{kG} \tag{14.23}$$

and $G = G_1 G_2 G_3$ to produce

$$T_r = \frac{N}{kG_1G_2G_3} \underbrace{=}_{(14.28)} \frac{1}{kG_1G_2G_3}\left(N_3 + N_2G_3 + N_1G_2G_3 + kT_aG_1G_2G_3\right)$$

or

$$T_r = T_a + \underbrace{\frac{N_1}{kG_1}}_{=T_1} + \underbrace{\frac{N_2}{kG_2}}_{=T_2}\frac{1}{G_1} + \underbrace{\frac{N_3}{kG_3}}_{=T_3}\frac{1}{G_1G_2}$$

Using (14.23) again, but only for each stage, we find that

$$\boxed{T_r = T_a + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1G_2} \quad \text{[K]}} \tag{14.29}$$

Notice that the noise temperatures of stages 2 and 3 are proportionally **reduced** by the gains of earlier stages.

Consequently, the receiver noise temperature *could* be dominated by the first stages in the chain of receiver subsystems if the gains of the following stages are appreciable.

As an example of this, we'll soon compute the noise temperature of the NorCal 40A.

Noise Figure

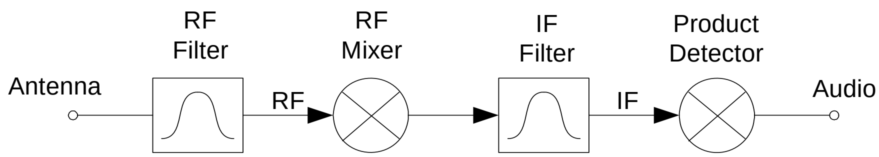
An alternative to noise temperature that is often used to quantify the noisiness of electrical components is the noise figure *F*.

By definition,
$$F = \frac{T_n}{T_0} + 1 \tag{14.30}$$
or
$$F = 10\log_{10}\left(\frac{T_n}{T_0} + 1\right) \ \text{[dB]}$$
where T_0 is a reference temperature, often 290 K.

For example, at 45 MHz from the SA602AN datasheet (p. 417)
$$F = 5.0 \text{ dB} \quad \Rightarrow \quad \frac{T_n}{T_0} + 1 = 3.162$$
or
$$T_n = 627 \text{ K} \tag{14.31}$$
which is the effective noise temperature of this active, double balanced mixer.

Noise Temperature of the NorCal 40A Receiver

As an application of this discussion on noise, we'll estimate the noise temperature of the NorCal 40A receiver, but only for the components shown in Fig. 14.5 (i.e., excluding the antenna):



- For the two mixers, the SA602AN datasheet specifies a gain of approximately 18 dB ($\Rightarrow G = 10^{18/10} = 63.1$) and a noise figure F = 5 dB ($\Rightarrow T_m = 627$ K).

- What about the filters? We'll assume a physical temperature of 290 K and a loss in the pass band of 5 dB ($\Rightarrow L = 10^{5/10} = 3.2$).

To compute the noise temperature of the filters, we need to assume that the losses in the passband are due to resistances in the filter. (Perhaps not completely true, but this will provide a worst-case scenario.)

In such a case, the filter in the passband acts as an attenuator. From Section 14.4 in the text, the noise temperature of an attenuator T_a is given as

$$\boxed{T_a = T\left(L-1\right)\ [\text{K}]} \tag{14.27}$$

where T is the physical temperature and L is the loss.

Using (14.27) for the two filters in Fig. 14.5, we find

$$T_a = 290\left(3.2-1\right) = 638\ \text{K}$$

Now, we are in a position to compute the noise temperature of the NorCal 40A. From (14.29), we start with T_1 and extend to a fourth stage

$$T_r = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \frac{T_4}{G_1 G_2 G_3}$$

Noting that $G_1 = G_3 = 1/L$, then the noise temperature of the NorCal 40A is approximately

$$T_r = T_{\text{RF Filter}} + T_{\text{RF Mixer}} L + T_{\text{IF Filter}} \frac{L}{G} + T_{\text{Prod Det}} \frac{L \cdot L}{G}$$

$$= T_f + T_m L + T_f \frac{L}{G} + T_m \frac{L^2}{G}$$

Now, with $T_f = T_a = 638$ K, $L = 3.2$, $T_m = T_n = 627$ K and $G = 63.1$ then

$$T_r = 638 + 627 \cdot 3.2 + 638 \frac{3.2}{63.1} + 627 \frac{3.2^2}{63.1}$$

or

$$\boxed{T_r = \underbrace{638 + 2006}_{\text{dominate terms}} + 32 + 102 = 2{,}778\ \text{K}} \tag{14.32}$$

From this last result we can deduce a very important fact: the receiver noise is wholly dominated by the noise generated by the RF Mixer (2,006 K) and the RF Filter (638 K).

Actually, once the receiver is connected to the antenna, you'll see that the noise temperature of 2,778 K is much, much smaller than the noise temperature of the antenna.