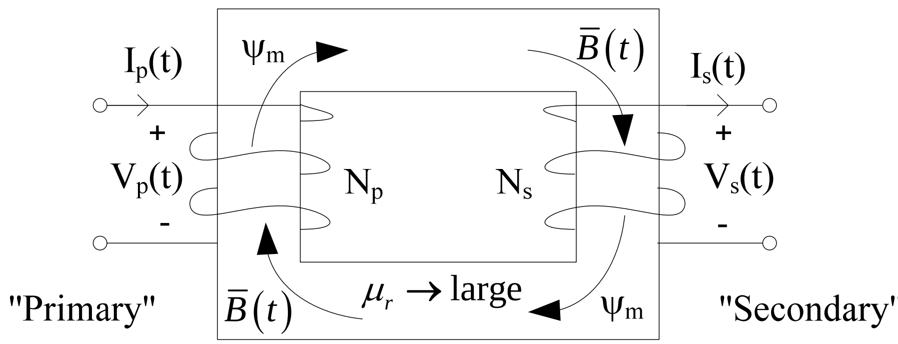
Lecture 14: Transformers. Ideal Transformers

In general, a transformer is a multi-port ac device that converts voltages, currents, and **impedances** from one value to another. This device only performs this transformation for time varying signals.

Here, we will consider the transformer circuit shown below:



The time varying current $I_p(t)$ in the "primary" circuit produces a magnetic flux density $\bar{B}_p(t)$ in and around coil *p*. Similarly, the "secondary" coil current $I_s(t)$ produces $\bar{B}_s(t)$. The total magnetic flux density is the sum $\bar{B}(t) = \bar{B}_p(t) + \bar{B}_s(t)$.

We will assume that the transformer core has a very large relative permeability μ_r. Consequently, $\bar{B}(t)$ will almost exclusively be contained within the core. This $\bar{B}(t)$ forms closed loops within the core. (We can think of this as a "self-shielded" core.)

The magnetic flux ψ_m is defined simply as the integral of $\bar{B}(t)$ over a cross section of the core:

$$\begin{aligned}\psi_m(t) &= \int_s \bar{B}(t)\cdot d\bar{s} = \int_s \left[\bar{B}_p(t) + \bar{B}_s(t)\right]\cdot d\bar{s} \\ &= \psi_{m,p}(t) + \psi_{m,s}(t)\end{aligned} \qquad (1)$$

With $\bar{B}(t)$ contained exclusively within the core, $\psi_m(t)$ will be the same throughout the transformer (though it will vary with time).

Assuming $\bar{B}(t)$ doesn't vary significantly over a cross section of the core, the magnetic flux will be proportional to the number of (identical) coil turns, the geometry of the coil, and the current in the coil:

$$\psi_{m,j}(t) = N_j A_l I_j(t) \quad \text{[Wb/turn]} \qquad (6.5),(2)$$

A_l is the inductance constant [H/turn^2] of the core and $j = p, s$.

This A_l is provided by the manufacturer of the cores that you use for your transformers (and inductors). Table D.2 (p. 356) in your text lists A_l for various cores used in the NorCal 40A.

Table D.2. The Cores in the NorCal 40A Transceiver.

Core	A_l, nH/turn^2	Q	ppm/°C	Material	Paint	Use
T37-2	4.0 (28 turns)	170	+100	iron powder	red	filter
T68-7	5.0 (60 turns)	200	+50	iron powder	white	oscillator
FT37-43	160 (14 turns)	1	−30	nickel–zinc ferrite	orange spot	transformer
FT37-61	66 (1 turns)	50	+500	nickel–zinc ferrite	none	tuned transformer

Note: There is a lot of information in the core number itself. For example, in FT37-43, "F" indicates a ferrite, "T" a toroidal core, "37" the outside diameter in hundredths of an inch, and "61" the manufacturing recipe. These are our measured values for A_l, Q, and the temperature coefficient. The values of A_l vary ±10% from lot to lot. All measurements are at 7 MHz, except for the T68-7 core, which is at 2 MHz. The values vary with frequency, and if the specific value of the inductance is critical, you should measure the inductance constant at the frequency you are interested in. Temperature coefficients are given for ferrites for completeness, but it is a poor idea to use ferrites in an application where temperature stability is important, because characteristics differ greatly over even a modest range of temperature.

Note that A_l can be a very strong function of frequency.

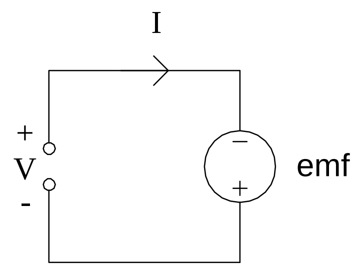
Induced Voltage

As we know, a time varying magnetic field through a coil of wire produces (or "induces") a voltage between the ends of the coil. This miraculous phenomenon was discovered and quantified by Michael Faraday and others and is mathematically stated in Faraday's law as

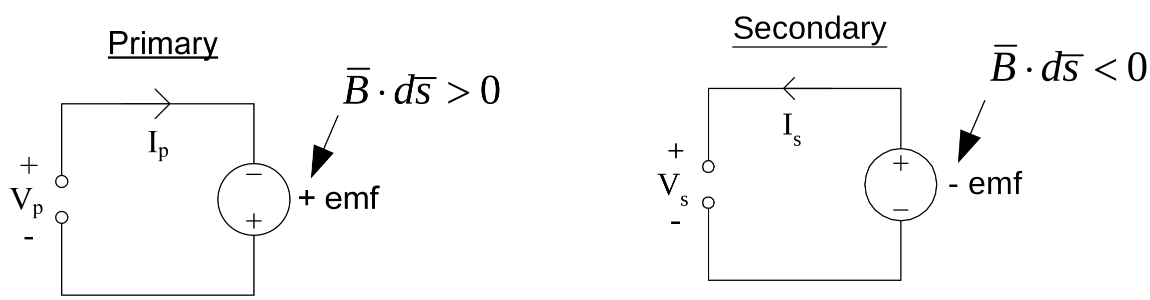
$$emf = -N \frac{d\psi_m}{dt} \quad [\text{V}] \tag{3}$$

where N is the number of (identical) turns of the coil.

This *emf* is a "net push" around a circuit that causes electrons to move. Voltage and *emf* are closely related concepts. We can determine the induced voltage $V(t)$ using the following equivalent circuit:



Applying this equivalent circuit to each side of the transformer shown on the first page gives:



The polarity of the lumped *emf* source is set by the direction of the current: a voltage source has current entering the negative terminal. The sign of the *emf* source is due to the direction of $d\bar{s}$ (by the RHR) and the assumed direction for $\bar{B}$ (and hence ψ_m).

From these equivalent circuits and applying (3), the sinusoidal steady state voltage at the primary and secondary are **both** of the form

$$V = j\omega N\psi_m \qquad (6.6),(4)$$

where V and ψ_m are now phasors. (Note carefully that both have the same sign.) Specifically, the primary and secondary voltages are

$$V_p = j\omega N_p \psi_m \quad (6.9),(5)$$
$$V_s = j\omega N_s \psi_m \quad (6.10),(6)$$

Dividing these two equations gives

$$\boxed{\frac{V_p}{V_s} = \frac{N_p}{N_s} = n} \quad (7)$$

where n is called the turns ratio.

Interestingly, we see here that the "output" voltage V_s can be different in amplitude than the "input" voltage V_p

$$V_s = \frac{N_s}{N_p} V_p \quad (6.11),(8)$$

Note that if $N_s > N_p$, the secondary voltage is larger in amplitude than the primary voltage. Very interesting.

- If $N_s > N_p$, called a **step-up** transformer,
- If $N_s < N_p$, called a **step-down** transformer.

Primary and Secondary Currents

Next we will consider the electrical currents in the primary and secondary of the transformer. From (1), the magnetic flux is the sum of the two magnetic fluxes from each coil

$$\psi_m = \psi_{m,p} + \psi_{m,s} \quad (9)$$

Using (2) and noting that $\psi_{m,s}$ will be negative since the direction of the current is assumed OUT of the secondary, then

$$\psi_m = N_p A_l I_p - N_s A_l I_s \quad (6.12),(10)$$

Solving for I_p we find that

$$I_p = \frac{\psi_m}{N_p A_l} + \frac{N_s}{N_p} I_s \qquad (6.13),(11)$$

The magnetic flux ψ_m is not a circuit quantity. To derive an equivalent circuit for the transformer we need to express ψ_m in terms of electrical circuit quantities.

To accomplish this, we use (5) in the first term of (11) yielding

$$\frac{\psi_m}{N_p A_l} = \frac{V_p}{j\omega \underbrace{N_p^2 A_l}_{L_p}} \qquad (12)$$

where

$$L_p \equiv N_p^2 A_l \qquad (6.23),(13)$$

is the inductance of the primary coil. Substituting this result (12) back into (11) gives

$$I_p = \underbrace{\frac{V_p}{j\omega L_p}}_{\text{magnetization}} + \underbrace{\frac{N_s}{N_p} I_s}_{\text{transformer}} \quad \text{[A]} \qquad (6.14),(14)$$

This last result is extremely illuminating. We see that the current in the primary is the sum of two parts: (1) magnetization current and (2) transformer current.

(1) **Magnetization Current**. The first term in (14) does not involve the secondary in any way. In other words, this is the current the transformer would draw regardless of the turns ratio of the transformer.

(2) **Transformer Current**. The second term in (14) directly depends on the secondary because of the N_s term. This component of the primary current is a transformed secondary current, in a manner similar to the voltage in (7), though inversely.

Ideal Transformer

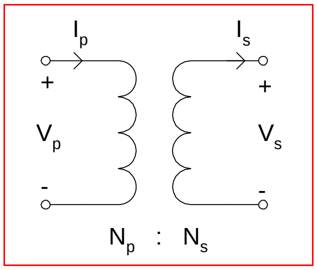
If the magnetization current $V_p/(j\omega L_p)$ in (14) is very small in magnitude relative to the transformer current $(N_s/N_p)I_s$ then such a device is called an ideal transformer. The equations for an ideal transformer are from (7) and (14):

$$V_s = \frac{N_s}{N_p} V_p \qquad (6.15),(15)$$

and

$$I_s = \frac{N_p}{N_s} I_p \qquad (6.16),(16)$$

The circuit symbol for an ideal transformer is



Discussion

1. We can surmise from (16) that for an ideal step-up transformer $I_s < I_p$. Therefore, while from (15) the voltage increases by N_s / N_p, the current has decreased by N_p / N_s.

   In the NorCal 40A, the transformer T1 is used to step up the current from the Driver Amplifier to the Power Amplifier. For T1, $N_p = 14$ and $N_s = 4$ so that $I_s = (N_p / N_s) I_p = 7/2 \cdot I_p$.

   Because of this current behavior, the power input to the primary equals the power output from the secondary:
   $$P_p(t) = V_p(t) I_s(t) \tag{17}$$
   $$P_s(t) = V_s(t) I_s(t) = \underbrace{\frac{N_s}{N_p} V_p(t)}_{(14)} \cdot \underbrace{\frac{N_p}{N_s} I_p(t)}_{(15)} = V_p(t) I_p(t) \tag{18}$$
   Therefore, the input power $P_p(t)$ equals the output power $P_s(t)$, as would be expected.

2. With an impedance Z_s connected to the secondary, then
   $$\frac{V_s}{I_s} = Z_s$$
   Substituting for V_s and I_s in this equation using (15) and (16)
   $$\frac{(N_s / N_p) \cdot V_p}{(N_p / N_s) \cdot I_p} = Z_s$$

or
$$\frac{V_p}{I_p} = \left(\frac{N_p}{N_s}\right)^2 Z_s$$

In other words, the **effective input impedance** $Z_{p,\text{eff}}$ at the primary terminals (the ratio V_p/I_p) is

$$Z_{p,\text{eff}} = \left(\frac{N_p}{N_s}\right)^2 Z_s \quad [\Omega] \qquad (6.19),(19)$$

The ideal transformer "transforms" the load impedance from the secondary to the primary. (Remember that this is only true for sinusoidal steady state signals.)

3. For maximum power transfer, we design a circuit so that the load is matched to the output resistance. We can use transformers as matching networks.

   For example, in the NorCal 40A, **if** T3 were an ideal transformer, it could be used to transform the output impedance from the RF Mixer (3 kΩ) to match the input impedance of the IF Filter (200 Ω). Using (19):

$$Z_s = \left(\frac{N_s}{N_p}\right)^2 Z_p = \left(\frac{6}{23}\right)^2 3000 = 204.2\,\Omega$$

   which is *very* close to the desired 200 Ω.

   But T3 is **not** an ideal transformer! What to do…☺