

Lecture 12: Bandpass Ladder Filters. Quartz Crystals

In addition to low- and high-pass filters as discussed in the previous lecture, ladder filters can also be used to construct higher-order **bandpass and bandstop** (i.e., “notch”) filters.

Circuit diagrams for a four element (i.e., second order) bandpass and bandstop filters are shown below.

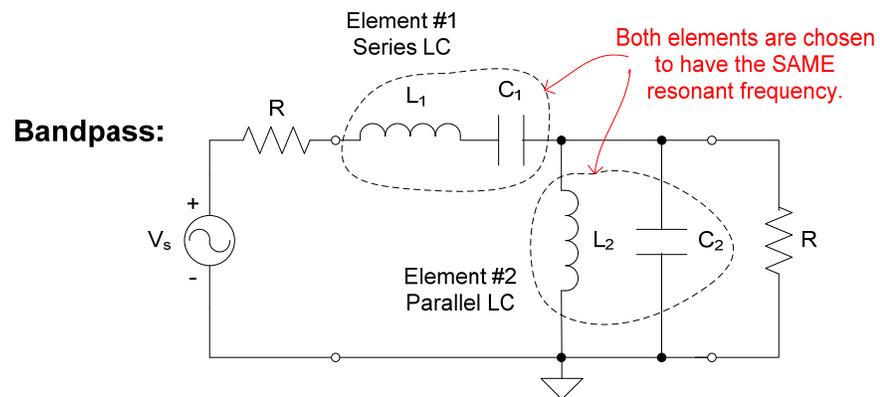


Fig. 5.7(a):

For bandstop filters, simply interchange the two sections:

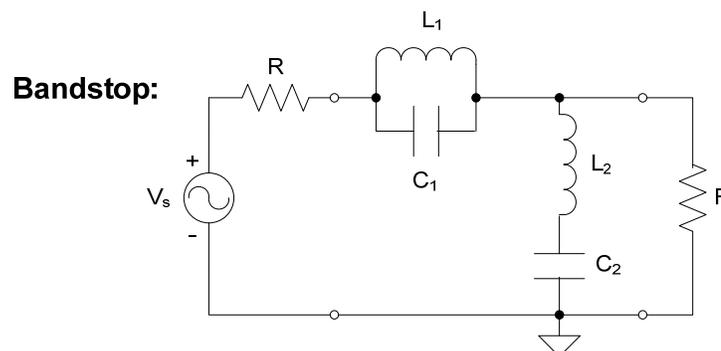


Fig. 5.7(b):

Amazingly, these filters can be **synthesized using the same filter tables** we used for low- and high-pass ladder filters. However, there are some important differences.

The design equations for a **bandpass filter** are listed below in equations (1)-(6) (ref. D. M. Pozar, *Microwave Engineering*) and are not given in your text. One begins using a filter table for a low pass prototype filter design and with those immittance coefficients, a_k , uses (1)-(6) to calculate the inductor and capacitor values needed for both the series LC and parallel LC sections of the bandpass filter:

Series LC combination with elements

$$L_k = \frac{a_k R}{\omega_g \Delta} \quad (\text{series element 1 of 2}) \quad (1)$$

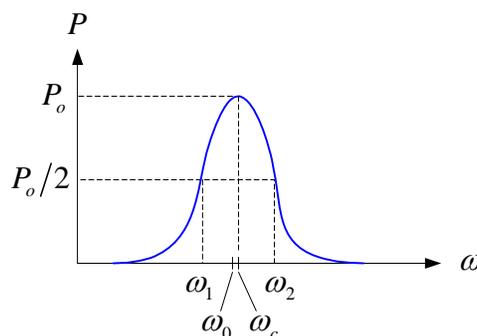
and

$$C_k = \frac{\Delta}{a_k R \omega_g} \quad (\text{series element 2 of 2}) \quad (2)$$

where

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_g} \quad (3)$$

is the fractional bandwidth of the passband:



Notice that ω_g in (1)-(3) is **not** the center frequency ω_c . Rather, it is defined as

$$\omega_g = \sqrt{\omega_1 \omega_2} \quad (4)$$

which is the geometric mean of ω_1 and ω_2 , rather than the more common arithmetic-mean definition. This was done to purportedly make the design equations simpler.

Parallel LC combination with elements

$$C_k = \frac{a_k}{R \omega_g \Delta} \quad (\text{shunt element 1 of 2}) \quad (5)$$

$$L_k = \frac{R \Delta}{a_k \omega_g} \quad (\text{shunt element 2 of 2}) \quad (6)$$

Example N12.1: Design a second order Butterworth bandpass filter (Fig. 5.7a) with a center frequency of 7 MHz and a 3-dB bandwidth of 750 kHz. For this design, assume the impedance level seen by the filter is 50 Ω .

From Table 5.1 for the design of the Butterworth low pass prototype filter and $n = 2$, $a_1 = a_2 = \sqrt{2}$. Using (4)

$$\begin{aligned} f_g &= \sqrt{\left(7 \times 10^6 + \frac{750}{2} \times 10^3\right) \left(7 \times 10^6 - \frac{750}{2} \times 10^3\right)} \\ &= 6.9899 \times 10^6 \text{ Hz} \approx f_0 \end{aligned}$$

With $\Delta f = 750$ kHz, then from (3)

$$\Delta = \frac{f_2 - f_1}{f_g} = \frac{750 \times 10^3}{6.9899 \times 10^6} = 0.1073$$

From (1) and (2):

$$L_1 = \frac{a_1 R}{\omega_g \Delta} \stackrel{(3),(4)}{=} \frac{\sqrt{2} \cdot 50}{\Delta \omega} = \frac{\sqrt{2} \cdot 50}{2\pi \cdot \Delta f} = \frac{\sqrt{2} \cdot 50}{2\pi \cdot 750 \times 10^3} = 15.0 \mu\text{H}$$

$$C_1 = \frac{\Delta}{a_1 R \omega_g} = \frac{0.1073}{\sqrt{2} \cdot 50 \cdot 2\pi \cdot 6.9899 \times 10^6} = 34.6 \text{ pF}$$

Referring to the NorCal 40A circuit on the inside flap of your text, these values correspond closely with L1 and C1 in the circuit, noting that C1 is a variable capacitor covering this range. (In Prob. 8 you will build and test this series LC network portion of the RF Filter.)

Continuing, we can calculate the shunt elements for the second order bandpass filter in Fig. 5.7(a) using (3) and (4):

$$C_2 = \frac{a_2}{R \omega_g \Delta} \stackrel{(3),(4)}{=} \frac{\sqrt{2}}{50 \cdot \Delta \omega} = \frac{\sqrt{2}}{50 \cdot 2\pi \cdot \Delta f} = \frac{\sqrt{2}}{50 \cdot 2\pi \cdot 750 \times 10^3} = 6.00 \text{ nF}$$

$$L_2 = \frac{R \Delta}{a_2 \omega_g} = \frac{50 \cdot 0.1073}{\sqrt{2} \cdot 2\pi \cdot 6.9899 \times 10^6} = 86.4 \text{ pF}$$

This completes the design of a second order Butterworth bandpass filter.

The method for designing bandpass filters described in your text is different than, but equivalent to, the procedure described above and used in Example N12.1.

Here is the procedure for a **bandpass filter** described in the text:

1. The filter tables are used to compute the series inductances and shunt capacitances as we did with the low-pass filter.
2. When un-normalizing, use the 3-dB bandwidth $\Delta\omega$ rather than ω_c to find the L 's and C 's.
3. Finally, compute the series C 's and parallel L 's using the resonant frequency condition $\omega_0 = 1/\sqrt{LC}$.

Example N12.2: Here we will perform a second design of the RF Filter (second order Butterworth bandpass filter) from the previous example, but we will use L1 and C1 as one section (as is done on pp. 104-105 of the text). We require that $f_0 = 7$ MHz and $R = 50 \Omega$.

With L1 and C1 specified, we have one half of the filter already:

$$L_1 = 15 \mu\text{H}, C_1 = \left(\omega_0^2 L_1\right)^{-1} = 34.5 \text{ pF}.$$

From Table 5.1 and $n = 2$, $a_1 = \sqrt{2}$. Recall that

$$a_1 = \frac{X_L}{R} = \frac{L\Delta\omega}{R}$$

Consequently, $\Delta\omega = a_1 R / 15 \times 10^{-6} = 4.71 \times 10^6$ rad or $\Delta f = \Delta\omega / 2\pi = 750$ kHz. This Δf is the 3-dB bandwidth of the filter. (At 7 MHz, this Δf is somewhat large, meaning this is a relatively low- Q filter.)

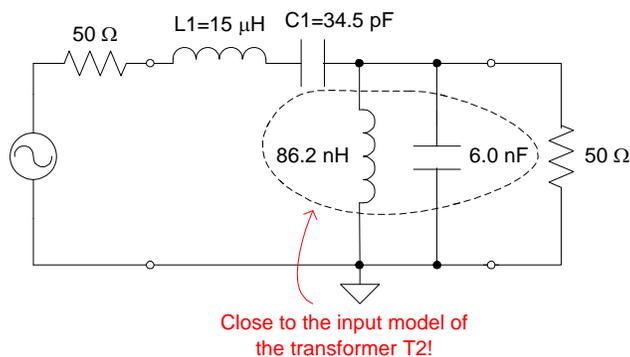
Next, using the filter table again, we will determine the shunt C_2 from

$$a_2 = \sqrt{2} = C_2 R \Delta\omega \quad \text{or} \quad C_2 = \frac{a_2}{R \Delta\omega} = \frac{\sqrt{2}}{50 \cdot 4.71 \times 10^6} = 6.00 \text{ nF.}$$

Finally, using $\omega_0 = 1/\sqrt{LC}$ then

$$L_2 = \frac{1}{\omega_0^2 C_2} = 8.62 \times 10^{-8} \text{ H} = 86.2 \text{ nH.}$$

The complete filter design is shown below (Fig. 5.8):



These values of L_1 , C_1 , L_2 , and C_2 are exactly the same as calculated in the previous example, as expected.

While these examples may have seemed to be just exercises, believe it or not, the actual RF Filter in the NorCal 40A is a **two-element** Butterworth bandpass filter! L_1 and C_1 are the series L and C , but where is the parallel L and C ?

This **second section is provided by the primary winding of T2** (≈ 66 nH). The transformer also transforms the impedance of C_2 to the primary side (you'll see this later in Lecture 15). Consequently, $C_p \lesssim 20$ nF or so. That's just what's needed for this second order Butterworth bandpass filter!

You'll see more with this aspect of the RF Filter in Prob. 16 when you construct and install T2.

Quartz Crystals

The maximum Q (minimum bandwidth) of LC ladder filters is **usually limited by the inductor “ Q ”** (i.e., the inductor losses).

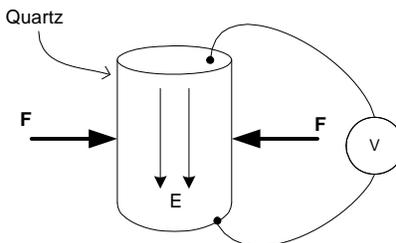
Other types of resonant elements must be used if Q 's higher than a few hundred are desired. Such Q 's are useful from audio to microwave frequencies.

Quartz crystals are one such element. These are made from silicon dioxide, which is cheap. The Q 's of such quartz crystal resonators range from 25,000 to 150,000!

Another advantage is that a **small temperature coefficient** can be obtained for quartz crystals. This is useful so that the resonant frequency drift with temperature is minimized as the transceiver warms up, or in other situations.

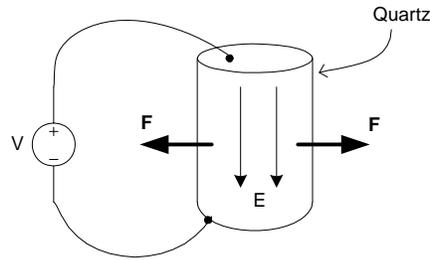
Quartz crystals make good electrical resonators because of the **piezoelectric effect**. This effect is a combination of a mechanical vibration and bound electric charges.

When a quartz crystal is squeezed, a voltage is produced:

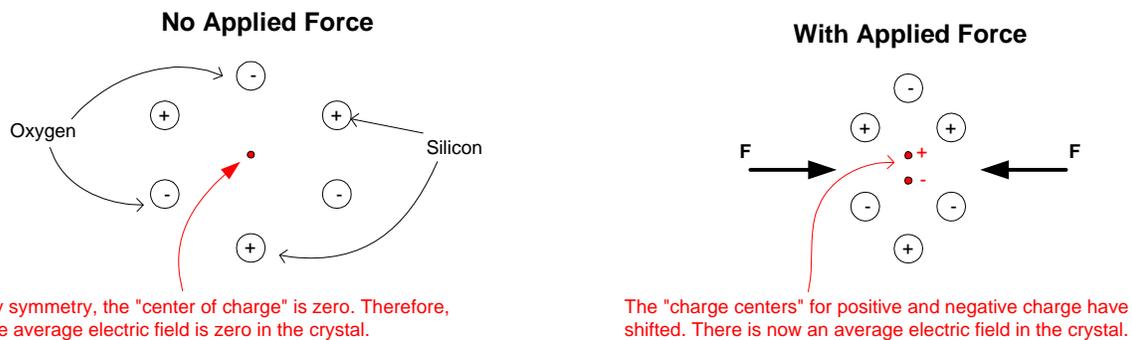


Electric starters for gas furnaces, water heaters, and grills use such a piezoelectric effect.

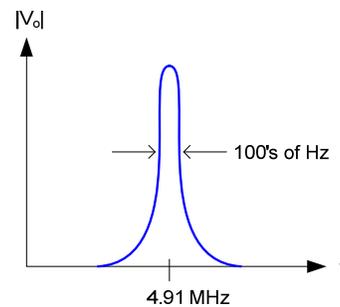
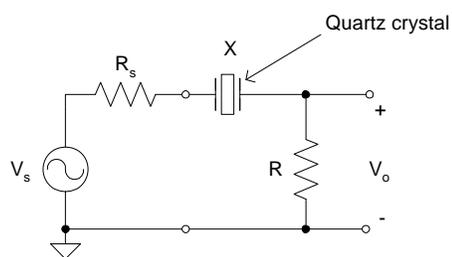
This piezoelectric effect also works the other way. **A voltage applied across a quartz crystal causes a small expansion of the crystal** (Fig. 5.12):



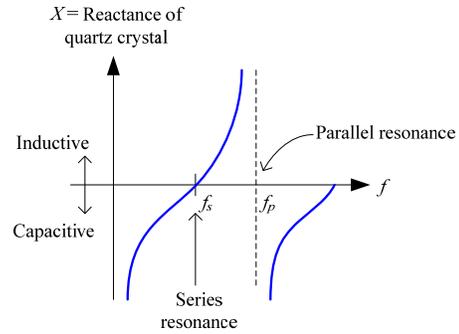
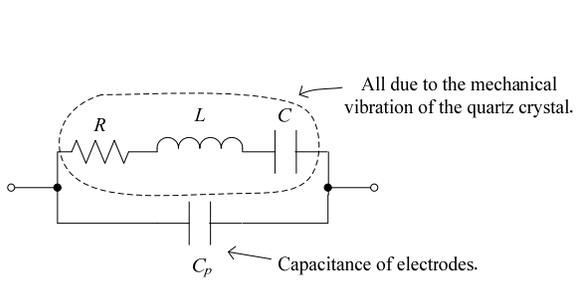
A microscopic view of the atoms in the quartz lattice helps us understand this piezoelectric effect (Fig. 5.12):



These quartz crystals can be modeled as RLC filters in electrical circuits:



The equivalent electrical circuit for the quartz crystal is



You will operate your quartz crystals in **series resonance** in the NorCal 40A.

Why is there variation with frequency? Because the mechanical vibrations of the lattice will not be as favorable for all frequencies of voltage excitation. At some frequencies, the lattice vibrations are maximum.