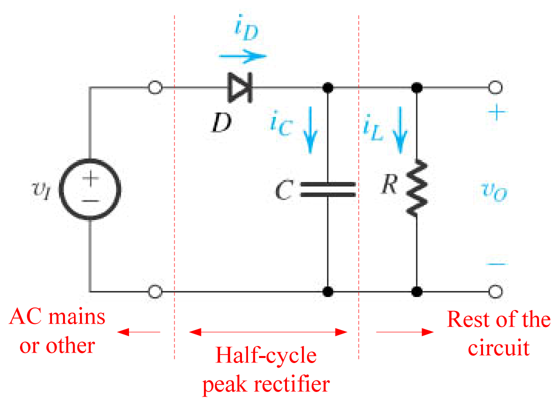
Lecture 8: Peak Rectifiers.

The output of the rectifier circuits discussed in the last lecture is pulsating significantly with time. Hence, it's not useful as the output from a DC power supply.

One way to reduce this ripple is to use a filtering capacitor.

Consider the half-cycle rectifier again, but now add a capacitor in parallel with the load:

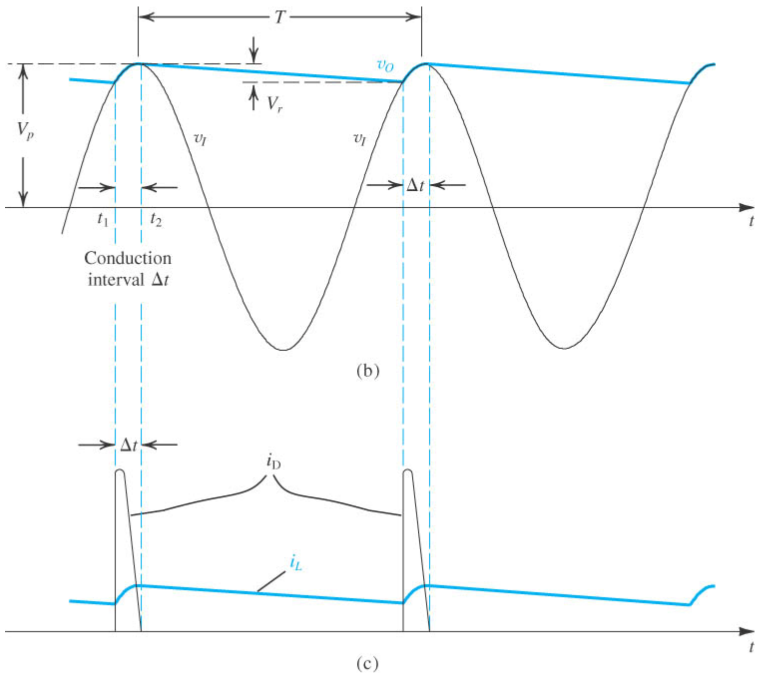


(Fig. 4.27a)

We expect that as soon as we turn on the source, the capacitor will charge up on "+" cycles of v_I and discharge on the "-" cycles.

To smooth out the voltage, we need this discharge to occur slowly in time. This means we need to choose C large enough to make this happen, presuming that R is a given quantity (the Thévenin resistance of the rest of the circuit).

The output voltage v_O will then be a smoothed-out signal that pulsates with time:

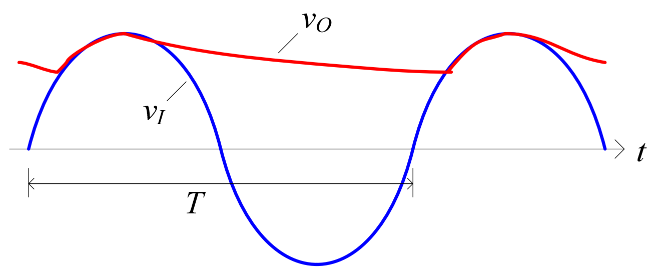


(Fig. 4.27)

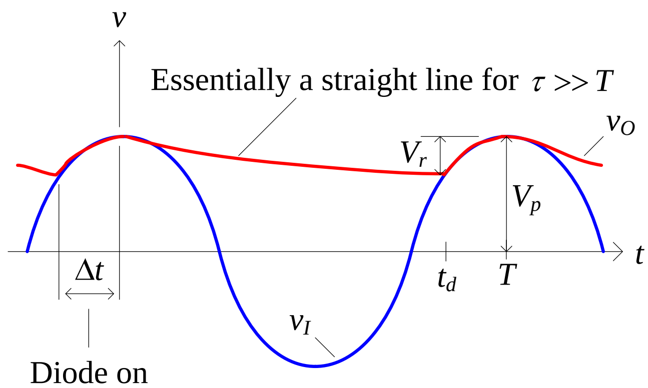
Notice the diode current and the capacitor voltage. They display behavior much different than what one would find in an AC circuit.

Analysis of Peak Rectifier Circuits

We'll require that $\tau = RC \gg T$, which means that the time constant of the RC circuit must be much greater that the period of the input sinusoidal signal:



Now, our quest is to approximately determine the ripple voltage V_r, assuming $\tau \gg T$:



Not sketched to scale.

When D is off, and assuming it is an ideal diode

$$v_O(t) = V_p e^{-t/\tau} \tag{1}$$

[If D is not ideal then $v_O(t) \approx (V_p - 0.7)e^{-t/\tau}$.]

At the end of the discharge time, t_d, the output voltage equals

$$v_O(t_d) = V_p - V_r \tag{2}$$

Substituting for v_O from (1) at this time t_d leads to

$$V_p e^{-t_d/\tau} = V_p - V_r \quad \text{or} \quad \frac{V_r}{V_p} = \left(1 - e^{-t_d/\tau}\right) \tag{3}$$

This equation has the two unknowns V_r and t_d, assuming τ is known. If we can determine t_d, then we can find V_r. Finding t_d can be done numerically by equating (1) to the expression for the input voltage

$$v_I(t) = V_p \cos(\omega t) \tag{4}$$

and solving for the time t_d when the two are equal as

$$V_p \cos(\omega t_d) = V_p e^{-t_d/\tau} \quad \text{or} \quad \cos(\omega t_d) = e^{-t_d/\tau} \tag{5}$$

This needs to be done numerically since (5) is a "transcendental equation."

Alternatively, if Δt is small compared to T (true when $\tau \gg T$, as assumed), then from (3)

$$\frac{V_r}{V_p} = 1 - e^{-(T-\Delta t)/\tau} \approx 1 - e^{-T/\tau} \tag{6}$$

Again, because $\tau \gg T$ then we can truncate the series expansion of the exponential function to two terms (see Lecture 4) giving

$$\boxed{\frac{V_r}{V_p} \approx \frac{T}{\tau}} \quad (\tau \gg T) \tag{7}$$

This simple equation gives the ratio of the ripple voltage to the peak voltage of the input sinusoidal signal for the half-cycle rectifier. It's worth memorizing, or knowing how to derive.

Often R and T are fixed quantities. So from (7)

$$\boxed{V_r \approx V_p \frac{T}{RC}} \quad (\tau \gg T) \qquad \text{(4.28),(8)}$$

to obtain a small ripple voltage we need a large C in this case.

Conduction Interval

Lastly, the conduction interval Δt is defined as the time interval in which the diode is actually conducting current. This time period is sketched in the preceding two figures.

The diode conducts current beginning at time td and ending at T, within each period. Using equation (4) at time t_d

$$V_p \cos\left[\omega\left(T - t_d\right)\right] = V_p - V_r \quad \text{or} \quad V_p \cos\left(\omega \Delta t\right) = V_p - V_r \quad (9)$$

We expect the conduction interval to be small. So truncating the series expansion of cosine to two terms, (9) gives

$$\omega \Delta t \approx \sqrt{\frac{2V_r}{V_p}} \quad (4.30),(10)$$

The factor $\omega \Delta t$ is sometimes called the conduction angle, θ. For $V_r \ll V_p$ this conduction angle (and conduction interval) will be small, as expected.

Discussion

To reiterate, the objective of the peak rectifier is to charge the shunt C when D is on, and slowly discharge it during those times when D is off.

When does D conduct? During the Δt periods in the previous figure. Also see Fig. 4.27(c).

Note that this peak rectifier is **not** a linear circuit. i_D is a very complicated waveform and not a sinusoid, as seen earlier in Fig. 4.27(c). There are no simple exact formulas for the solution to this problem. The text only shows approximate solutions for peak i_D:

$$i_D\big|_{\max} \approx \frac{V_p}{R}\left(1+2\pi\sqrt{\frac{2V_p}{V_r}}\right) \text{ [A]} \quad (V_r \ll V_p) \qquad (4.32),(11)$$

Example N8.1 (similar to text Example 4.8). A half-cycle peak rectifier with R = 10 kΩ is fed by a 60-Hz sinusoidal voltage with a peak amplitude of 100 V.

(a) Determine C for a ripple voltage of 2 V_{pp}. From (8):

$$C = \frac{T}{R}\frac{V_p}{V_r} = \frac{1}{60 \cdot 10{,}000}\frac{100}{2}$$

or

$$C = 83.3 \ \mu\text{F}.$$

For a "factor of safety" of two, make C twice as large. Remember, a bigger C translates to smaller ripple.

(b) Determine the peak diode current. Using (11):

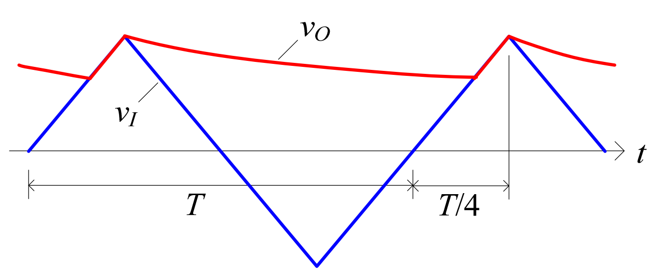
$$i_D\big|_{\max} \approx \frac{V_p}{R}\left(1+2\pi\sqrt{\frac{2V_p}{V_r}}\right) = \frac{100}{10,000}\left(1+2\pi\sqrt{\frac{2\cdot 100}{2}}\right)$$

or

$$i_D\big|_{\max} \approx 638 \text{ mA}.$$

When specifying a diode for your circuit design, you would need to find one that could safely handle this amount of current.

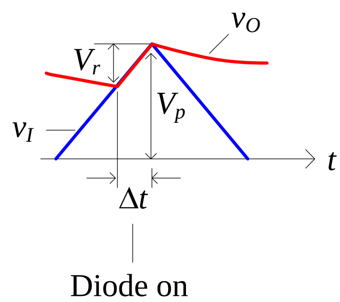
Example N8.2. A half-cycle peak rectifier with R = 10 kΩ is fed by a 60-Hz triangular voltage with a peak amplitude of 100 V.



(a) Determine C for a ripple voltage of 2 V_{pp}. If you go back and look at the derivation of (8) you'll find that there were no approximations made that required a sinusoidal waveform. Consequently, (8) applies to this triangular waveform as well, provided $\tau \gg T$. Hence, as before

$$C = 83.3 \ \mu\text{F}.$$

(b) Determine the diode conduction time, Δt. Referring to this sketch of the region near the positive peak voltage for v_I:



Because the rising portion of the waveform is a straight line:

$$v_I = \frac{\text{rise}}{\text{run}} t = \frac{V_p}{T/4} t$$

To find Δt, equate

$$\Delta v_I = \frac{4V_p}{T}\Delta t \quad \text{or} \quad V_r = \frac{4V_p}{T}\Delta t$$

Therefore, for a triangular waveform

$$\Delta t = \frac{T}{4}\frac{V_r}{V_p} \tag{12}$$

In this particular case,

$$\Delta t = \frac{1/60}{4}\frac{2}{100} = 83.3\ \mu\text{s}$$

Compare this time to a sinusoidal waveform:

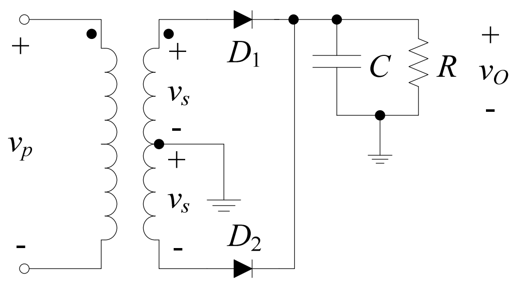
$$\Delta t = \frac{T}{2\pi}\sqrt{\frac{2V_r}{V_p}} = \frac{1/60}{2\pi}\sqrt{\frac{2\cdot 2}{100}} = 530.5\ \mu\text{s}$$

This time is much longer than for the triangular waveform. Consequently, we would expect $i_D\big|_{\max}$ for D to be much larger for the triangular waveform than for the sinusoid!

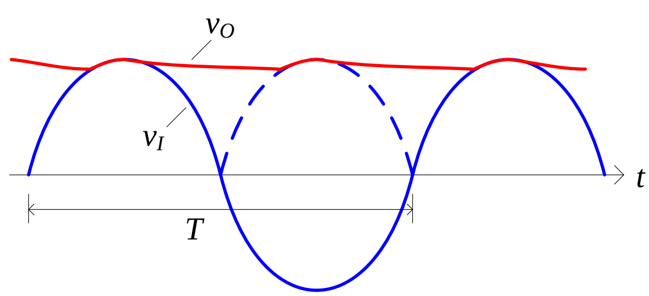
Full-Cycle Peak Rectifiers

In a similar fashion, we can also add a shunt C to full cycle and bridge rectifiers to convert them to peak rectifiers.

For example, for a full-cycle peak rectifier:



The output voltage has less ripple than from a half-cycle peak rectifier (actually one half less ripple).



The "ripple frequency" is twice that of a half-cycle peak rectifier. Using the same derivation procedure as before with the half cycle, but with $T \rightarrow T/2$ gives from (7)

$$\boxed{\frac{V_r}{V_p} \approx \frac{T}{2\tau}} \quad (\tau \gg T) \qquad (4.33),(13)$$

Lastly, it can be shown that the $i_D\big|_{\max}$ for the full-cycle peak rectifier:

$$i_D\big|_{\max} \approx \frac{V_p}{R}\left(1+2\pi\sqrt{\frac{V_p}{2V_r}}\right) \quad \text{[A]} \qquad (4.35),(14)$$

is approximately one-half that of the half-cycle peak rectifier when $V_r \ll V_p$.