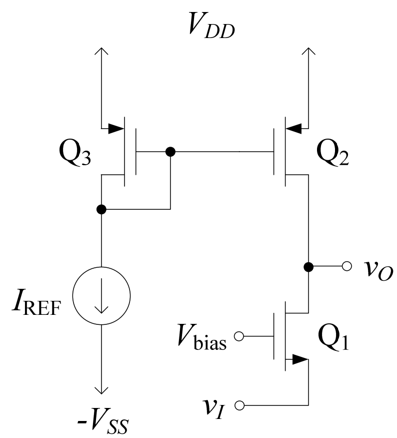
# Lecture 35: CMOS Common Gate Amplifier.

The IC version of the common gate amplifier with an active load is shown below implemented in CMOS:




(Fig. 1)

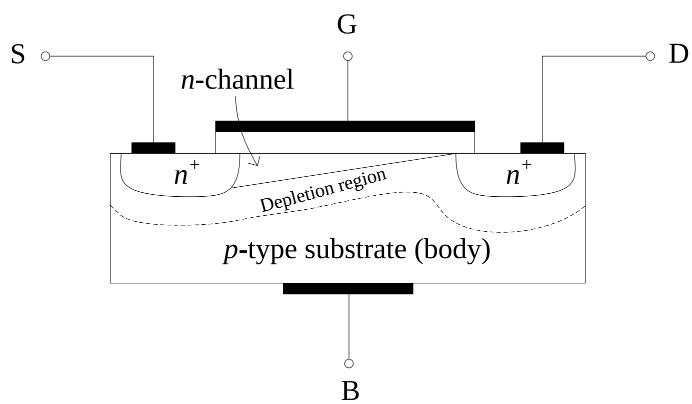
The common gate amplifier functions similar to a BJT common base amplifier as we discovered in Lecture 34.

In the CMOS implementation of this amplifier shown above, $Q_2$ and $Q_3$ function as a current mirror (as with the CMOS common source amplifier we studied in Lecture 33) with $Q_2$ further serving as an active load.

There is one subtlety with this IC amplifier that can affect its performance that's not found in the other two types of CMOS amplifiers. It's called the **body effect**. We'll first study this MOSFET body effect and then come back to the analysis of the CMOS CG amplifier.

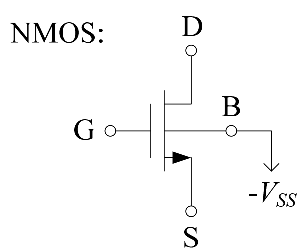
# MOSFET Body Effect

In ICs, the body or substrate is shared amongst all (or most) of the transistors.




(Fig. 2)

To guarantee that the body-to-source junction (and the other parts of the channel) remain reversed biased for all transistors in the IC, the substrate is connected to $-V_{SS}$, which is the smallest voltage in the circuit:




The reason is that if any body-to-source *pn* junctions (or any other parts of the channel) become forward biased, there would be a catastrophic failure in the transistor operation since current would now flow from the body into the source (or other parts of the channel).

To avoid this problem, one idea is to just connect the body terminal to the source terminal for all transistors in the IC.

For the common gate amplifier of Fig. 1, however, we see there is a problem in doing this. For circuits with input at the source (such as the common gate amplifier), we don't want the source connected to the most negative voltage in the circuit (or whatever voltage is assigned to the body).

So instead, we'll connect $V_B$ to $-V_{SS}$. The question now is what affect will this have on the device operation? Referring to Fig. 2, the resulting reverse-bias voltage between the source and body ($V_{SB}$ for NMOS):
1. Will widen the depletion region.
2. This, in turn, will decrease the channel depth. (To return the channel to its original depth, $v_{GS}$ would need to be increased.)

It can be shown (see Section 5.4.1 of the text) that this effect of $V_{SB}$ on the channel can be modeled as simply a **change in the threshold** voltage according to

$$V_t = V_{t0} + \gamma\left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right) \text{ [V]} \qquad (5.30),(1)$$

where
- $V_{t0}$ is the threshold voltage for $V_{SB} = 0$
- $2\phi_f$ is the material dependent Fermi potential (often $2\phi_f \approx 0.6$ V for NMOS)

- $\gamma$ is a fabrication process parameter (the body-effect parameter) given as

$$\gamma = \frac{\sqrt{2qN_A\varepsilon_s}}{C_{ox}} \ [\text{V}^{1/2}] \qquad (5.31),(2)$$

(often $\gamma \approx 0.4$ V$^{1/2}$).

Notice in (1) that with this body effect a change in $V_{SB}$ produces a change in $V_t$. This change in $V_t$ will change $i_D$, even though $v_{GS}$ doesn't change. (Why? Because the channel depth changes with changing $V_{SB}$, as we've just learned.)

Consequently, **this body voltage $V_{SB}$ controls $i_D$ like $v_{GS}$ does**. In other words, the body is acting like a second gate (of sorts), sometimes called the backgate.

This phenomenon is called the body effect. It can degrade the performance of MOSFET amplifiers in some cases.

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## Small-Signal Modeling of the Body Effect

A change in the body-to-source voltage $v_{BS}$ will cause a change in the drain current $i_D$, as we've just seen. This behavior can be quantified for time varying signals by the relationship

$$i_d = g_{mb} v_{bs} \qquad (3)$$

where $g_{mb}$ is the so-called body conductance

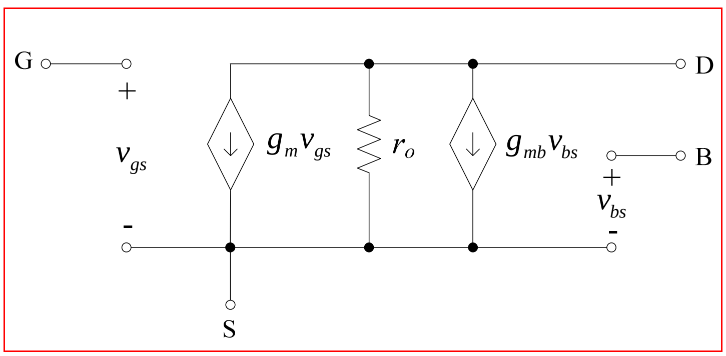
$$g_{mb} \equiv \left.\frac{\partial i_D}{\partial v_{BS}}\right|_{\substack{v_{GS}=\text{constant}\\ v_{DS}=\text{constant}}} \qquad (7.49),(4)$$

It can be shown that

$$\boxed{g_{mb} = \chi g_m \text{ [S]}} \qquad (7.50),(5)$$

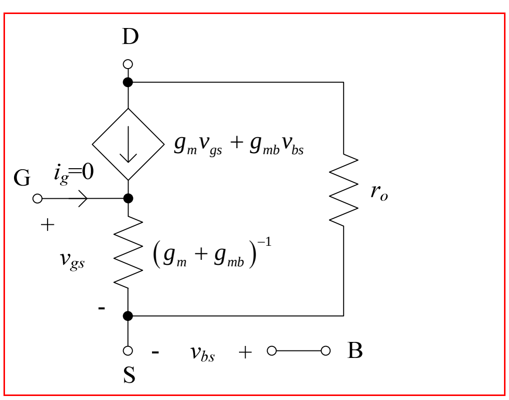
where $\chi$ is dimensionless and typically lies in value between 0.1 and 0.3.

For small-signal modeling, in light of (3) a second dependent current source is added to the MOSFET equivalent circuit, as shown in Fig 7.19, which is dependent on $v_{bs}$.




(Fig. 7.19b)

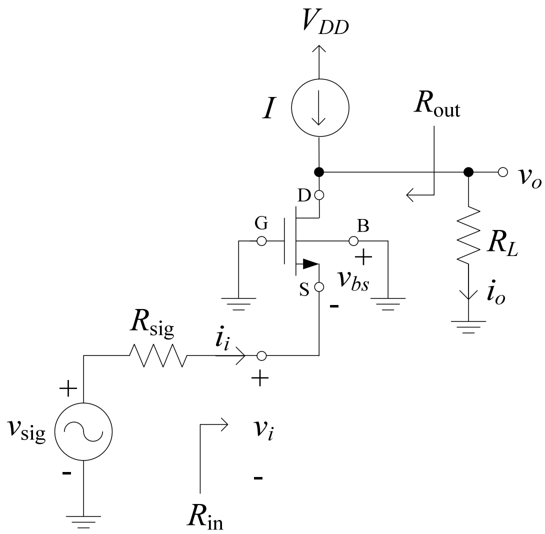
Alternatively, the small-signal T model for the MOSFET amplifier including the body effect is:




(Fig. 3)

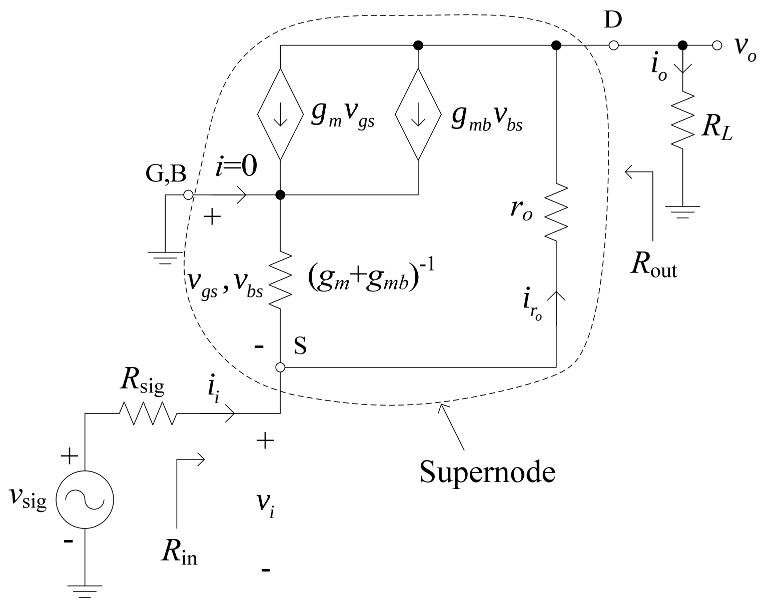
# Common Gate Amplifier with Active Load

We will model the active load for the CG amplifier in Fig. 1 as a simple ideal current source as shown in Fig. 4:




(Fig. 4)

Note here that the body terminal is not connected to the source terminal, but rather is connected to the lowest voltage in the circuit (ground). Because of this we need to account for the body effect in the small-signal T model of this amplifier:




(Fig. 5)

Notice that we've incorporated the body effect into the T small-signal model of the MOSFET. Because the gate and body are both grounded in Fig. 5, then

$$v_{gs} = v_{bs} \tag{6}$$

Consequently, the body effect in this CG amplifier can be completely accounted for by simply replacing $g_m$ of the MOSFET with $g_m + g_{mb}$. That is,

$$g_m \rightarrow g_m + g_{mb} = (1+\chi) g_m \tag{7}$$

(Refer to Section 8.4.3 of the text.)

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## Small-Signal Amplifier Characteristics

We'll calculate the following small-signal quantities for this CMOS common gate amplifier: $R_{in}$, $A_v$, $G_v$, $G_i$, and $R_{out}$.

- Input resistance, $R_{\text{in}}$. Using KCL at the source terminal in Fig. 5 as well as (6) we find

$$i_i + \left(g_m + g_{mb}\right)v_{gs} = i_{r_o} \tag{8}$$

An important insight into this circuit can be realized by examining the "supernode" contained in the dashed outline in the small-signal circuit. Because $i_g = i_b = 0$, applying KCL to this supernode

$$i_o = i_i \tag{9}$$

Since $v_{gs} = -v_i$ and

$$i_{r_o} = \frac{v_i - v_o}{r_o} = \frac{v_i - i_o R_L}{r_o} \underbrace{=}_{(9)} \frac{v_i - i_i R_L}{r_o} \tag{10}$$

then (8) becomes

$$i_i = -\left(g_m + g_{mb}\right)\left(-v_i\right) + \frac{v_i}{r_o} - i_i \frac{R_L}{r_o} \tag{11}$$

From this result, we can quickly determine that

$$R_{\text{in}} \equiv \frac{v_i}{i_i} = \frac{r_o + R_L}{1 + \left(g_m + g_{mb}\right) r_o} \tag{12}$$

We see from this expression that the body effect tends to reduce the input resistance.

Notice that $R_{\text{in}}$ depends on $R_L$. This type of amplifier is called a bilateral amplifier, in contrast to a unilateral amplifier in which $R_{\text{in}}$ is not dependent on $R_L$.

- Partial small-signal voltage gain, $A_v$. At the output side of the small-signal circuit

$$v_o = i_o R_L \underbrace{=}_{(9)} i_i R_L \tag{13}$$

while at the input

$$v_i = i_i R_{\text{in}} \tag{14}$$

Dividing these two equations we arrive at the partial small-signal voltage gain

$$A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_{\text{in}}} \underbrace{=}_{(12)} \frac{R_L\left[1+\left(g_m + g_{mb}\right)r_o\right]}{r_o + R_L} \tag{15}$$

Here we see that the body effect tends to increase $A_v$.

- Overall small-signal voltage gain, $G_v$. As we have done many times in the past for other types of amplifiers

$$G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_v \tag{16}$$

Substituting for $A_v$ from (15) gives

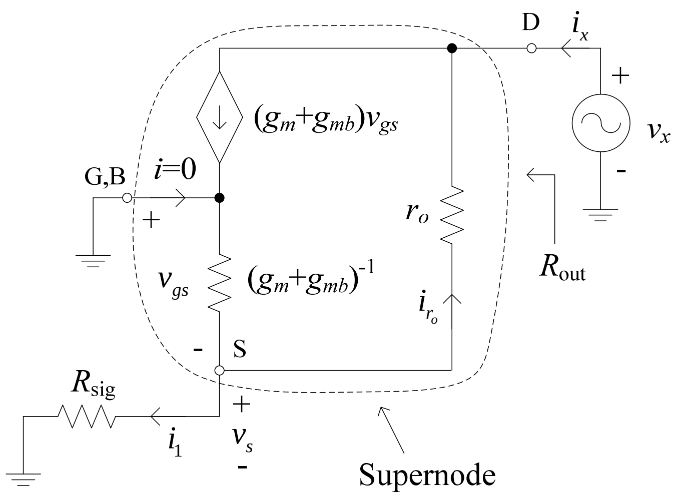
$$G_v = \frac{R_L}{R_{\text{in}} + R_{\text{sig}}} \tag{17}$$

Since the body effect tends to reduce $R_{\text{in}}$, we see from this expression that it tends to increase $G_v$.

- Overall small-signal current gain, $G_i$. From (9) we see directly that

$$G_i \equiv \frac{i_o}{i_i} = 1 \tag{18}$$

- <u>Output resistance, $R_{\text{out}}$</u>. To determine $R_{\text{out}}$ from the small-signal circuit above we set $v_{sig} = 0$ and apply a fictitious AC voltage source $v_x$ at the output as shown:




(Fig. 6)

We can see that with $v_x$ attached, the voltage $v_{gs}$ will not usually be zero. This means the current in the dependent current source is also not zero. Consequently, we need to analyze this circuit – including the effects of the dependent current source – to determine the output resistance.

Employing KCL at the drain terminal

$$i_x + i_{r_o} = (g_m + g_{mb}) v_{gs} \tag{19}$$

It is easy to see in Fig. 6 that

$$i_{r_o} = \frac{v_s - v_x}{r_o} \tag{20}$$

Applying KCL to the supernode indicated in Fig. 6, we find

$$i_1 = i_x \tag{21}$$

Because

$$v_s = i_1 R_{\text{sig}} \underbrace{=}_{(21)} i_x R_{\text{sig}} \tag{22}$$

then using this in (20) and substituting into (19)

$$i_x + i_x \frac{R_{\text{sig}}}{r_o} = \left(g_m + g_{mb}\right) v_{gs} + \frac{v_x}{r_o} \tag{23}$$

From the small-signal circuit in Fig. 6

$$v_{gs} = -v_s \underbrace{=}_{(22)} -i_x R_{\text{sig}} \tag{24}$$

Substituting (24) into (23) gives

$$i_x + i_x \frac{R_{\text{sig}}}{r_o} + \left(g_m + g_{mb}\right) i_x R_{\text{sig}} = \frac{v_x}{r_o}$$

or rearranging

$$R_{\text{out}} \equiv \frac{v_x}{i_x} = r_o + \left[1 + \left(g_m + g_{mb}\right) r_o\right] R_{\text{sig}} \tag{25}$$

We see from this result that the body effect tends to increase $R_{\text{out}}$.

Since $R_{\text{out}}$ depends on $R_{\text{sig}}$, this is another reason why this amplifier is a bilateral rather than a unilateral amplifier.

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## Summary of the Small-Signal Characteristics

In summary, we find for the CG small-signal amplifier:

- A non-inverting amplifier [see (17) and (18)].

- Moderately large input resistance. From (12), with $r_o = \mathcal{O}\left(10^5\right)$, $R_L = \mathcal{O}\left(10^3\right)$, $g_m, g_{mb} = \mathcal{O}\left(10^{-3}\right)$, then from (12), $R_{\text{in}} = \mathcal{O}\left(10^3\right)$.
- Relatively modest small-signal voltage, depending on the choices for $R_L$ and $R_{\text{sig}}$ [see (17)].
- Small-signal current gain equal to one [see (18)].
- Potentially large output resistance [see (25)].

These are characteristics similar to the discrete version of the common gate amplifier we discussed in the previous lecture. No surprise.

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**Example N35.1.**