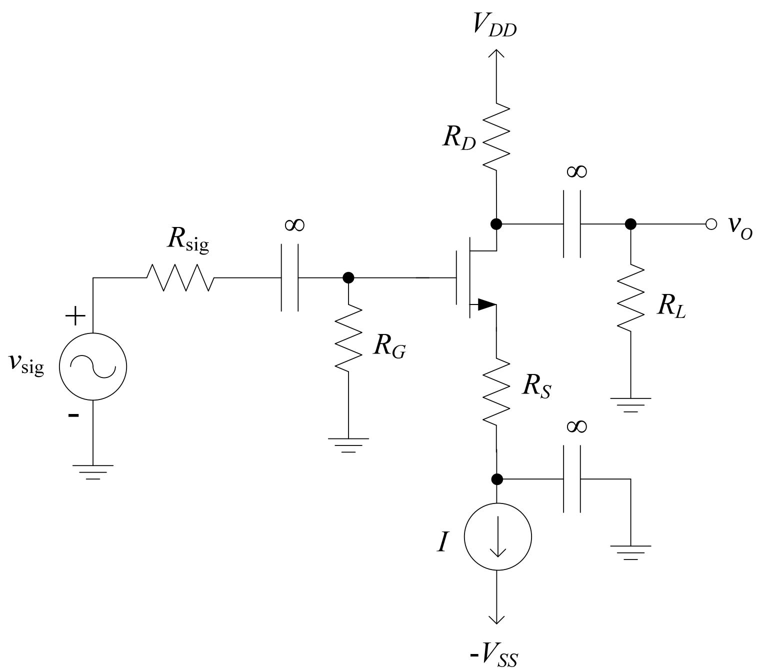
Lecture 32: Common Source Amplifier with Source Degeneration.

The small-signal amplification performance of the CS amplifier discussed in the previous lecture can be improved by including a series resistance in the source circuit. (This is very similar – if not identical – to the effect of adding emitter degeneration to the BJT CE amplifier.) This so-called CS amplifier with source degeneration circuit is shown in Fig. 1.

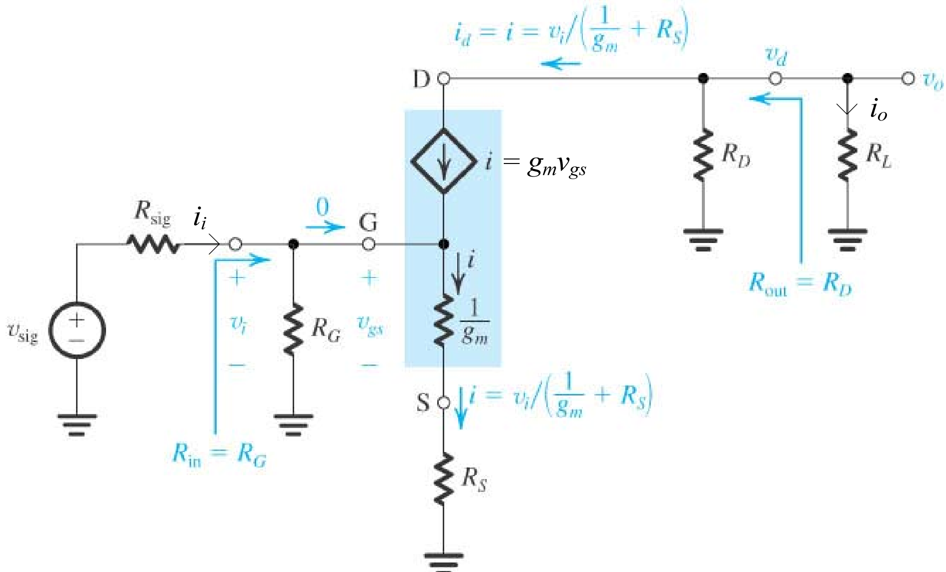


(Fig. 1)

In addition to the methods we discussed in Lecture 30 for biasing MOSFET amplifiers, this is yet another method using a current source. The current source in Fig. 1 could perhaps be the output of a current mirror that is replicating a "golden current" produced elsewhere in the circuit. The bypass capacitor acts to place an AC ground at the input of the current source, thus removing the effects of the current source from the AC operation of the amplifier.

We have a choice of small-signal models to use for the MOSFET. A T model will simplify the analysis, on one hand, by allowing us to incorporate the effects of R_S by simply adding this value to $1/g_m$ in the small-signal model, if we ignore r_o.

This small-signal circuit is shown in Fig. 2.



(Fig. 2)
(Sedra and Smith, 5th ed.)

On the other hand, using the T model makes the analysis more difficult when r_o is included. (The hybrid π model is better at easily including the effects of r_o.) However, r_o in the MOSFET amplifier is large so we can reasonably ignore its effects for now in the expectation of making the analysis more tractable.

Small-Signal Amplifier Characteristics

We'll now calculate the following small-signal quantities for this MOSFET common source amplifier with source degeneration: R_{in}, A_v, G_v, G_i, and R_{out}.

- Input resistance, R_{in}. Referring to the small-signal equivalent circuit above in Fig. 2, with $i_g = 0$, then

$$R_{\text{in}} = R_G \quad (4.84),(1)$$

  Here we see the direct benefit of adding the explicit boundary condition $i_g = 0$ to the small-signal model, as discussed in Lecture 28. Without it, we would need to set up a system of equations, solve them, and then find out that the effects of the circuit to the right of gate make no contribution to R_{in}. That's a lot more effort.

- Partial small-signal voltage gain, A_v. We see at the output side of the small-signal circuit in Fig. 2

$$v_o = -g_m v_{gs} \left(R_D \parallel R_L \right) \quad (2)$$

which is the same result (ignoring r_o) as we found for the CS amplifier without source generation. At the gate, however, we find through voltage division that

$$v_{gs} = \frac{1/g_m}{1/g_m + R_S} v_i = \frac{v_i}{1 + g_m R_S} \qquad (7.98),(3)$$

This is a different result than for the CS amplifier in that v_{gs} is only a fraction of v_i here, whereas $v_{gs} = v_i$ without R_S.

Substituting (3) into (2), gives the partial small-signal AC voltage gain to be

$$A_v \equiv \frac{v_o}{v_i} = \frac{-g_m \left(R_D \,\|\, R_L\right)}{1 + g_m R_S} \qquad (4)$$

- <u>Overall small-signal voltage gain, G_v</u>. As we did in the previous lecture, we can derive an expression for G_v in terms of A_v. By definition,

$$G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \underbrace{\frac{v_o}{v_i}}_{=A_v} = \frac{v_i}{v_{\text{sig}}} A_v \qquad (5)$$

Applying voltage division at the input of the small-signal equivalent circuit in Fig. 2,

$$v_i = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} v_{\text{sig}} \underbrace{=}_{(1)} \frac{R_G}{R_G + R_{\text{sig}}} v_{\text{sig}} \qquad (6)$$

Substituting (6) into (5) we the overall small-signal AC voltage gain for this CS amplifier with source degeneration to be

$$G_v = \frac{-R_G}{R_G + R_{\text{sig}}} \frac{g_m \left(R_D \| R_L \right)}{1 + g_m R_S} \tag{7}$$

- Overall small-signal current gain, G_i. Using current division at the output in the small-signal model above in Fig. 2

$$i_o = \frac{-R_D}{R_D + R_L} g_m v_{gs} \tag{8}$$

while at the input,

$$i_i = \frac{v_i}{R_G} \underbrace{=}_{(3)} \frac{1 + g_m R_S}{R_G} v_{gs} \tag{9}$$

Substituting (9) into (8) we find that the overall small-signal AC current gain is

$$G_i \equiv \frac{i_o}{i_i} = \frac{-g_m R_D}{R_D + R_L} \frac{R_G}{1 + g_m R_S} \tag{10}$$

- Output resistance, R_{out}. From the small-signal circuit in Fig. 2 with $v_{\text{sig}} = 0$ then i must be zero leading to

$$R_{\text{out}} = R_D \tag{11}$$

Discussion

Adding R_S has a number of effects on the CS amplifier. (Notice, though, that it doesn't affect the input and output resistances.)

First, observe from (3)

$$v_{gs} = \frac{v_i}{1+g_m R_S} \tag{3}$$

that we can employ R_S as a tool to lower v_{gs} relative to v_i and lessen the effects of nonlinear distortion.

This R_S also has the effect of lowering the small-signal voltage gain, which we can directly see from (7).

A major benefit, though, of using R_S is that the small-signal voltage (and current) gain can be made much less dependent on the MOSFET device characteristics. (We saw a similar effect in the CE BJT amplifier with emitter degeneration.)

We can see this here for the MOSFET CS amplifier using (7)

$$G_v = \frac{-R_G}{R_G + R_{\text{sig}}} \frac{g_m \left(R_D \,\|\, R_L \right)}{1+g_m R_S} \tag{7}$$

The key factor in this expression is the second one. In the case that $g_m R_S \gg 1$ then

$$G_v \approx \frac{-R_G}{R_G + R_{\text{sig}}} \frac{R_D \,\|\, R_L}{R_S} \tag{12}$$

which is no longer dependent on g_m.

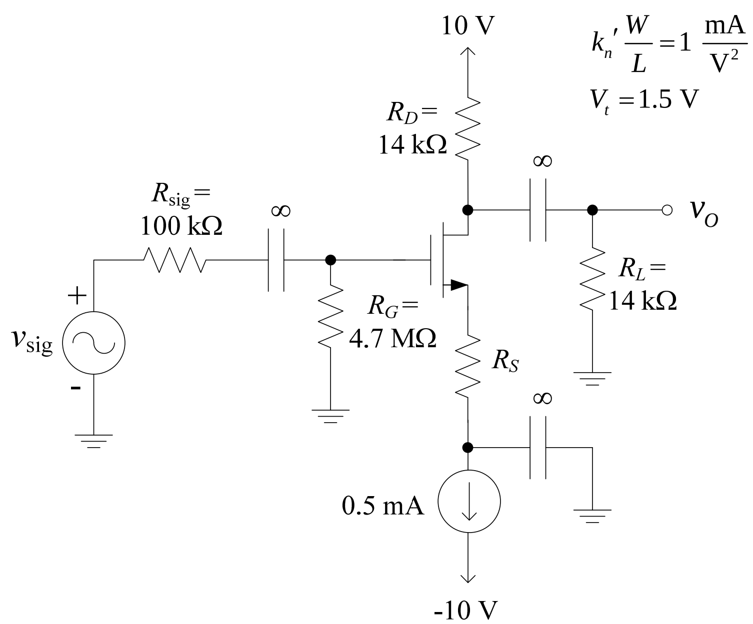
Conversely, without R_S in the circuit ($R_S = 0$), we see from (7) that $G_v \propto g_m$ and is directly dependent on the physical properties of the transistor (and the biasing) because

$$g_m \equiv \frac{i_d}{v_{gs}} = k_n' \frac{W}{L}\left(V_{GS} - V_t\right) \qquad (7.32),(13)$$

in the case of an NMOS device.

The "price" we pay for this desirable behavior in (12) – where G_v is not dependent on g_m – is a reduced value for G_v. This G_v is largest when $R_S = 0$, as can be seen from (7).

Example N32.1 (loosely based on text Exercises 7.37. 7.38, and 7.39). Compute the small-signal voltage gain for the circuit below with $R_S = 0$, $k_n' W/L = 1$ mA/V^2, and V_t = 1.5 V. For a 0.4-V_{pp} sinusoidal input voltage, what is the amplitude of the output signal?



For the DC analysis, we see that $V_G = 0$ and $I_D = I_S = 0.5$ mA. (Why is $V_G = 0$?) Consequently,

$$V_D = 10 - R_D I_D = 10 - 14\text{k} \cdot 0.5\text{m} = 3 \text{ V}$$

Assuming MOSFET operation in the saturation mode

$$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

such that $0.5 \text{ mA} = \frac{1}{2} 1 \times 10^{-3} (V_{GS} - 1.5)^2$

or $V_{GS} - 1.5 = \pm 1 \quad \Rightarrow \quad V_{GS} = 2.5 \text{ V}$ or 0.5 V

Therefore, $V_S = -2.5 \text{ V}$

for operation in the saturation mode.

For the AC analysis, from (13)

$$g_m = 10^{-3} (2.5 - 1.5) = 1 \text{ mS}$$

Using this result in (7) with $R_S = 0$ gives

$$G_v = \frac{-4.7\text{M}}{4.7\text{M} + 100\text{k}} 10^{-3} (14\text{k} \| 14\text{k}) = -6.85 \ \frac{\text{V}}{\text{V}}$$

For an input sinusoid with 0.4-V_{pp} amplitude, then

$$V_o = G_v \cdot V_{\text{sig}} = 6.85 \cdot 0.4 \text{ V}_{\text{pp}} = 2.74 \text{ V}_{\text{pp}}$$

Will the MOSFET remain in the saturation mode for the entire cycle of this output voltage? For operation in the saturation mode, $v_{DG} = v_D > V_t = 1.5 \text{ V}$. On the negative swing of the output voltage,

$$v_D\big|_{\min} = V_D - \frac{v_{o,\text{pp}}}{2} = 3 - \frac{2.74}{2} = 1.63 \text{ V}$$

which is greater than V_t, so the MOSFET will not leave the saturation mode on the negative swings of the output voltage. On the positive swings,

$$v_D\Big|_{\max} = V_D + \frac{v_{o,\text{pp}}}{2} = 3 + \frac{2.74}{2} = 4.37 \text{ V}$$

which is less than $V_{DD} = 10$ V so the MOSFET will not cutoff and leave the saturation mode.

(Alternatively, the MOSFET **does** leave the saturation mode on the negative swings if $R_D = R_L = 15 \text{ k}\Omega$.)

Lastly, imagine that for some reason the input voltage is increased by a factor of 3 (to 1.2 V_{pp}). What value of R_S can be used to keep the output voltage unchanged?

From (7), we can choose R_S so that the so-called feedback factor $1 + g_m R_S$ equals 3. The output voltage amplitude will then be unchanged with this increased input voltage.

Hence, for

$$1 + g_m R_S = 3 \quad \Rightarrow \quad R_S = \frac{3-1}{g_m} = \frac{2}{10^{-3}} = 2 \text{ k}\Omega.$$

With $R_S = 2 \text{ k}\Omega$ the new overall small-signal AC voltage gain is from (7)

$$G_v = \frac{-6.85}{1 + g_m R_S} = \frac{-6.85}{3} = -2.28 \ \frac{\text{V}}{\text{V}}$$

The overall small-signal voltage gain has gone down, but the amplitude of the output voltage has stayed the same since the input voltage amplitude was increased.