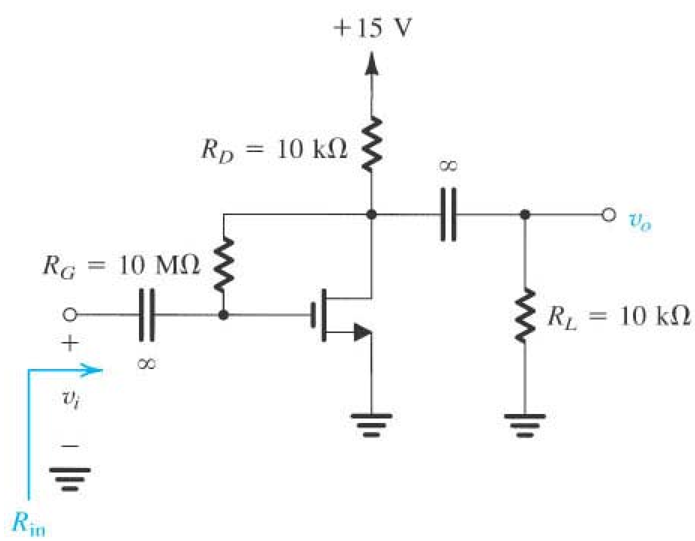
Lecture 29: MOSFET Small-Signal Amplifier Examples.

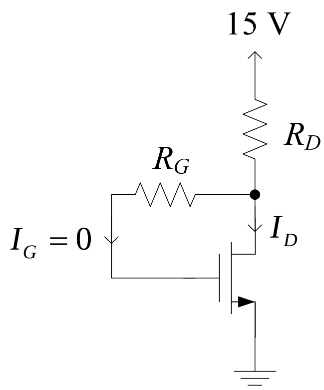
We will illustrate the analysis of small-signal MOSFET amplifiers through two examples in this lecture.

Example N29.1 (text Example 7.3). Determine A_v (neglecting the effects of R_G), R_{in}, and R_{out} for the circuit below given that $V_t = 1.5$ V, $k_n' W/L = 0.25$ mA/V^2, and $V_A = 50$ V.



(Fig. 7.15)

The first step is to determine the DC operating point. The DC equivalent circuit is:



Since $V_{GD} = 0 < V_t$ the MOSFET is operating in the saturation mode if $I_D \neq 0$. Assuming operation in the saturation mode the DC drain current from (5.23) is

$$I_D = \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS} - V_t\right)^2 \left(1 + \lambda V_{DS}\right) \tag{5.23}$$

Notice in the circuit that $V_{GS} = V_{DS}$, so we will eventually create a triatic equation in V_{GS} for I_D.

However, the last factor in (5.23) will be quite small for large V_A (small λ). So, for simplicity we will neglect r_o in (5.23) giving

$$I_D \approx \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS} - V_t\right)^2 \tag{7.44}$$

For this DC circuit

$$I_D = \frac{1}{2} \cdot 0.25 \times 10^{-3} \left(V_{GS} - 1.5\right)^2 = 1.25 \times 10^{-4} \left(V_{GS} - 1.5\right)^2$$

Notice in the circuit that $V_{GS} = V_{DS}$ so that this last equation becomes

$$I_D = 0.125 \left(V_{DS} - 1.5\right)^2 \text{ mA} \tag{1}$$

Also, by KVL

$$V_{DS} = 15 - R_D I_D = 15 - 10{,}000 I_D \qquad (7.43),(2)$$

Substituting (2) into (1)

$$I_D = 1.25\times10^{-4}\left(15 - 10{,}000 I_D - 1.5\right)^2$$

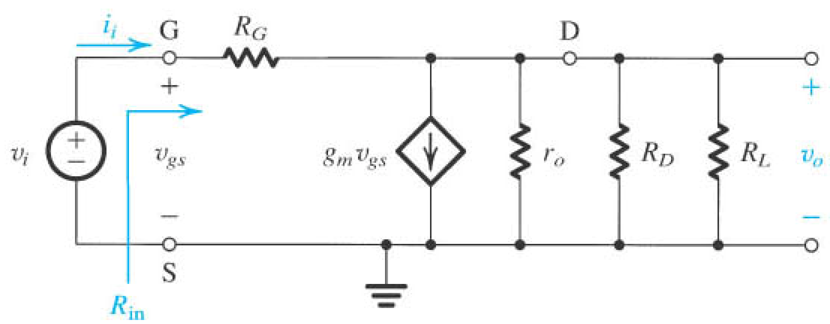
Solving this equation gives

$$I_D = 1.06 \text{ mA} \Rightarrow V_{DS} = 4.4 \text{ V} (= V_{GS})$$

or $$I_D = 1.72 \text{ mA} \Rightarrow V_{DS} = -2.2 \text{ V} (= V_{GS})$$

This latter result is not consistent with the assumption of operation in the saturation mode since $V_{GS} < V_t = 1.5$ V. So the proper solution for I_D is the first ($I_D = 1.06$ mA).

Next, we construct the small-signal equivalent circuit. We'll use the π small-signal model of the MOSFET with r_o included:



(Fig. 7.15c)

✓ $g_m = k_n' \dfrac{W}{L}\left(V_{GS} - V_t\right) = 0.25\times10^{-3}\left(4.4 - 1.5\right) = 0.725 \text{ mS}$

✓ $r_o = \dfrac{V_A}{I_D} = \dfrac{50}{1.06 \text{ mA}} = 47.2 \text{ k}\Omega$

Recall from the previous lecture (and also Lecture 26) that the proper I_D in the r_o calculation is that with $\lambda = 0$, which is what we ended up calculating earlier.

To compute the small-signal voltage gain, we start at the output (assuming R_G is extremely large $R_G \gg r_o \parallel R_D \parallel R_L$)

$$v_o \approx -g_m v_{gs}\left(r_o \parallel R_D \parallel R_L\right)$$

At the input notice that $v_{gs} = v_i$. Therefore

$$A_v = \frac{v_o}{v_i} \approx -g_m\left(r_o \parallel R_D \parallel R_L\right) = -g_m(4{,}521) = -3.28 \text{ V/V}$$

Notice that the assumption $R_G \gg r_o \parallel R_D \parallel R_L$ is met and hugely exceeded since 10 MΩ >> 4,521 Ω.

For the input resistance R_{in} calculation, we cannot set $v_{gs} = 0$ – and subsequently open circuit the dependent current source – since this would artificially force $R_{\text{in}} = 0$. Rather, we need to determine i_i as a function of v_i and use this in the definition

$$R_{\text{in}} \equiv \frac{v_i}{i_i}$$

The dependent current source will remain in these calculations.

Proceeding, at the input of the small-signal equivalent circuit shown above

$$i_i = \frac{v_i - v_o}{R_G} = \frac{v_i}{R_G}\left(1 - \frac{v_o}{v_i}\right) = \frac{v_i}{R_G}\left(1 - A_v\right)$$

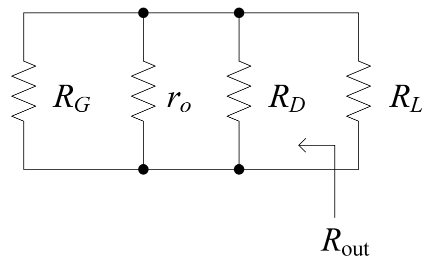
Therefore,

$$i_i = \frac{v_i}{R_G}(1 + 3.28)$$

Consequently, using this expression we find that

$$R_{\text{in}} = \frac{v_i}{i_i} = \frac{R_G}{4.28} = 2.34 \text{ M}\Omega$$

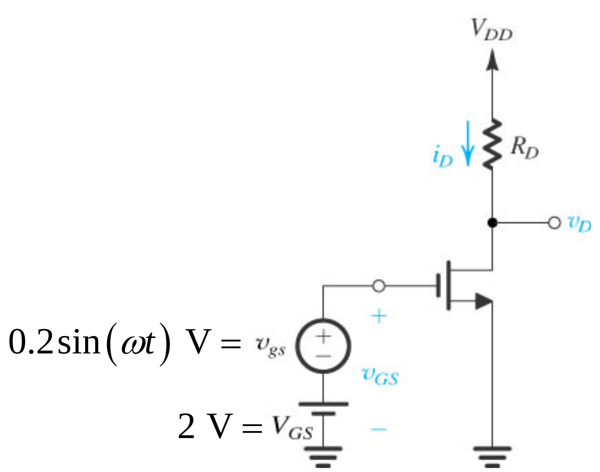
Lastly, to determine the output resistance, we can set $v_{gs} = 0$ in the small-signal equivalent circuit above, which will open circuit the dependent current source leading to the equivalent circuit:



from which we see that

$$R_{\text{out}} = R_G \parallel r_o \parallel R_D = 8.24 \text{ k}\Omega$$

Example N29.2 (text Exercise 7.6). Determine the following quantities for the conceptual MOSFET small-signal amplifier of Fig. 7.4 given that $V_{DD} = 5$ V, $R_D = 10$ kΩ, and $V_{GS} = 2$ V.



(Fig. 7.4)

The MOSFET characteristics are $V_t = 1$ V, $k_n' = 20$ μA/V^2, W/L = 20, and $\lambda = 0$.

(a) Determine I_D and V_D. We see from the circuit that $V_{GS} > V_t$. Therefore, the MOSFET is operating in the saturation or triode mode. We'll assume saturation. In that case

$$I_D = \frac{1}{2} k_n' \frac{W}{L} \left(V_{GS} - V_t\right)^2 = \frac{1}{2} \cdot 20 \times 10^{-6} \cdot 20 (2-1)^2 = 0.2 \text{ mA}$$

and
$$V_D = V_{DD} - I_D R_D = 3 \text{ V}$$

Let's check if the MOSFET is operating in the saturation mode:

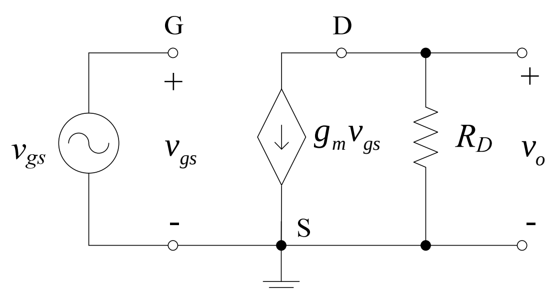
$$V_{GD} = 2 - 3 = -1 < V_t$$

Therefore, the MOSFET is indeed saturated, as assumed.

(b) Determine g_m. Using (7.32)

$$g_m = k_n' \frac{W}{L} \left(V_{GS} - V_t\right) = 20 \times 10^{-6} \cdot 20 \cdot (2-1) = 4.0 \text{ mS}$$

(c) Determine the voltage gain A_v. We begin by first constructing the small-signal equivalent circuit



Directly from this circuit,

$$v_o = -g_m v_{gs} R_D$$

so $A_v = \dfrac{v_o}{v_{gs}} = -g_m R_D = -0.4\times 10^{-3}\cdot 10\times 10^{3} = -4 \text{ V/V}$

(d) If $v_{gs} = 0.2\sin(\omega t)$ V, find v_d and the max/min v_D.

$$A_v \equiv \frac{v_o}{v_{gs}} \Rightarrow v_d = A_v v_{gs} = -4\cdot 0.2\sin(\omega t)$$

Therefore, $v_d = -0.8\sin(\omega t) \text{ V}$

Hence, $v_D\big|_{\max} = V_D + V_d = 3 + 0.8 = 3.8 \text{ V}$

while $v_D\big|_{\min} = V_D - V_d = 3 - 0.8 = 2.2 \text{ V}$

Let's check that the MOSFET stays in saturation at the two extremes of the drain voltage:

- $V_{GS} - v_D\big|_{\max} = 2 - 3.8 = -1.8 < V_t \Rightarrow$ saturated
- $V_{GS} - v_D\big|_{\min} = 2 - 2.2 = -0.2 < V_t \Rightarrow$ saturated

Therefore the transistor stays saturated for the entire cycle of the output voltage.

(e) Determine the second harmonic distortion. From (7.28) or (6) in the previous lecture notes, the total drain current is given as

$$i_D = I_D + k_n'\frac{W}{L}(V_{GS} - V_t)v_{gs} + \frac{1}{2}k_n'\frac{W}{L}v_{gs}^2$$

or $$i_D = I_D + 20\times 10^{-6}\cdot 20(2-1)v_{gs} + \frac{1}{2}\cdot 20\times 10^{-6}\cdot 20 v_{gs}^2$$

$$= I_D + 0.4\times 10^{-3} v_{gs} + 0.2\times 10^{-3} v_{gs}^2$$

Substituting $v_{gs} = 0.2\sin(\omega t)$ into this equation gives

$$i_D = I_D + 80\times 10^{-6}\sin(\omega t) + 8\times 10^{-6}\sin^2(\omega t)$$

Using the trigonometry identity

$$\sin^2(\omega t) = 1/2 - 1/2\cos(2\omega t)$$

this last expression becomes

$$i_D = 200 + 80\sin(\omega t) + 4 - 4\cos(2\omega t)\ \mu\text{A}$$

or

$$i_D = 204 + 80\sin(\omega t) - 4\cos(2\omega t)\ \mu\text{A}$$

The first term in i_D is I_D, the DC current. We see that there is a slight shift upward in value by 4 μA.

The third term in i_D is the second harmonic term because it varies with time at twice the frequency of the input signal. The second harmonic distortion is

$$\frac{4}{80}\times 100\% = 5\ \%$$