Lecture 22: BJT Internal Capacitances. High Frequency Circuit Model.

The BJT amplifiers we have examined so far are all mid-band frequency amplifiers. For large valued DC blocking capacitors and for frequencies of tens to hundreds of kHz, the simple small-signal models we used will work well.

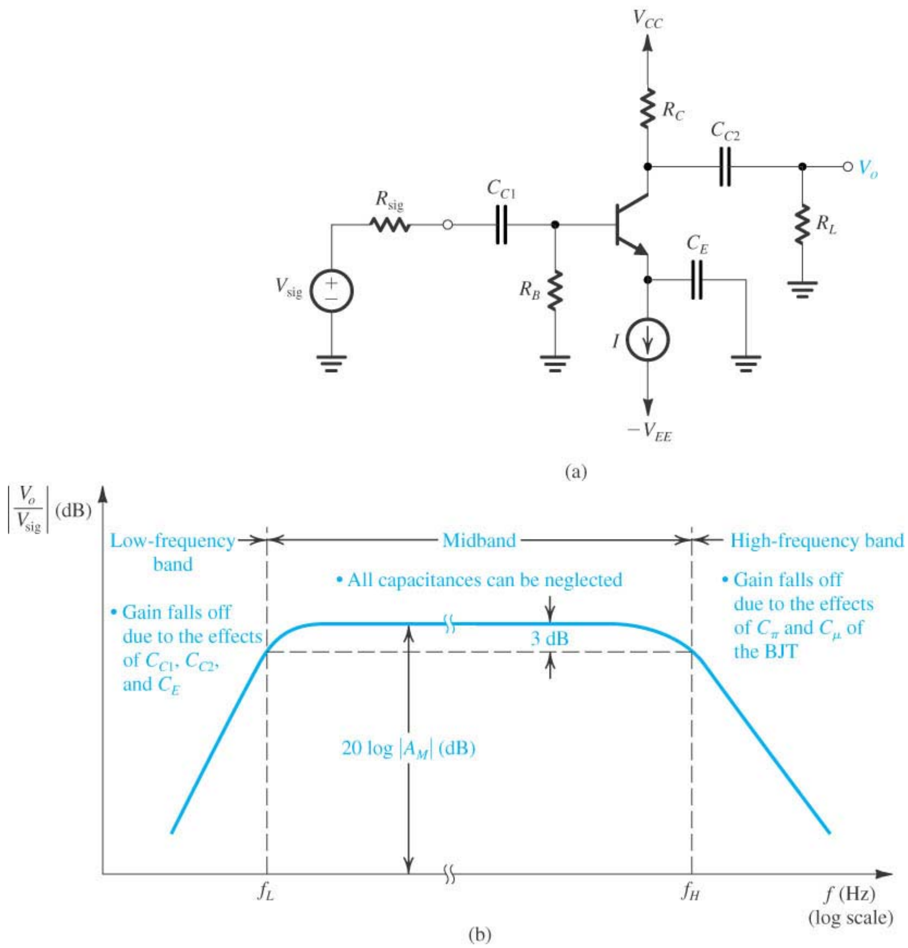
As the frequency increases, though, there are multiple sources of effects that will limit the performance of these amplifiers including:

1. Internal capacitances of the BJT. These are due to charge storage effects at and near the two *pn* junctions.
2. Parasitic effects. These are due to packaging and transistor construction that create additional capacitances, lead inductances, and resistances.

Additionally, the performance of many BJT amplifiers we've already examined will be sharply curtailed by DC blocking capacitors that have finite value (i.e., less than infinity).

For these reasons, all real transistor amplifiers operate effectively only over a limited (but hopefully large) range of signal frequencies.

Referring to Fig. 10.1, our analysis of small-signal BJT amplifiers up to this point has focused on the "Midband" frequency region. This frequency band is bounded by the frequencies f_L and f_H, which are the -3-dB gain frequencies, or half-power frequencies.



(Fig. 10.1)

The roll off in gain near f_L and lower is due to effects of the DC blocking capacitors C_{C1} and C_{C2}, and the bypass capacitor C_E. It's not possible to eliminate this effect, though f_L can be moved about by choosing different values for these capacitors. But large capacitors take up lots of space and can be expensive.

The primary focus of this lecture, however, is the origin of the roll off in gain experienced at higher frequencies near f_H.

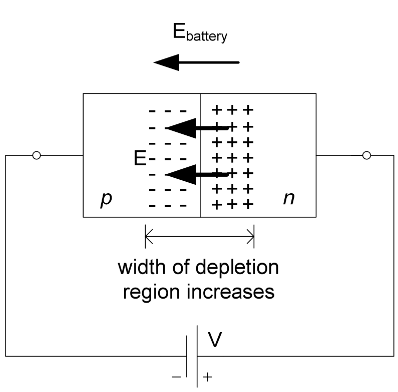
Capacitance of *pn* Junctions

There are basically two types of capacitances associated with *pn* junctions:

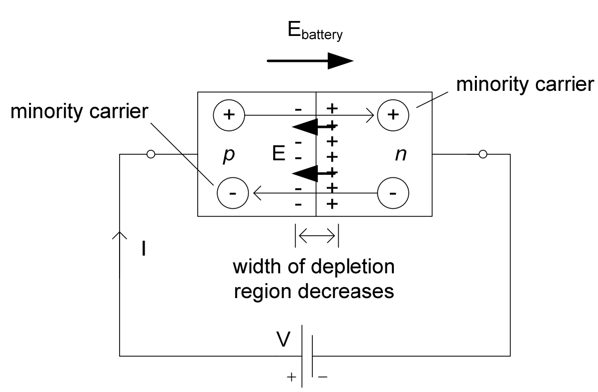
1. Junction capacitance. This is related to the space charge that exists in the depletion region of the *pn* junction.
2. Diffusion capacitance, or charge storage capacitance. This is a new phenomenon we haven't yet considered in this course.

The junction capacitance effect was briefly mentioned earlier in this course in Lecture 4. The width of the depletion region will change depending on the applied voltage and whether the junction is reversed or forward biased:

Reversed Biased

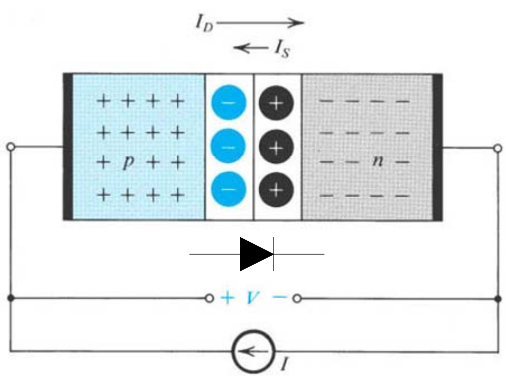


Forward Biased



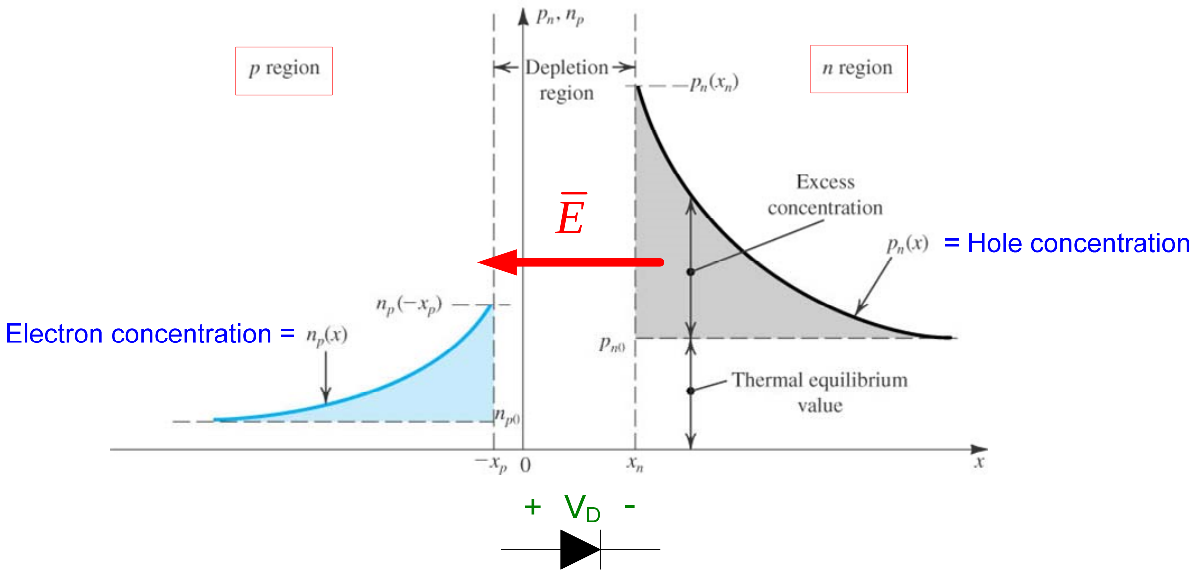
The time-varying $\bar{E}$ due to the space charge in the depletion region is a so-called **displacement current** that can be modeled by a junction capacitance.

The second basic type of capacitance, diffusion capacitance, is associated with *pn* junctions that are forward biased.



(Fig. 1)
(Sedra and Smith, 5th ed.)

In this state, current will flow across the junction, of course. Because of the current source in Fig. 1 and the voltage drop V, holes are injected across the junction into the n region while electrons are injected across the junction into the p region.



(Fig. 3.12)

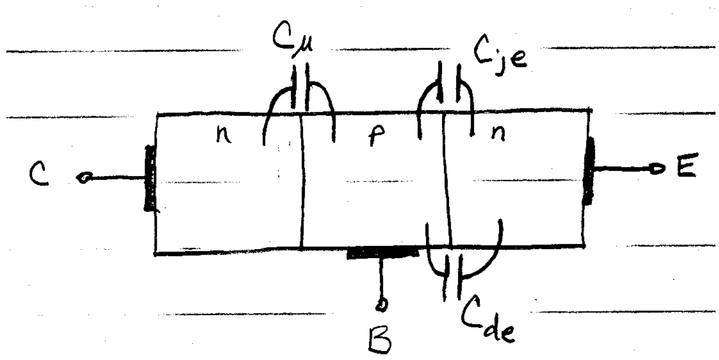
The concentrations of these electrons and holes decrease in value away from the junction, as shown in Fig. 3.12, due to recombination effects.

The important point here is that these concentrations of charges create an electric field $\bar{E}$ across the *pn* junction that will vary with time when a signal source is connected to this device. This electric field is directed from the *n* to *p* region, and the overall effect can be modeled by what is called the **charge storage capacitance, or diffusion capacitance**.

To summarize, the capacitive effects of a reversed biased *pn* junction are described by the junction capacitance while those of a forward biased *pn* junction are described by both a junction and a diffusion capacitance. In the latter case, though, the diffusion capacitance usually dominates.

BJT High Frequency Small-Signal Model

The active mode BJT has one forward biased *pn* junction (the EBJ) and one reversed biased *pn* junction (the CBJ). In the case of an *npn* BJT the capacitances associated with the *pn* junctions in the device are labeled as:

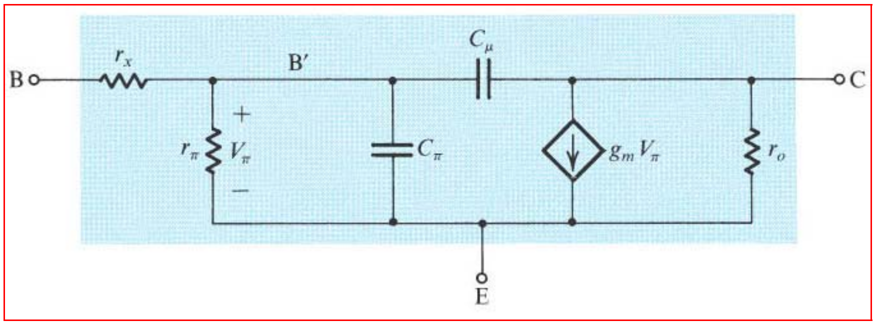


As we just discussed, there is a junction capacitance associated with the reversed biased CBJ, which is labeled C_μ as shown above. There will be a junction capacitance, C_{je}, associated with the forward biased EBJ as well as a diffusion capacitance labeled C_{de}. These latter two capacitances appear in parallel and so can be combined as

$$C_\pi \equiv C_{je} + C_{de} \tag{1}$$

Typically C_μ ranges from a fraction of pF to a few pF while C_π ranges from a few pF to tens of pF, which is dominated by C_{de}.

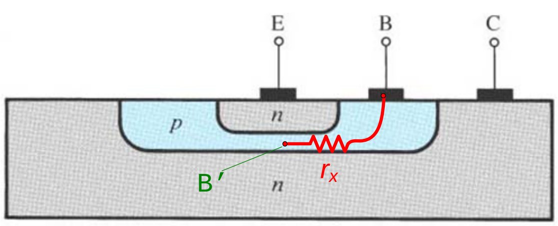
With these capacitances, the high frequency small-signal model of the BJT becomes



(Fig. 10.14a)

Note the use of the V_π notation in this small-signal model. Your textbook has switched to sinusoidal steady state notation for this high frequency discussion.

The high frequency small-signal model in Fig. 10.14a also includes the resistance r_x, which is mostly important at high frequencies. It's there to approximately model the resistance of the base region from the terminal to a point B′ somewhere directly below the emitter:



(Fig. 6.7)

C_μ is sometimes referred to as C_{ob} (or C_{obo}) in datasheets. This designation reflects the fact that C_μ can be the output to base capacitance.

The values of these small-signal circuit model elements may or may not be available in a datasheet for your transistor. For example, from the Motorola P2N2222A datasheet:

SMALL–SIGNAL CHARACTERISTICS				
Current–Gain — Bandwidth Product(2) (I_C = 20 mAdc, V_{CE} = 20 Vdc, f = 100 MHz)	f_T	300	—	MHz
Output Capacitance (V_{CB} = 10 Vdc, I_E = 0, f = 1.0 MHz)	C_{obo}	—	8.0	pF
Input Capacitance (V_{EB} = 0.5 Vdc, I_C = 0, f = 1.0 MHz)	C_{ibo}	—	25	pF

Actually, we would expect these capacitances to vary with the voltage across the respective *pn* junction. In the following figure from the Motorola P2N2222A datasheet, we see the dependence of "C_{eb}" (= C_π?) and "C_{cb}" (= C_μ) for a range of junction voltages. (Perhaps the labeled voltage for C_{eb} should be "forward voltage"?)

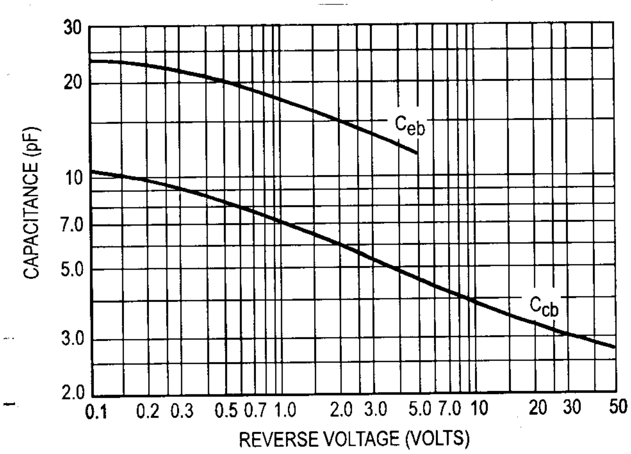


Figure 9. Capacitances

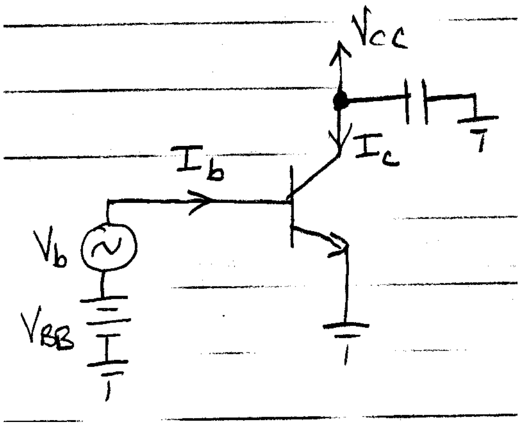
Unity-Gain Bandwidth

An important high frequency characteristic of transistors that is usually specified is the unity-gain bandwidth, f_T. This is defined as the frequency at which the short-circuit current gain

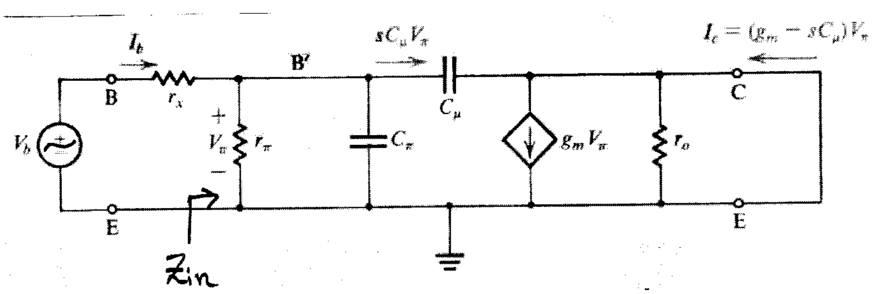
$$h_{fe} \equiv \left.\frac{I_c}{I_b}\right|_{\text{s.c. load}} \tag{2}$$

has decreased to a value of one.

A test circuit for this measurement would look something like:



The small-signal high frequency model of this test circuit is:



(Fig. 10.15)

Applying KCL at the collector terminal provides an equation for the short-circuit collector current

$$I_c = g_m V_\pi - j\omega C_\mu V_\pi = \left(g_m - j\omega C_\mu\right) V_\pi \qquad (10.35),(3)$$

At the input terminal B’

$$V_\pi = I_b \cdot Z_{\text{in}} = I_b \cdot r_\pi \,||\left(Z_{C_\pi} + Z_{C_\mu}\right)$$

or

$$V_\pi = I_b \cdot \left[\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)\right]^{-1} \qquad (10.36),(4)$$

Substituting (4) into (3) gives

$$I_c = \left(g_m - j\omega C_\mu\right) \cdot I_b \cdot \left[\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)\right]^{-1}$$

Using the definition of h_{fe} from (2) we find from this last equation that

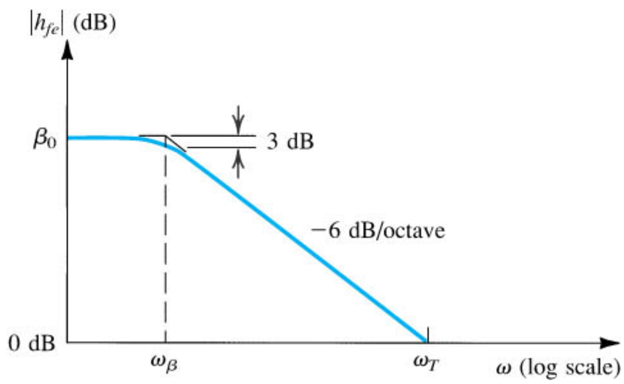
$$h_{fe} = \left.\frac{I_c}{I_b}\right|_{\text{s.c. load}} = \frac{g_m - j\omega C_\mu}{\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)} \qquad (5)$$

It turns out that C_μ is typically quite small and for the purposes of determining the unity-gain bandwidth, $g_m \gg |\, j\omega C_\mu \,|$ for the frequencies of interest here. In other words, the frequency at which ωC_μ is important relative to g_m is much higher than what is of interest here.

Consequently, from (5)

$$h_{fe} \approx \frac{g_m}{\frac{1}{r_\pi} + j\omega\left(C_\pi + C_\mu\right)} = \frac{g_m r_\pi}{1 + j\omega r_\pi\left(C_\pi + C_\mu\right)} \qquad (6)$$

We can recognize this frequency response of h_{fe} in (6) as that for a single pole low pass circuit [$h_{fe} = \beta_0 / \left(1 + j\,\omega/\omega_\beta\right)$]:



(Fig. 10.16)

$\beta_0 = g_m r_\pi$ in this plot is the low frequency value of h_{fe}, as we've used in the past [see eqn. (7.68)], while the 3-dB frequency of $\left|h_{fe}\right|$ is given by

$$\omega_\beta = \frac{1}{r_\pi\left(C_\pi + C_\mu\right)} \qquad (10.39),(7)$$

The frequency at which $\left|h_{fe}\right|$ in (6) declines to a value of 1 is denoted by ω_T, which we can determine from (6) to be

$$\left|h_{fe}\right| = 1 \approx \frac{\beta_0}{\left|1 + j\omega_T r_\pi\left(C_\pi + C_\mu\right)\right|}$$

or

$$\beta_0 \approx \left|1 + j\omega_T r_\pi\left(C_\pi + C_\mu\right)\right| = \left|1 + j\frac{\omega_T}{\omega_\beta}\right|$$

such that

$$\beta_0^2 = 1 + \left(\frac{\omega_T}{\omega_\beta} \right)^2 \approx \left(\frac{\omega_T}{\omega_\beta} \right)^2 \quad \text{for } \omega_T \gg \omega_\beta .$$

Therefore

$$\omega_T \approx \beta_0 \omega_\beta = \frac{g_m r_\pi}{\left(C_\pi + C_\mu \right) r_\pi} = \frac{g_m}{C_\pi + C_\mu} \qquad (10.40),(8)$$

so that

$$f_T \approx \frac{g_m}{2\pi \left(C_\pi + C_\mu \right)} \qquad (10.41),(9)$$

This **unity-gain frequency f_T** (or bandwidth) is often specified on transistor datasheets. On page 8, for example, $f_T = 300$ MHz for the Motorola P2N2222A. Using (9), this f_T can be used to determine $C_\pi + C_\mu$ for a particular DC bias current.

Lastly, the high frequency, hybrid-π, small-signal model of Fig. 10.14a is fairly accurate up to frequencies of about 0.2 f_T.

Furthermore, at frequencies above 5f_β to 10 f_β, the effects of r_π are small compared to the impedance effects of C_π. Above that, r_x becomes the only resistive part of the input impedance at high frequencies. Consequently, r_x is a very important element of the small-signal model at these high frequencies, but much less so at low frequencies.