

## Lecture 15: BJT Small-Signal Amplifier Examples.

We will now consider three examples in this lecture of BJTs used as linear amplifiers. Here are the **steps to follow** when solving small-signal transistor amplifier problems:

1. Determine the  $Q$  point of the BJT using DC analysis. Compute  $I_C$ .

2. Calculate the small-signal model parameters for the BJT:

$$\bullet \quad g_m = \frac{I_C}{V_T}, \text{ and} \quad (7.62),(1)$$

$$\bullet \quad r_\pi = \frac{\beta}{g_m}, \text{ or} \quad (7.68),(2)$$

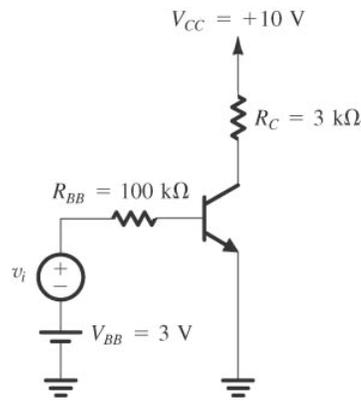
$$\bullet \quad r_e = \frac{\alpha}{g_m} = \frac{V_T}{I_E} \quad (7.74),(3)$$

3. Sketch the small-signal circuit: short out DC sources and open DC current sources. Use the small-signal model for the BJT.

4. Analyze the small-signal circuit for the desired quantities such as voltage, small-signal voltage gain, etc.

---

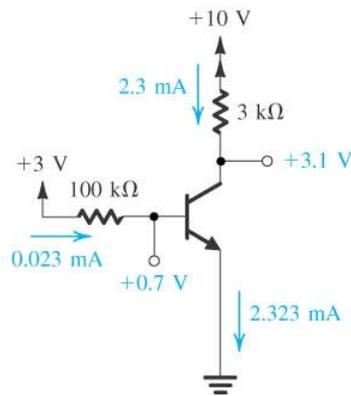
**Example N15.1** (text Example 7.5). Determine the small-signal AC voltage gain for the circuit below, assuming  $\beta = 100$  and the output voltage taken at the collector terminal.



(Fig. 7.28a)

The first step in the solution is to determine the  $Q$  point through DC analysis. By **superposition**, we'll force  $v_i = 0$  for this analysis.

Assuming the BJT is in the active mode, the results of the DC analysis are:



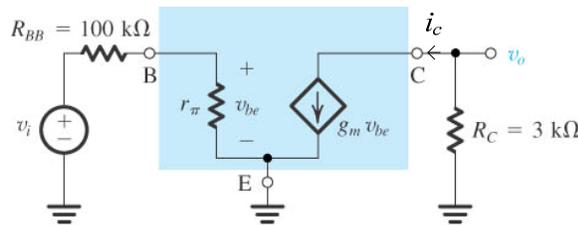
(Fig. 7.28b)

We see that the CBJ is reversed biased so this *npn* BJT is in the active mode because of this and that the EBJ is forward biased.

Next, we determine the BJT small-signal model parameters for the hybrid- $\pi$  model:

- From (1),  $g_m = \frac{I_C}{V_T} = \frac{2.3 \times 10^{-3}}{25 \times 10^{-3}} = 0.092 \text{ S}$
- From (2),  $r_\pi = \frac{\beta}{g_m} = \frac{100}{0.092} = 1,087 \text{ } \Omega$

Now, we **insert a small-signal equivalent model** of the BJT into the circuit of Fig. 7.28(a) after shorting the DC voltage sources ( $V_{BB}$  and  $V_{CC}$ ). This gives the small-signal equivalent circuit:



(Fig. 7.28c)

Notice the **AC ground** at  $R_C$ . This is an “AC ground” because the voltage at this node does not vary with time. For the purposes of the AC signal analysis, we can set this node to an AC ground. (As a side note, in the lab power supplies have a finite internal resistance. This Thévenin equivalent resistance must be included in the AC circuit for analysis purposes.)

Next, we perform the **small-signal analysis** referring to Fig. 7.28c. At the input

$$v_{be} = \frac{r_\pi}{r_\pi + R_{BB}} v_i \quad (7.81), (4)$$

while at the output

$$v_o = -R_C i_c = -R_C g_m v_{be} \quad (5)$$

Substituting for  $v_{be}$  from (4) gives

$$v_o = -R_C g_m \frac{r_\pi}{r_\pi + R_{BB}} v_i \quad (6)$$

Therefore, the **small-signal AC voltage gain,  $A_v$** , is

$$A_v \equiv \frac{v_o}{v_i} = -R_C g_m \frac{r_\pi}{r_\pi + R_{BB}} \quad (7)$$

For this particular problem

$$A_v = -3,000 \cdot 0.092 \frac{1,087}{1,087 + 100,000}$$

or

$$A_v = -2.97 \text{ V/V}$$

The negative sign indicates this is an **inverting** amplifier: the AC output signal is inverted with respect to the input AC signal.

---

**Example N15.2** (text Example 7.6). Repeat the analysis of the previous example but with a **triangular** input waveform of small amplitude.

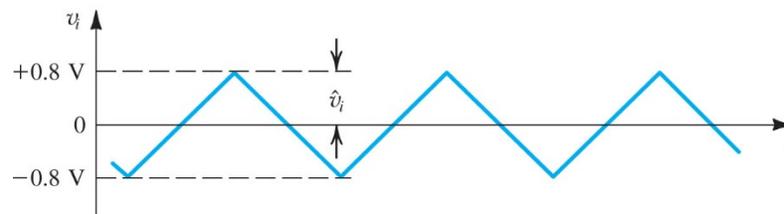
In the text,  $v_{i,p} = 0.8 \text{ V}$  is the peak amplitude of the triangular input voltage ( $=\hat{v}_i$  in the text).

Then from (4) above (and the fact that there are only resistors in the circuit) the peak base-to-emitter small-signal voltage is

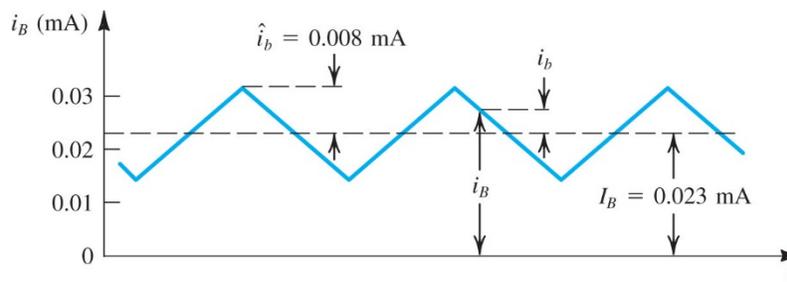
$$\hat{v}_{be} = \frac{r_\pi}{r_\pi + R_{BB}} \cdot 0.8 = 8.60 \text{ mV}$$

which is less than 10 mV, which is a rule-of-thumb for the largest  $v_{be}$  in small-signal analysis. This value is fairly small with respect to  $2V_T = 50$  mV so we'll go ahead and use the [small-signal](#) analysis and models.

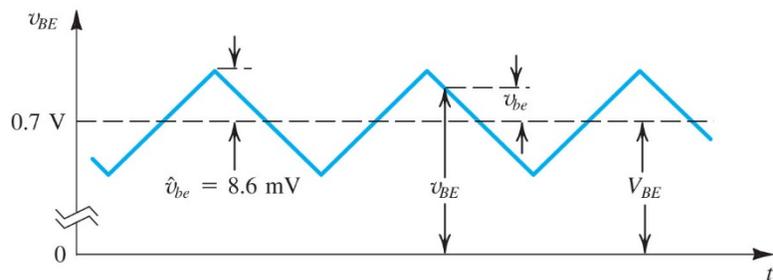
Sketches of the total voltages and currents from this circuit are shown in Fig. 7.29:



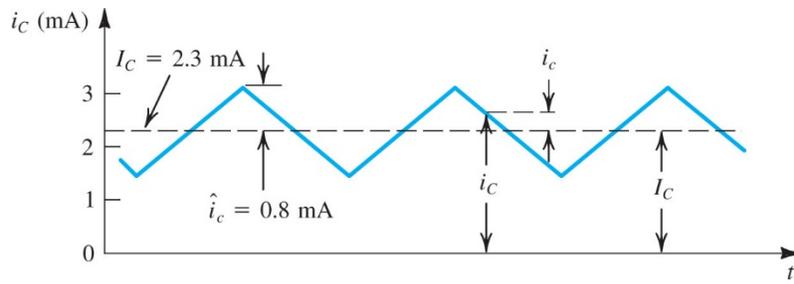
(a)



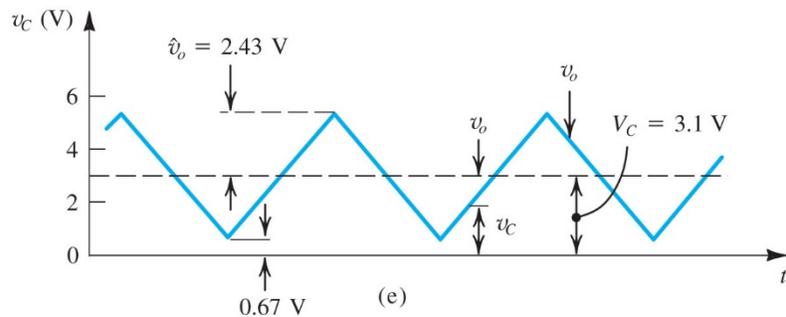
(b)



(c)



(d)



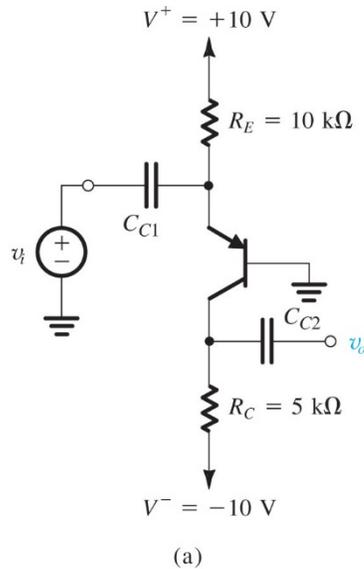
(e)

A few things to take special notice:

- In Fig. 7.29c,  $v_{BE}$  has a DC part and an AC part (see Fig. 7.29a) that is “riding” on the former. Notice the enlarged vertical scale in Fig. 7.29c.
- In Fig. 7.29d,  $i_c$  is in-phase with the input voltage.
- In Fig. 7.29e,  $v_c = V_C - i_c R_C$  is  $180^\circ$  out-of-phase with the input. As  $v_i \uparrow$ ,  $i_c \uparrow \Rightarrow v_c \downarrow$ . We can see how the AC ground works here.

---

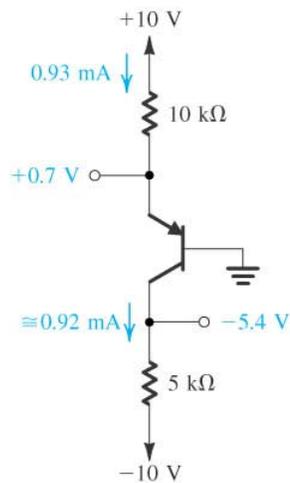
**Example N15.3** (text Example 7.7). Determine the small-signal AC voltage gain for the BJT amplifier circuit shown in Fig. 7.30a.



(Fig. 7.30a)

The two capacitors in this circuit serve as **DC blocks**. They have a large enough  $C$  so that  $X_C \approx 0$  at the operating frequency. With these capacitors, the DC bias is unchanged by the source or load attachments. We call this “**capacitively coupled**” input and output.

As always, we first determine the **DC bias**. We’ll assume the BJT is in the active mode and that  $\beta = 100$ :



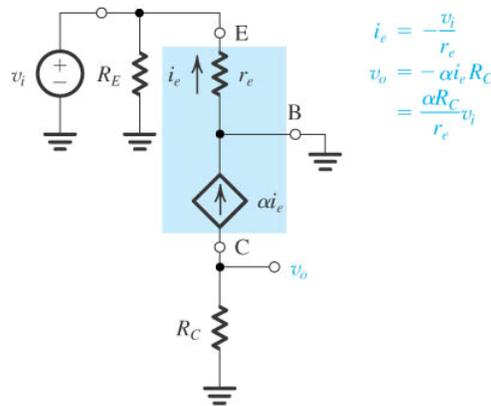
(Fig. 7.30b)

From this result

$$I_C = 0.92 \text{ mA} \Rightarrow V_C = -10 + I_C R_C = -5.4 \text{ V}$$

Since  $V_C < V_B$  (and  $V_{EB} = 0.7 \text{ V}$ ), the *pn*p BJT is operating in the active mode.

Next, we construct the **small-signal equivalent circuit** and analyze the circuit to determine the voltage gain. We'll use the T model, though the hybrid- $\pi$  model would work as well.



(Fig. 7.30c)

Notice the **two AC grounds** in this circuit: one at  $R_E$  and the other at  $R_C$ .

Also notice that this is the **first small-signal model of the *pn*p transistor** we have used. The small-signal model of the *pn*p transistor is exactly the same as that for the *n*pn with **no change** in the polarities of the currents or voltages. This can be a little confusing. Here, for example,  $i_e$  is a negative quantity.

Using (3) for the small-signal equivalent model of the BJT

$$r_e = \frac{V_T}{I_E} = \frac{25 \times 10^{-3}}{0.93 \times 10^{-3}} = 26.9 \, \Omega$$

From the small-signal AC circuit:

$$\bullet \quad v_o = -\alpha i_e R_C \quad (8)$$

$$\bullet \quad \text{Because the base is grounded, } i_e = -v_i / r_e. \quad (9)$$

Therefore,

$$v_o = -\alpha R_C \left( \frac{-v_i}{r_e} \right) \Rightarrow A_v = \frac{v_o}{v_i} = \alpha \frac{R_C}{r_e} \quad (10)$$

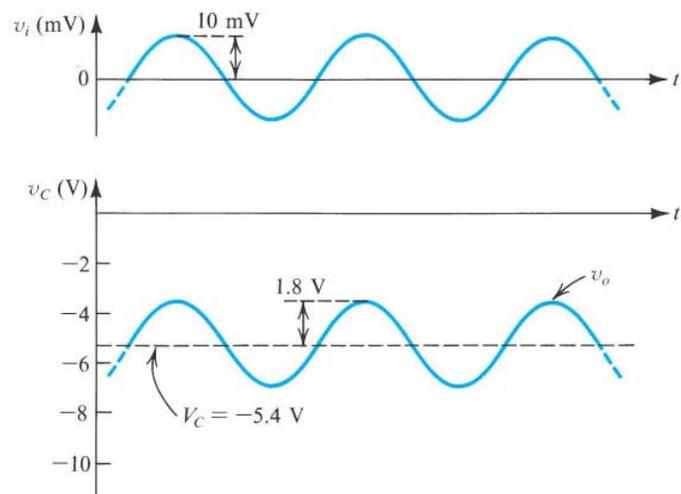
Notice that this small-signal voltage gain is a positive quantity. The reason for this is the input is tied to the emitter. (Note that this positive gain did not occur just because this is a *pn*p BJT.)

Now, with  $\alpha = 0.99$  then from (10)

$$A_v = 0.99 \frac{5,000}{26.9} = 184.0 \text{ V/V.}$$

Lastly, for linear operation of this amplifier,  $v_{eb} \lesssim 10 \text{ mV}$ . With  $v_{eb} = -v_i$  then  $v_i \lesssim 10 \text{ mV}$  for linear operation of the amplifier, which implies that  $v_o \lesssim 1.8 \text{ V}$ .

A sketch of the output from this small-signal amplifier is shown in Fig. 7.31 for a sinusoidal input voltage:



We're assuming the output remains linear and the BJT in the active mode at all times for the entire voltage swing in  $v_C$ .

If this input voltage were set to a larger value, this would no longer be the case and the BJT would **first encounter nonlinear behavior and eventually saturate**. Both of these effects would distort the output voltage and it would no longer be an amplified copy of the input voltage.