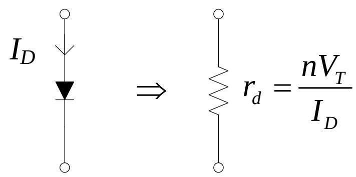
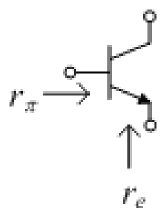
Lecture 14: BJT Small-Signal Equivalent Circuit Models.

Our next objective is to develop small-signal circuit models for the BJT. We'll focus on the *npn* variant in this lecture.

Recall that we did this for the diode back in Lecture 4:



In order to develop these BJT small-signal models, there are two small-signal resistances that we must first determine. These are:

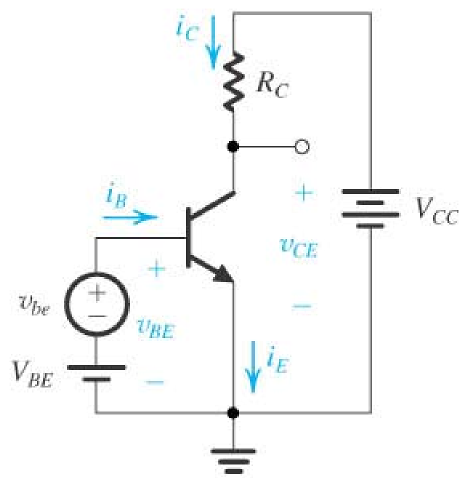


1. ***r_π***: the small-signal, active mode input resistance between the base and emitter, as "seen looking into the base."
2. ***r_e***: the small-signal, active mode output resistance between the base and emitter, as "seen looking into the emitter."

These resistances are NOT the same. Why? Because the transistor is not a so-called reciprocal device. Like a diode, the behavior of the BJT in the circuit changes if we interchange the terminals (i.e., a non-reciprocal device).

Determine r_π

Assuming the transistor in this circuit



(Fig. 7.20a)

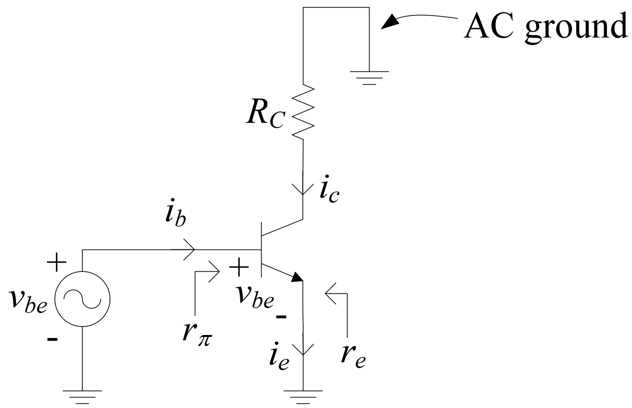
is operating in the active mode, then

$$i_B = \frac{i_C}{\beta} \underbrace{=}_{(7.59)} \frac{1}{\beta}\left(\underbrace{I_C}_{\text{DC}} + \underbrace{\frac{I_C}{V_T} v_{be}}_{\text{AC}} \right) \quad (1)$$

so that

$$i_b = \frac{I_C}{\beta V_T} v_{be} = \frac{g_m}{\beta} v_{be} \quad (7.65),(7.66),(2)$$

The AC small-signal equivalent circuit from Fig. 7.20a is

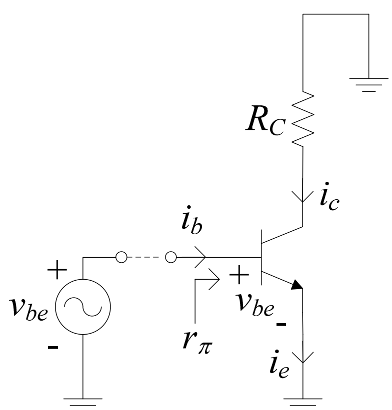


(Fig. 1)

Notice the "AC ground" in the circuit. This is an **extremely important concept**. Since the voltage at this terminal is held constant at V_{CC}, there is no time variation of the voltage. Consequently, we can set this terminal to be an "AC ground" in the small-signal circuit.

For AC grounds, we "kill" the DC sources at that terminal: short circuit voltage sources and open circuit current sources.

So, from the small-signal equivalent circuit above:



(Fig. 2)

we see that

$$r_\pi = \frac{v_{be}}{i_b} \qquad (7.67),(3)$$

Hence, using (2) in (3)

$$\boxed{r_\pi = \frac{\beta}{g_m} \quad [\Omega]} \qquad (7.68),(4)$$

This r_π is the BJT active mode small-signal input resistance of the BJT between the base and the emitter as seen looking into the base terminal. (Similar to a Thévenin resistance, this statement means we are fictitiously separating the source from the base of the BJT and observing the input resistance, as indicated by the dashed line in Fig. 2.)

Determine r_e

We'll determine r_e following a similar procedure as for r_π, but beginning with

$$i_E = \frac{i_C}{\alpha} = \underbrace{\frac{I_C}{\alpha}}_{\text{DC}} + \underbrace{\frac{i_c}{\alpha}}_{\text{AC}} \qquad (5)$$

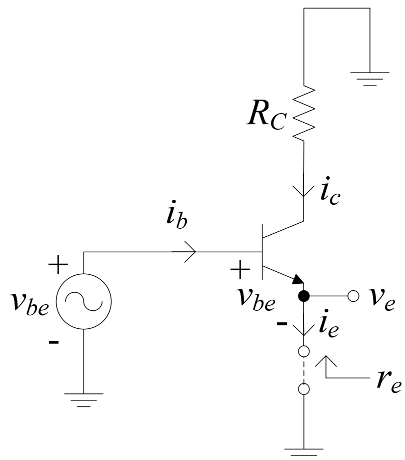
The AC component of i_E in (5) is

$$i_e = \frac{i_c}{\alpha} \underbrace{=}_{(7.60)} \frac{I_C}{\alpha V_T} v_{be} \qquad (7.71),(6)$$

or with $I_E = I_C / \alpha$,

$$i_e = \frac{I_E}{V_T} v_{be} \qquad (7.71),(7)$$

As indicated in Fig. 1 above, r_e is the BJT small-signal resistance between the emitter and base seen looking into the emitter:



Mathematically, this is stated as

$$r_e \equiv \frac{v_e}{-i_e} \qquad (8)$$

Assuming an ideal signal voltage source, then $v_e = -v_{be}$ and

$$r_e \equiv \frac{v_{be}}{i_e} \qquad (7.72),(9)$$

Using (7) in this equation we find

$$r_e = \frac{V_T}{I_E} \qquad (7.73),(10)$$

But from (7.62)

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} \quad \Rightarrow \quad \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

Therefore, using this last result in (10) gives

$$r_e = \frac{\alpha}{g_m} \approx \frac{1}{g_m} \; [\Omega] \qquad (7.74),(11)$$

This is the BJT active mode small-signal resistance between the base and emitter seen looking into the emitter.

It can be shown that

$$r_\pi = (\beta + 1) r_e \; [\Omega] \qquad (7.75),(12)$$

It is quite apparent from this equation that $r_\pi \neq r_e$. This result is not unexpected because the active mode BJT is a non-reciprocal device, as mentioned on page 1 of these notes.

BJT Small-Signal Equivalent Circuit Models

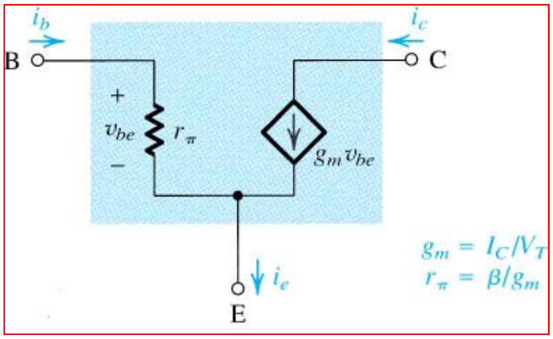
We are now in a position to construct the equivalent active mode, small-signal circuit models for the BJT. There are two families of such circuits:

1. Hybrid-π model
2. T model.

Both are equally valid models, but choosing one over the other sometimes leads to simpler analysis of certain circuits.

Hybrid-π Model

- <u>Version A</u>.



(Fig. 7.24a)

Let's verify that this circuit incorporates all of the necessary small-signal characteristics of the BJT:

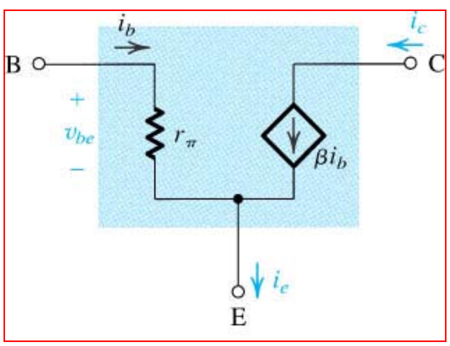
- ✓ $i_b = v_{be}/r_\pi$ as required by (3).
- ✓ $i_c = g_m v_{be}$ as required by (7.61), which we saw in the last lecture.
- ✓ $i_b + i_c = i_e$ as required by KCL.

We can also show from these relationships that $i_e = v_{be}/r_e$.

- Version B. We can construct a second equivalent circuit by using

$$i_c \underbrace{=}_{(7.61)} g_m v_{be} \underbrace{=}_{(3)} g_m \left(i_b r_\pi\right) = \underbrace{g_m r_\pi}_{(4)} i_b = \beta i_b$$

Hence, using this result and (3), the second hybrid-π model is



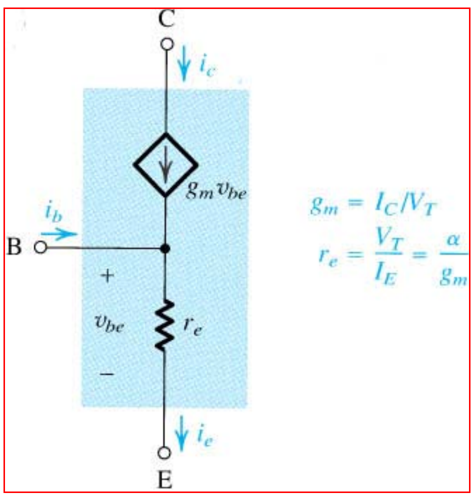
(Fig. 7.24b)

T Model

The hybrid-π model is definitely the most popular small-signal model for the BJT. The alternative is the T model, which is useful in certain situations.

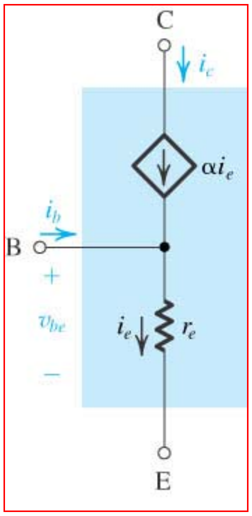
The T model also has two versions:

- Version A.



(Fig. 7.26a)

- Version B.



(Fig. 7.26b)

Small-Signal Models for *pnp* BJTs

The small-signal models for *pnp* BJTs are identically the same as those shown here for the *npn* transistors. It is important to note that there is no change in any polarities (voltage or current) for the *pnp* models relative to the *npn* models. Again, these small-signal models are identically the same.