Section 14.3 The Decibel Scale

In the next section, we will begin an exhaustive study on plotting transfer functions versus frequency for many different types of circuits.

But before doing that, it is important to first introduce the decibel (dB).

The gain of a circuit is commonly expressed in decibels:

$$G = 10 \log_{10} \frac{P_2}{P_1} \qquad (14.5)$$

where P_2 is an output power and P_1 is an input power.

So, we use a logarithm in this definition because the range in value of gain G can change hugely over a wide range of frequency. On a linear plot, this may be difficult to distinguish or read accurately. log plots, & semi log or log-log) helps with that.

So, we will be working w/ logarithms. Some properties to recall are:

$$\log_{10} P_1 P_2 = \log_{10} P_1 + \log_{10} P_2$$

$$\log_{10} \frac{P_1}{P_2} = \log_{10} P_2 - \log_{10} P_2$$

$$\log_{10} P^n = n \log_{10} P$$

$$\log_{10} 1 = 0$$

$$\log_{10} 0 = -\infty$$

Some special values of gain in (14.5) include

$$10 \log_{10} 2 = 3.010 \text{ dB} \approx 3 \text{ dB} \qquad (14.6)$$

and $10 \log_{10} 0.5 = -3.010 \text{ dB} \approx -3 \text{ dB}$

Alternatively, we can calculate gain of ratios of voltage or current as well.

For example, if the output voltage is V_2 & the input voltage is V_1, then

$$G_{dB} = 10 \log_{10}\left(\frac{P_2}{P_1}\right) = 10 \log_{10}\left(\frac{V_2^2/R}{V_1^2/R}\right) = 10 \log_{10}\left(\frac{V_2^2}{V_1^2}\right)$$

$$= 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2$$

$$\therefore \quad G_{dB} = \underset{\uparrow}{20} \log_{10}\left(\frac{V_2}{V_1}\right) \qquad (14.10)$$

Similarly, can show

$$G_{dB} = \underset{\uparrow}{20} \log_{10}\left(\frac{I_2}{I_1}\right) \qquad (14.11)$$

So we see $20 \log_{10}$ for ratios of voltages or currents, but $10 \log_{10}$ for ratios of powers.

Note that the definition of gain is the ratio of similar quantities such as P_2/P_1 or V_2/V_1 or I_2/I_1. These are dimensionless. We don't express dB of quantities of different units such as V/I or I/V.