Section 13.2

Equivalent ckt model for mutually coupled inductors

Now that we've discussed the electrical behavior of coupled inductors, we'll now move to the analysis of circuits containing such elements. To that end, consider the circuit of Fig. 13.7(a)

Fig. 13.7(a)

Applying KVL to coil 1 $\quad v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + \underbrace{M \frac{di_2}{dt}}_{(13.1k)}$ (13.20a)

and KVL to coil 2: $\quad v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_2}{dt}$ (13.20b)

Much of all our circuit analysis will be for sinusoidal steady state, so in the frequency domain, these two eqns read

$$\bar{V}_1 = R_1 \bar{I}_1 + j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2 \qquad (13.21a)$$

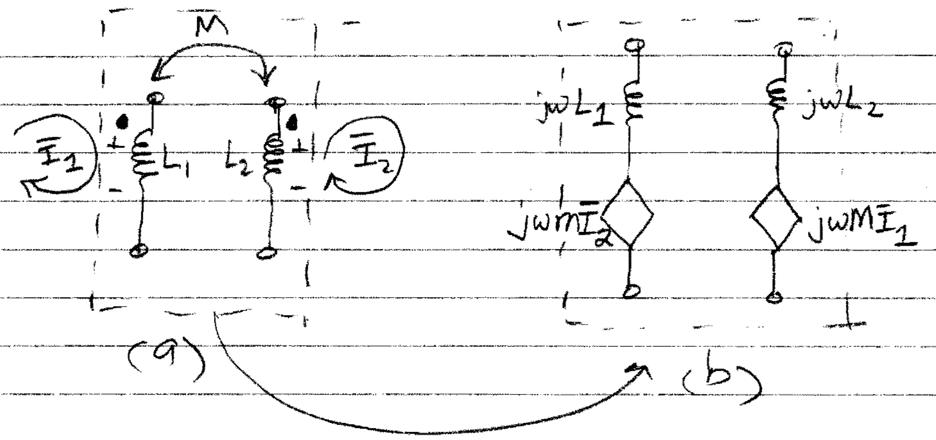
$$\bar{V}_2 = R_2 \bar{I}_2 + j\omega L_2 \bar{I}_2 + j\omega M \bar{I}_1 \qquad (13.21b)$$

The second term in each eqn. is the voltage produced by the impedance of the inductor itself while the third represents a voltage produced by the current in the other coil.

We could analyze such a system of equations in (13.21) in a brute force manner. There are two linearly independent equations. Provided we are supplied with sufficient information (enough $\bar{V}$'s & $\bar{I}$'s) we could solve for up to two unknowns.

But things are tricky with ckts. containing coupled inductors. Including the $M\frac{di}{dt}$ term isn't obvious, and the dot convention business can easily lead to important sign errors.

To aid us in properly incorporating both of these concepts, there is a very, very useful equivalent circuit for coupled inductors shown in Fig 13.8:



In this equivalent circuit model for coupled inductors, the M symbol has been replaced with something far more valuable and helpful: 2 dependent voltage sources. These come from the last terms in eqns (13.2a) & (13.2b).

→ So as a first step in ckt analysis, we replace the coupled L's with equiv. ckt of Fig. 13.8b)
But notice the polarity of the dependent voltage sources are not yet shown.
This is where the dot convention comes into play.

Remember from Fig 13.5(a), but in frequency domain

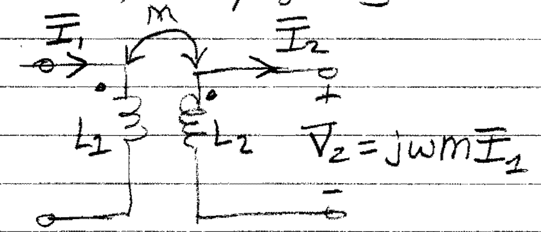
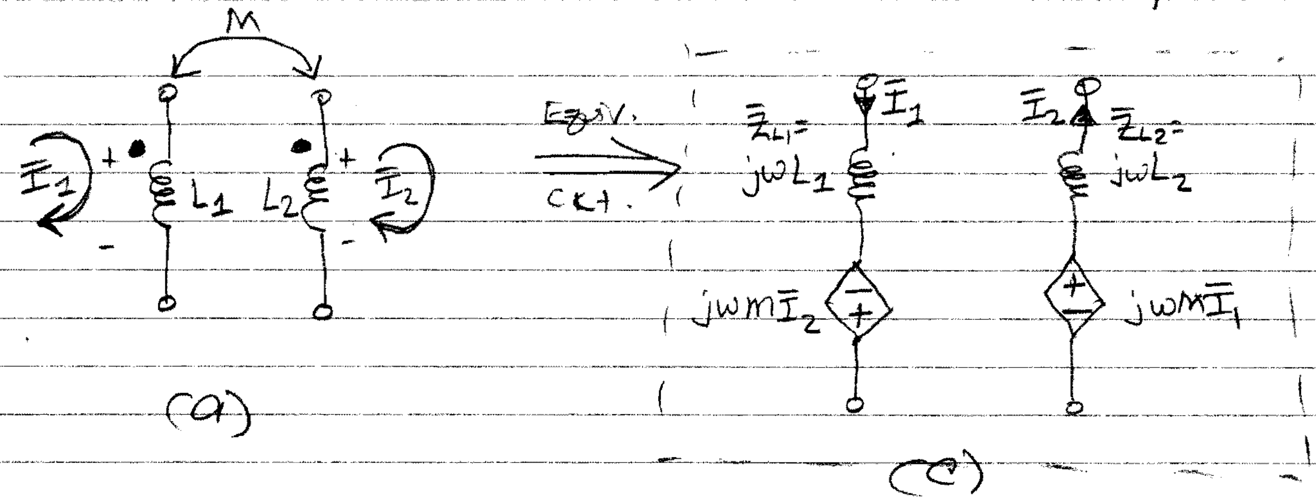


Fig. 1

With I_1 into dotted terminal, V_2 positive with polarity of V_2 as shown.
That is, w/ '+' at dotted terminal for V_2.

In such a circuit, the current I_1 induces a voltage V_2 in "port" 2.
This induced voltage V_2 acts as a source trying to "push" or expel charges from port 2, with I_2 positive as shown in the fig above, Fig. 1.

Knowing this behavior allows us to properly assign the signs of the dependent voltage sources in Fig 13.8(b).
Because I_1 enters the dotted terminal in Fig. 1, the effects of mutual inductance will work to expel charges from the dotted terminal in port 2.
induce a voltage in coil 2 that tries to

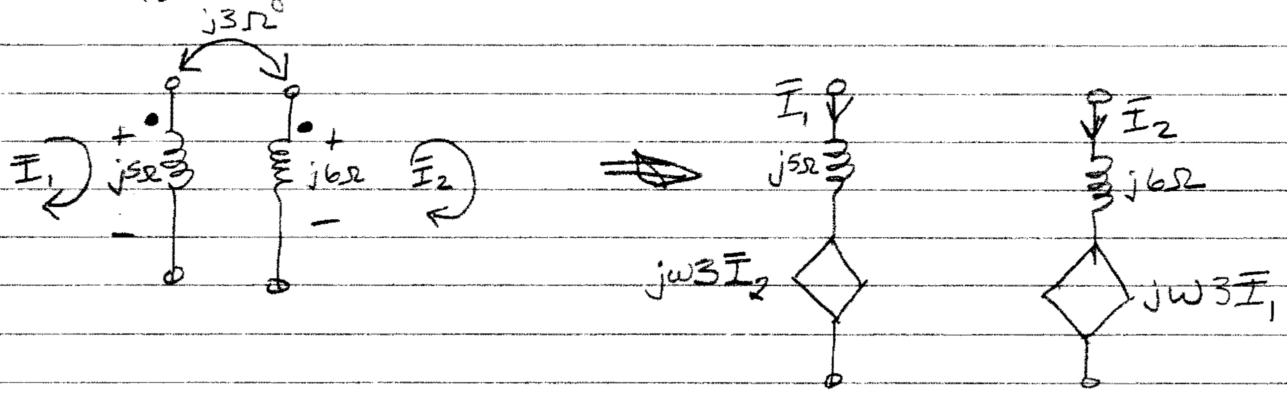


Now we simply insert this equivalent circuit into the original circuit in place of the original mutual inductance and solve the circuit in a straightforward manner as we would in Circuits I.

The effects of mutual inductance and the complexities and pitfalls of the dot convention have all been incorporated into the equivalent circuit of Fig. 13.8(c).

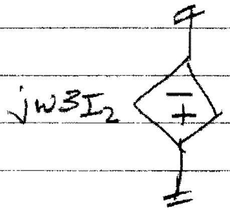
<u>Example 13.1</u>. Calculate the currents I_1 & I_2 in the circuit of Fig. 13.9(a).

The first step is to replace the mutual inductance with the equivalent circuit model of Fig. 13-8(b) — <u>w/o</u> the signs in the dependant voltage sources

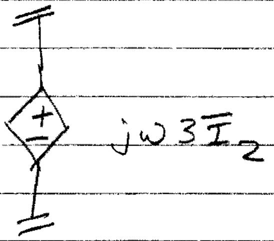


Then add the signs to the dependant voltage sources:

- I_2 into non-dotted terminal induces '−' voltage in port 1 ⇒ pushing current out of <u>non</u>-dotted terminal:



- I_1 into dotted terminal induces '+' voltage in port 2 ⇒ pushing current out of <u>dotted</u> terminal:



This gives the equivalent circuit of Fig. 13.9(b).

Fig 13.9(b)

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The remaining part of the problem is to simply solve for $\bar{I}_1$ & $\bar{I}_2$ in Fig. 13.9(b) using circuit analysis skills from Circuits I.

KVL in Loop 1: $12 = (-j4 + j5)\bar{I}_1 - j3\bar{I}_2$

or $j\bar{I}_1 - j3\bar{I}_2 = 12$ (13.1.1)

KVL in Loop 2: $j3\bar{I}_1 = j6\,\bar{I}_2 + 12\bar{I}_2 = (12+j6)\bar{I}_2$

$\Rightarrow \quad \bar{I}_1 = \frac{12+j6}{j3}\bar{I}_2 = (2-j4)\,\bar{I}_2$ (13.1.2)

Sub this into 13.1.1: $j(2-j4)\bar{I}_2 - j3\bar{I}_2 = 12$

or $j2 + 4\bar{I}_2 - j3\bar{I}_2 = (4-j1)\bar{I}_2 = 12$

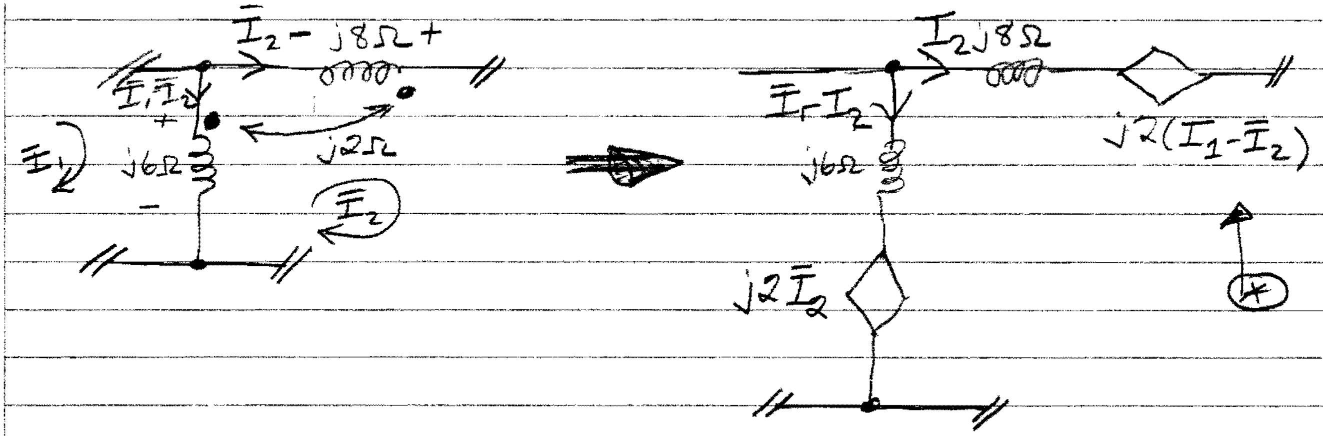
$\therefore \quad \underline{\underline{\bar{I}_2 = 2.91 \angle 14.04^\circ \text{ A}}}$ (13.1.3)

& $\bar{I}_1 = (2-j4)\bar{I}_2 = \underline{\underline{13.01 \angle -49.39^\circ \text{ A}}}$

(13.1.2)

Example 13.2 Calculate the mesh currents $\bar{I}_1$, $\bar{I}_2$ in the ckt. of Fig. 13.11.
Fig. 13.11

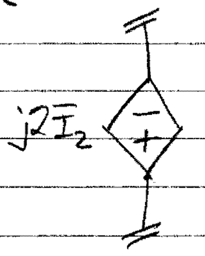
To begin, we replace the mutual inductance with the equivalent circuit model:



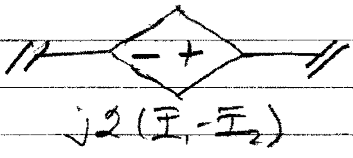
Notice the $\bar{I}_1 - \bar{I}_2$ current: we're solving for mesh currents, not branch currents, so the current in the j6Ω inductor is $\bar{I}_1 - \bar{I}_2$.

For the polarity of the dependent voltage sources:

- $\bar{I}_2$ into non-dotted terminal of j8 induces negative voltage in j6 w/ polarity as indicated, which creates tendency to push charges out of '−' non-dotted terminal



- $\bar{I}_1 - \bar{I}_2$ into dotted terminal of j6 induces positive voltage w/ polarity indicated across j8. This creates propensity to push charges out of the dotted terminal of j8:



Applying KVL to loop 1 in Fig 13.11 using equivalent circuit of Fig 13.12

$$100\angle 0^\circ = (4-j3)\bar{I}_1 + j6(\bar{I}_1-\bar{I}_2) - j2\bar{I}_2$$

or $$100 = (4+j3)\bar{I}_1 - j8\bar{I}_2 \qquad (13.2.2)$$

Then applying KVL to loop 2,

$$j2\bar{I}_2 + j6(\bar{I}_2-\bar{I}_1) + j8\bar{I}_2 - j2(\bar{I}_1-\bar{I}_2) + 5\bar{I}_2 = 0$$

or $$\bar{I}_1(-j6-j2) + \bar{I}_2(j2+j6+j8+j2+5) = 0$$

or $$0 = -j8\bar{I}_1 + (5+j18)\bar{I}_2 = 0 \qquad (13.2.2)$$

$$\Rightarrow \quad \bar{I}_1 = \frac{5+j18}{j8}\bar{I}_2 = (2.250 - j0.625)\bar{I}_2 \qquad (1)$$

Sub this into (13.2.2) gives

$$100 = (4+j3)\cdot(2.250-j0.625)\bar{I}_2 - j8\bar{I}_2$$

$$= (10.875+j4.25)\bar{I}_2 - j8\bar{I}_2 = (10.875-j3.75)\bar{I}_2$$

$$\therefore \underline{\bar{I}_2} = 8.218 + j2.834 = \underline{8.693\angle 19.03^\circ \text{ A}} \qquad (2)$$

and Sub (2) → (1)

$$\underline{\bar{I}_1} = (2.250-j0.625)\cdot(8.218+j2.834) = 20.$$

$$= 20.262 + j1.240 = \underline{\underline{20.300\angle 3.50^\circ \text{ A}}}$$