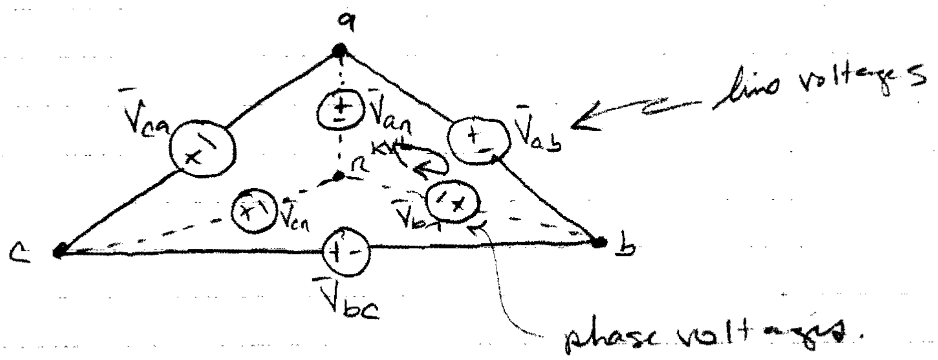
12.6 Balanced Delta-Wye Connection

Fig. 12.18.

Once again, we wish to solve for the line currents in the problem.

There are many ways to accomplish this analysis. We'll transform the Δ connected source to a Wye source, then analyze each phase separately.

The Δ connected source voltages can be related to the Wye connected source voltages as sketched in Fig. 12.19:



We're given the line voltages to be

$$\bar{V}_{ab} = V_p \angle 0^\circ, \quad \bar{V}_{bc} = V_p \angle -120^\circ, \quad \& \quad \bar{V}_{ca} = V_p \angle -240^\circ \quad (12.34)$$

KVL around loop above: $\bar{V}_{an} = \bar{V}_{ab} + \bar{V}_{bn}$ or $\bar{V}_{an} - \bar{V}_{bn} = \bar{V}_{ab}$ (1)

With positive abc sequence phasing, then $\bar{V}_{bn} = \bar{V}_{an} \angle -120^\circ$

$$\therefore \bar{V}_{an} - \bar{V}_{bn} = \bar{V}_{an} - \bar{V}_{an} \angle -120^\circ = \bar{V}_{an}(1 - 1\angle -120^\circ) = \bar{V}_{an}\left(1 + \tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right)$$

$$= \bar{V}_{an} \cdot \sqrt{3} \angle 30^\circ$$

Sub → (1): $\bar{V}_{an}\sqrt{3} \angle 30^\circ = \bar{V}_{ab} = V_p \angle 0^\circ$

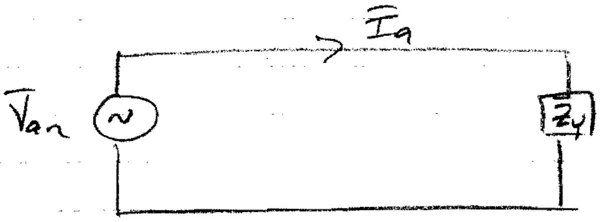
$$\therefore \bar{V}_{an} = \frac{V_p}{\sqrt{3}} \angle -30^\circ \quad (12.38)$$

For positive abc phasing,

$$\bar{V}_{an} = \frac{V_p}{\sqrt{3}} \measuredangle -30^\circ \quad , \quad \bar{V}_{bn} = \frac{V_p}{\sqrt{3}} \measuredangle -150^\circ \quad , \quad \bar{V}_{cn} = \frac{V_p}{\sqrt{3}} \measuredangle 270^\circ \qquad (12.38)$$

With these important formulas, we can transform the Δ connected source into an equivalent Y connected one.

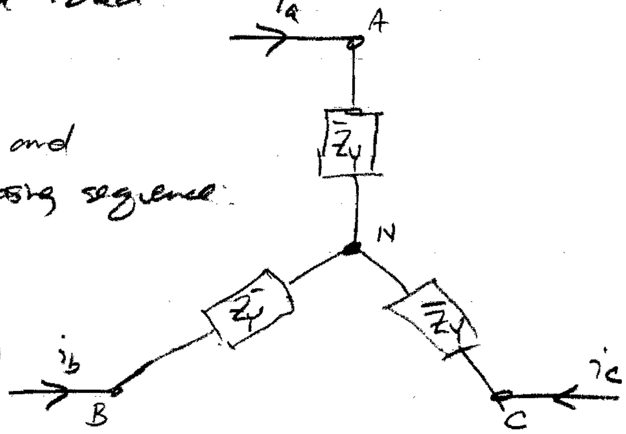
For the a-phase, the equivalent ckt is in Fig. 12.20:



Therefore, the phase current is $\quad \bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}_Y} = \frac{V_p \measuredangle -30^\circ}{\sqrt{3} \cdot \bar{Z}_Y} \qquad (12.39)$

12.7 Power in a Balanced 3ϕ System

Let's consider the power delivered to a balanced three phase load. We'll consider a Y-connected load.



To calculate instantaneous power we'll work with time domain voltages and currents, and we'll assume positive abc phasing sequence.

$$v_{AN} = \sqrt{2}\, V_p \cos(\omega t)$$

$$v_{BN} = \sqrt{2}\, V_p \cos(\omega t - 120°) \quad (12.41)$$

$$v_{CN} = \sqrt{2}\, V_p \cos(\omega t - 240°)$$

where V_p is an RMS voltage.

If $\bar{Z}_Y = Z\angle\theta$, then the line (= phase) currents are

$$i_a = \sqrt{2}\, I_p \cos(\omega t - \theta), \quad i_b = \sqrt{2}\, I_p \cos(\omega t - \theta - 120°), \; \&$$

$$i_c = \sqrt{2}\, I_p \cos(\omega t - \theta - 240°) \quad (12.42)$$

Notice we're not relating V_p : I_p as of yet.

The total instantaneous power delivered to the load is the sum of the instantaneous powers delivered to each $\bar{Z}_Y$:

$$p(t) = p_a + p_b + p_c = v_{AN} \cdot i_a + v_{BN} \cdot i_b + v_{CN} \cdot i_c \quad (12.43)$$

$$= 2 V_p I_p \left[\cos(\omega t) \cdot \cos(\omega t - \theta) + \cos(\omega t - 120°)\cos(\omega t - \theta - 120°)\right.$$

$$\left. + \cos(\omega t - 240°)\cos(\omega t - \theta - 240°)\right]$$

Using the trig id $\quad \cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right] \quad (12.44)$

Then:

$$\cos(\underbrace{\omega t}_{A})\cos(\underbrace{\omega t-\theta}_{B}) = \frac{1}{2}\left[\cos(\underbrace{2\omega t-\theta}_{A+B}) + \cos(\underbrace{\theta}_{A-B})\right]$$

$$\cos(\omega t-120^\circ)\cos(\omega t-\theta-120^\circ) = \frac{1}{2}\left[\cos(2\omega t-\theta-240^\circ)+\cos(\theta)\right]$$

$$\cos(\omega t-240^\circ)\cos(\omega t-\theta-240^\circ) = \frac{1}{2}\left[\cos(2\omega t-\theta-480^\circ)+\cos(\theta)\right]$$

Sub. these into (2.43) gives

$$p^{(t)} = 2V_pI_p\cdot\frac{1}{2}\left[3\cos(\theta) + \cos(2\omega t-\theta) + \cos(2\omega t-\theta-240^\circ) + \cos(2\omega t-\theta-480^\circ)\right] \qquad (1)$$

Let's focus on these last three terms: with $\gamma \equiv 2\omega t - \theta$:

$$\cos(\gamma) + \cos(\gamma-240^\circ) + \cos(\gamma-120^\circ) \qquad (2)$$

$$= \cos(\gamma) + \cos(\gamma-240^\circ) + \cos(\gamma+240^\circ)$$

Trig i-d's:

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\therefore \quad \cos(\alpha-\beta)+\cos(\alpha+\beta) = 2\cos\alpha\cos\beta.$$

w/ $\alpha=\gamma$ in (2); $\beta = 240^\circ$, then (2) becomes

$$\cos(\gamma) + \cos(\gamma-240^\circ) + \cos(\gamma+240^\circ) = \cos\gamma + 2\cos\gamma\underbrace{\cos(240^\circ)}_{=-1/2}$$

$$= \cos\gamma - \cos\gamma = \underline{\underline{0}}$$

Hence (1) becomes

$$\underline{p(t)} = V_p I_p \cdot 3\cos\theta = \underline{3 V_p I_p \cos\theta} \qquad (12.45)$$

Hugely important result: this total instantaneous power delivered to all three loads doesn't vary w/ time! Can show this is the same result for a Δ connected balanced 3ϕ load.

Since the instantaneous power $p(t)$ in (12.45) is independant of time, the time-average power delivered to the load P is equal to that:

$$P = 3 V_p I_p \cos\theta$$

While the real power delivered to each phase of the load, P_p, is simply one third of this since it is a balanced load:

$$P_p = \frac{P}{3} = V_p I_p \cos\theta \qquad (12.46)$$

The associated reactive power per phase is then

$$Q_p = V_p I_p \sin\theta \qquad (12.47)$$

Such that the apparent power per phase is

$$S_p = V_p I_p \qquad (12.48)$$

and the complex power per phase is

$$\bar{S}_p = P_p + jQ_p = \bar{V}_p \bar{I}_p^{*} \qquad (12.49)$$

We can write the total apparent power (12.46) in a slightly different form

$$P = \sqrt{3}\, V_L I_L \cos\theta \qquad (12.50)$$

where V_L & I_L are <u>line</u> voltage & current, respectively.

Where for a Y-Y connected load $I_L = I_p$ & $V_L = \sqrt{3}\, V_p$.

While for a Δ-Δ connected load $I_L = \sqrt{3}\, I_p$ & $V_L = V_p$

Similarly, the total reactive load power can be expressed as

$$Q = 3Q_p = 3V_p I_p \sin\theta = \sqrt{3}\, V_L I_L \sin\theta \quad (12.51)$$

Consequently, the total complex load power can be expressed as

$$\bar{S} = 3\bar{S}_p = 3\bar{V}_p \bar{I}_p^*$$

Alternatively, $\bar{S} = \underset{\uparrow\ (12.50);\ (2.51)}{P + jQ} = \sqrt{3}\, V_L I_L \angle\theta \quad (12.53)$

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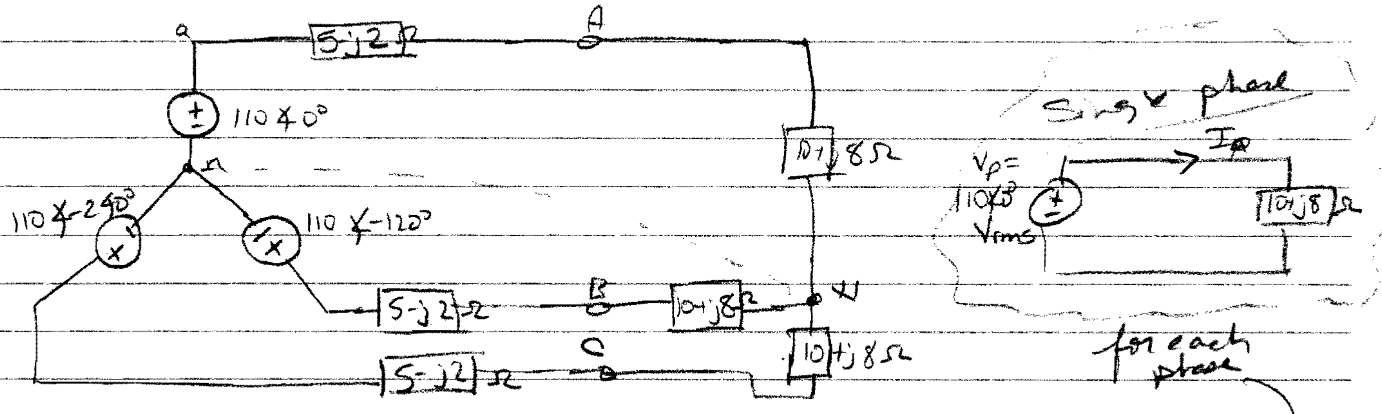
Example 12.6 – Y-Y connected balanced 3φ System of Example 12.2.



From Example 12.2, $\bar{V}_p = 110 \angle 0^\circ \, V_{rms}$ & $I_p = 6.81 \angle -21.8^\circ \, A_{rms}$

Determine the total average power, reactive power, and complex power supplied by the source, and delivered to the load.

The source complex power is

$$\bar{S}_s = \underset{\substack{\uparrow \\ \text{source}}}{-} 3 \bar{V}_p \bar{I}_p^* = -3 \, (110 \angle 0^\circ \, V_{rms}) \cdot (6.81 \angle +21.8^\circ \, A_{rms})$$

$$= \underbrace{-2086.59}_{= P_s \text{ total time avg. power supplied}} - j \, \underbrace{834.57}_{= Q_s = \text{ total reactive power supplied.}} \; VA =$$

At the load

$$\bar{S}_L = 3 \bar{V}_{rms} \cdot \bar{I}_{rms}^* \qquad \text{but} \quad \bar{V}_{rms} = \bar{I}_{rms} \cdot \bar{Z}_p$$

not line

$$\therefore \bar{S}_L = 3 \bar{I}_{rms} \cdot \bar{I}_{rms}^* \bar{Z}_p = 3 |\bar{I}_{rms}|^2 \bar{Z}_p.$$

$$= 3 \, (6.81)^2 \cdot (10 + j8)$$

$$= \underbrace{1391.28}_{\text{total time avg. power delivered to } 3\phi \text{ load}} + j \, \underbrace{1{,}113.03}_{\text{total reactive power delivered to load.}} \; VA$$

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Notice that $\bar{S}_S \neq \bar{S}_L$. There is also complex power delivered to the line impedances, $\bar{S}_\ell$.

$$\bar{S}_\ell = 3|\bar{I}_p|^2 Z_\ell = 3\cdot(6.8))^2\cdot(5-j2)$$

$$= 695.64 - j278.26 \text{ VA}.$$

Now, $\bar{S}_L + \bar{S}_\ell = 2086.92 + j834.77$ VA $= -\bar{S}_S$, as expected.

Example 12.7

V_L

A three phase motor can be regarded as a balanced Y connected load. Suppose a 3ϕ motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor. I_L

$$V_L = 220\ V_{rms} \quad \& \quad I_L = 18.2\ A_{rms}$$

The apparent power $S = |\bar{S}| \underset{(12.53)}{=} |\sqrt{3}\, V_L I_L \angle\theta|$

$$\therefore \quad S = \sqrt{3}\, V_L I_L = \sqrt{3}\,(220)(18.2) = 6935.13\ VA$$

The real (time avg) power was given to be 5600 W.

$$\therefore \quad \underline{\underline{pf}} = \cos\theta = \frac{P}{S} = \frac{5600}{6935.13} = \underline{\underline{0.8075}}$$