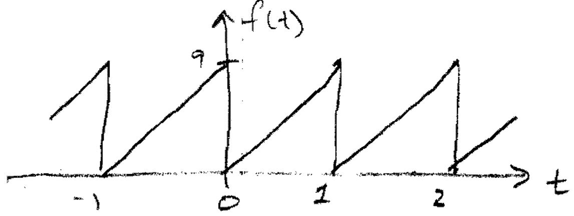
Example 17.11. Calculate the complex Fourier series representation of the Sawtooth waveform of Fig. 17.9. Plot the complex amplitude and phase spectra.



We've had quite a bit of experience calculating the Fourier series of this waveform, and working w/ it in the lab.

$T = 1 \Rightarrow \omega_0 = \frac{2\pi}{T} = 2\pi$ rad/s.

From (17.59), $$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}\,dt = \frac{1}{1}\int_0^1 t\,e^{-jn2\pi t}\,dt \quad (1)$$

Let $u = t \Rightarrow du = dt$

$dv = e^{-jn2\pi t}dt \Rightarrow v = -\frac{1}{jn2\pi}e^{-jn2\pi t}$ provided $\underline{\underline{n \neq 0}}$

$$c_n = uv - \int v\,du = t\left(-\frac{1}{jn2\pi}\right)e^{-jn2\pi t}\Big|_0^1 - \int_0^1\left(-\frac{1}{jn2\pi}\right)e^{-jn2\pi t}\,dt$$

$$= -\frac{1}{jn2\pi}\left[1\cdot e^{-jn2\pi} - 0\right] + \frac{1}{jn2\pi}\left(\frac{-1}{jn2\pi}\right)e^{-jn2\pi t}\Big|_0^1$$

$$= -\frac{1}{jn2\pi}e^{-jn2\pi} + \frac{1}{(n2\pi)^2}\left(e^{-jn2\pi} - 1\right) \quad (2)$$

From Euler's identity, $e^{jx} = \cos x + j\sin x$

$\therefore\ e^{-jx} = \cos x - j\sin x$

$$\therefore\ e^{-jn2\pi} = \underbrace{\cos(n2\pi)}_{=1} - j\underbrace{\sin(n2\pi)}_{=0} = 1$$

So, (2) becomes

$$\underline{\underline{c_n}} = -\frac{1}{jn2\pi} = \underline{\underline{\frac{j}{n2\pi}}} \qquad \forall n,\ n \neq 0. \quad (3), (17.11.3)$$

The integration by parts was not valid for $n=0$, since n was in the denominator leading to an infinite result. So, we'll perform the calculation of c_0 directly.

using (1) w/ $n=0$ gives

$$\underline{\underline{c_0}} = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{1}\cdot\int_0^1 t\,dt = \frac{t^2}{2}\Big|_0^1 = \underline{\underline{\frac{1}{2}}}. \qquad (4), (17.11.4)$$

substituting c_0 & c_n into the complex form of the Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad (5), (17.58)$$

gives

$$\underline{\underline{f(t) = 0.5 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j}{n2\pi} e^{jn2\pi t}}} \qquad (6), (17.11.5)$$

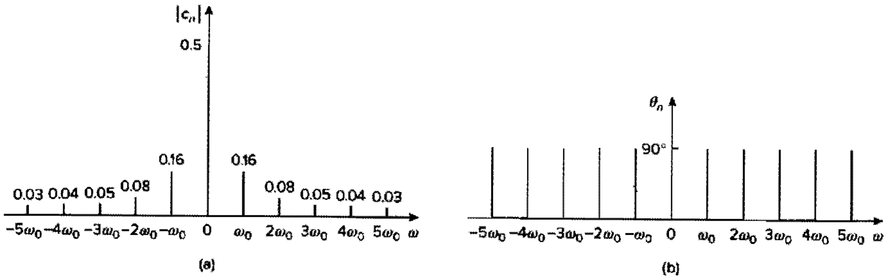
Even though complex #'s this yields a purely real function f(t).

The amplitude spectra from (3) & (4) is where

$$|c_n| = \begin{cases} 0.5 & n=0 \\ \frac{1}{n2\pi} & n\neq 0 \end{cases} \qquad (7), (17.11.6)$$

$$\theta_n = \measuredangle c_n = \begin{cases} 0^\circ & n=0 \\ 90^\circ & n\neq 0 \end{cases}$$

Fig. 17.31: complex frequency spectrum:



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In[7]:= f[nmax_, t_] := 1 / 2 + Sum[i / (n * 2 * Pi) * Exp[i * n * 2 * Pi * t], {n, -1, -nmax, -1}] +
          Sum[i / (n * 2 * Pi) * Exp[i * n * 2 * Pi * t], {n, 1, nmax}]

In[8]:= Plot[f[3, t], {t, -2, 2}, AxesLabel → {"t", "f(t)"}]
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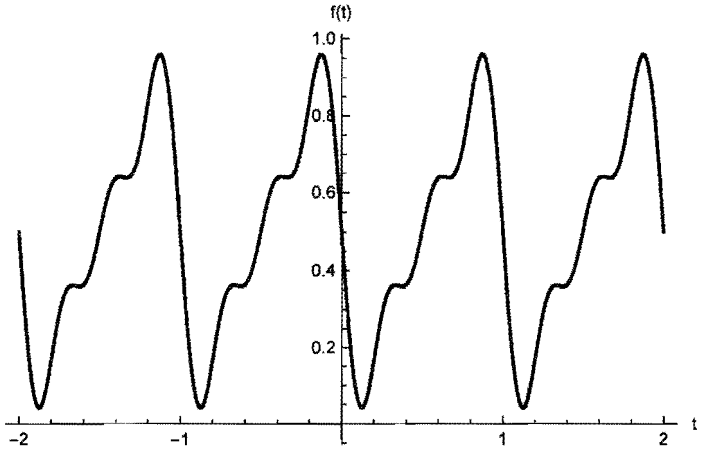
Out[8]=

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In[10]:= Plot[f[20, t], {t, -2, 2}, AxesLabel → {"t", "f(t)"}]
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Out[10]=