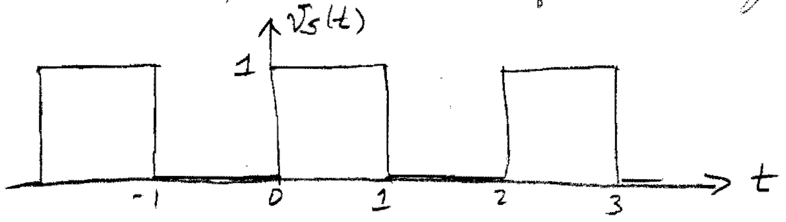
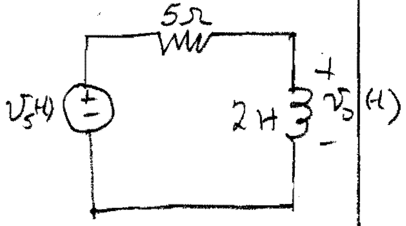
Example 17.6 Determine $v_o(t)$ in the circuit below if the input source voltage is the square wave of Example 17.1:





To analyze this circuit for $v_o(t)$, we note that while $v_s(t)$ is a steady state periodic function, we cannot use sinusoidal steady state (phasor) analysis since v_s is not a sinusoid.

A very useful approach here is to expand $v_s(t)$ in a Fourier series, which we did in Example 17.1.

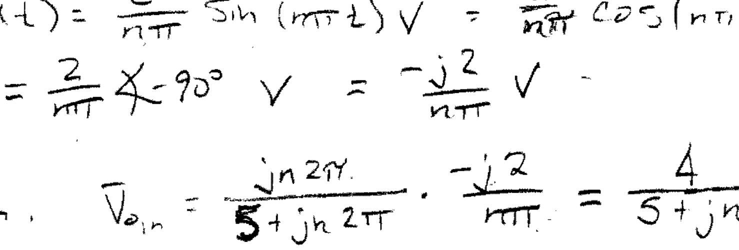
$$v_s(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{\substack{n=1 \\ (n\ odd)}}^{\infty} \frac{1}{n}\sin(n\pi t) \quad V \qquad (1),(17.1.7)$$

Because the circuit is linear, we can use superposition. We note that (1) has a DC component and a sum of sinusoids of different frequencies $n\omega_0 = n\pi$. Each of these "harmonics" can be used as a phasor input voltage to the phasor domain ckt and we can solve for the phasor output voltage at that frequency.

- DC analysis: Replace L w/ short circuit $\Rightarrow$ $V_o\big|_{DC} = 0\ V$.

$V_s\big|_{DC} = \frac{1}{2}$ V

- AC analysis: The phasor domain circuit at frequency $\omega = n\omega_0 = n\pi$ rad/s is:



From (1)

$$v_{s,n}(t) = \frac{2}{n\pi}\sin(n\pi t)\ V = \frac{2}{n\pi}\cos(n\pi t - 90^\circ)\ V$$

$$\therefore\quad \bar{V}_{s,n} = \frac{2}{n\pi}\angle -90^\circ\ V = \frac{-j2}{n\pi}\ V$$

By voltage division. $$\bar{V}_{o,n} = \frac{jn2\pi}{5 + jn2\pi}\cdot\frac{-j2}{n\pi} = \frac{4}{5 + jn2\pi}\ V$$

So this output voltage will change depending on the frequency of the harmonic that is input from the source.

In magnitude angle form:

$$\bar{V}_{o,n} = \frac{4}{|5+jn2\pi|} \measuredangle -\tan^{-1}\left(\frac{n2\pi}{5}\right)$$

$= \sqrt{(5+jn\pi)\cdot(5+jn\pi)^*}$

$$\rightarrow |5+jn2\pi| = \sqrt{(5+jn2\pi)(5-jn2\pi)} = \sqrt{25+n^2 4\pi^2}$$

$$\therefore \bar{V}_{on} = \frac{4}{\sqrt{25+4n^2\pi^2}} \measuredangle -\tan^{-1}\left(\frac{n2\pi}{5}\right)$$

This is the phasor output voltage for the sinusoidal steady state input voltage at the frequency of $n\omega_0 = n\pi$ rad/s.

In the time domain, notice ω_n!

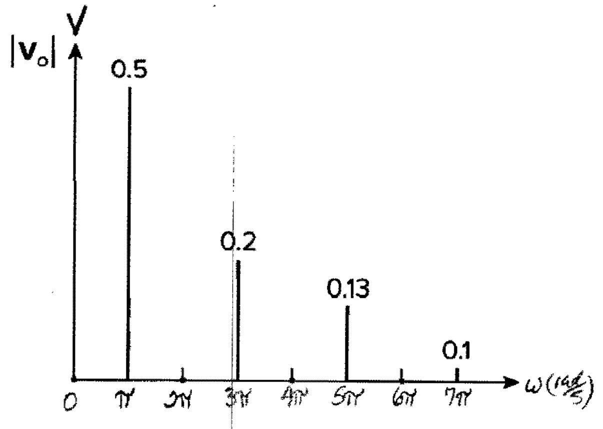
$$v_{o,n}(t) = \mathrm{Re}\left\{\bar{V}_{on} e^{j\omega_n t}\right\} = \mathrm{Re}\left\{\frac{4}{\sqrt{25+4n^2\pi^2}} \angle -\tan^{-1}\left(\frac{2n\pi}{5}\right) \cdot e^{j\omega_n t}\right\}$$

$$\therefore v_{o,n}(t) = \frac{4}{\sqrt{25+4n^2\pi^2}} \cos\left[n\pi t - \tan^{-1}\left(\frac{2n\pi}{5}\right)\right] \quad (n \text{ odd})$$

Because this circuit is linear, we can use superposition and construct the total voltage - the actual solution - by summing all of these harmonics together w/ the dc soln:

$$v_o(t) = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \frac{4}{\sqrt{25+4n^2\pi^2}} \cos\left[n\pi t - \tan^{-1}\left(\frac{2n\pi}{5}\right)\right] \quad V$$

Amplitude Spectrum - Fig. 17.21



Add Example 17.7

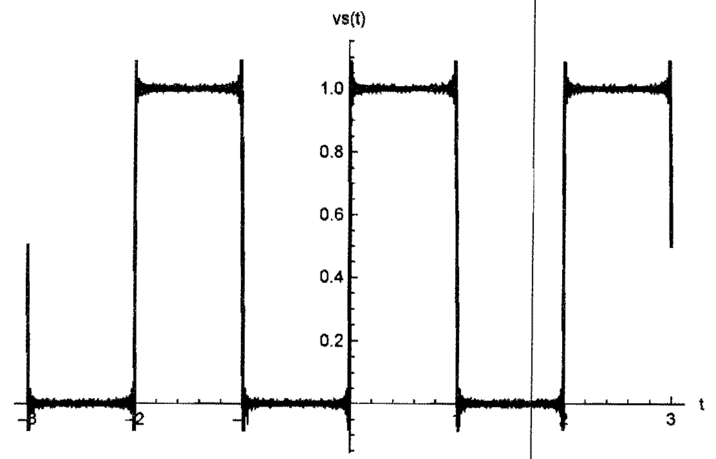
Whites EE 221 - Circuits II

In[5]:=
```
vs[nmax_, t_] := 1 / 2 + 2 / Pi * Sum[1 / n * Sin[n * Pi * t], {n, 1, nmax, 2}]
vo[nmax_, t_] :=
 Sum[4 / Sqrt[25 + 4 * n^2 * Pi^2] * Cos[n * Pi * t - ArcTan[2 * n * Pi / 5]], {n, 1, nmax, 2}]
```

n=odd

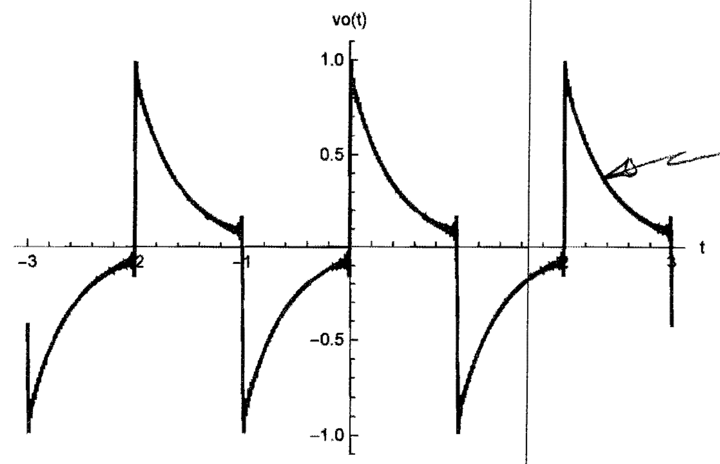
In[7]:=
```
Plot[vs[101, t], {t, -3, 3}, AxesLabel → {"t", "vs(t)"}]
```

Out[7]=



In[8]:=
```
Plot[vo[101, t], {t, -3, 3}, AxesLabel → {"t", "vo(t)"}]
```

Out[8]=



makes sense: This is an STC ckt ; $\tau = \frac{L}{R} = \frac{2}{5} = 0.4$ sec.