Amplitude - Phase Form of the Fourier Series

There is more than one way of expressing a Fourier Series expansion. The trigonometric form we just saw is very popular. Another form that is very popular is the Amplitude - Phase form we'll discuss now.

In amplitude-phase form, the Fourier Series is expressed as

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n) \qquad (17), (17.10)$$

The coefficients A_n & phases ϕ_n are the two coefficients we must determine for each frequency harmonic n.

We can easily derive these formulas for A_n & ϕ_n in terms of the coefficients a_n & b_n.

We begin w/ the trig i.d. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

and applying this to each term in (17) giving (w/ $\alpha = n\omega_0 t$ & $\beta = \phi_n$)

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n\left[\cos(n\omega_0 t)\cos(\phi_n) - \sin(n\omega_0 t)\sin(\phi_n)\right]$$

$$= a_0 + \sum_{n=1}^{\infty}\left[A_n\cos(\phi_n)\cos(n\omega_0 t) - A_n\sin(\phi_n)\sin(n\omega_0 t)\right]$$

We can equate these coefficients with those from the trig. form in (3):

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\right] \qquad (3)$$

yielding $\quad a_n = A_n\cos\phi_n \quad$ & $\quad b_n = -A_n\sin\phi_n \qquad (18)$

To solve for A_n, we square both equations and sum:

$$a_n^2 = A_n^2\cos^2\phi_n \quad ; \quad b_n^2 = A_n^2\sin^2\phi_n$$

$$\Rightarrow \quad a_n^2 + b_n^2 = A_n^2\underbrace{\left(\cos^2\phi_n + \sin^2\phi_n\right)}_{=1} = A_n^2$$

$$\therefore \boxed{A_n = \pm\sqrt{a_n^2 + b_n^2}} \qquad (19), (17.13b)$$

while dividing the two yields

$$\frac{b_n}{a_n} = \frac{-A_n \sin(\phi_n)}{A_n \cos(\phi_n)} = -\tan(\phi_n)$$

or

$$\boxed{\phi_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)} \qquad (20), (17.13b)$$

Another way to express these relationships is through this single complex form:

$$\boxed{A_n \measuredangle \phi_n = a_n - jb_n.} \qquad (21), (17.14)$$

Types of Spectra

- A plot of A_n versus n or $n\omega_0$ is called the amplitude spectrum of $f(t)$
- A plot of ϕ_n versus n or $n\omega_0$ is called the phase spectrum of $f(t)$.
- Both the amplitude & phase spectrum form the frequency spectrum of $f(t)$

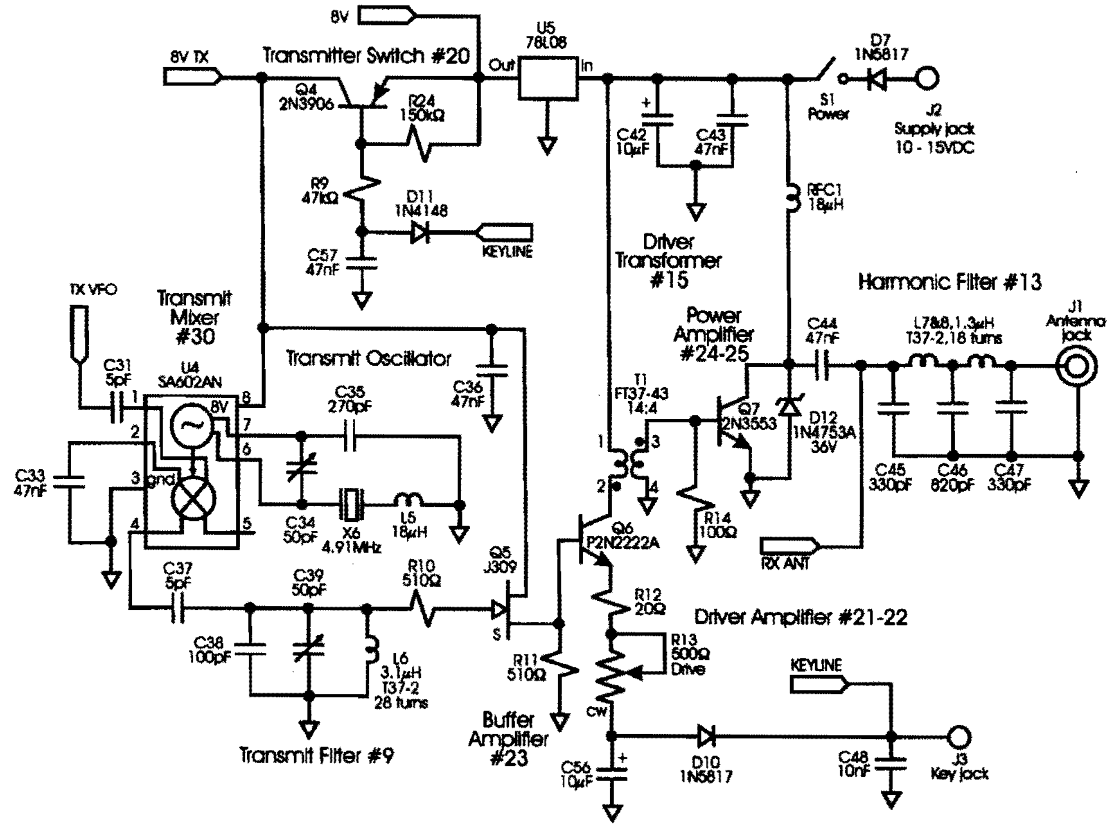
Consequently the Fourier series is a mathematical tool that allows us to calculate the frequency spectrum of a periodic signal.

A spectrum analyzer allows us to measure this spectrum in the laboratory.

NorCal 40A transmitter Spectrum (next page)

Example 17.1 Determine frequency spectrum of the square wave of Fig. 17.1, P101.

Anagostou notes.



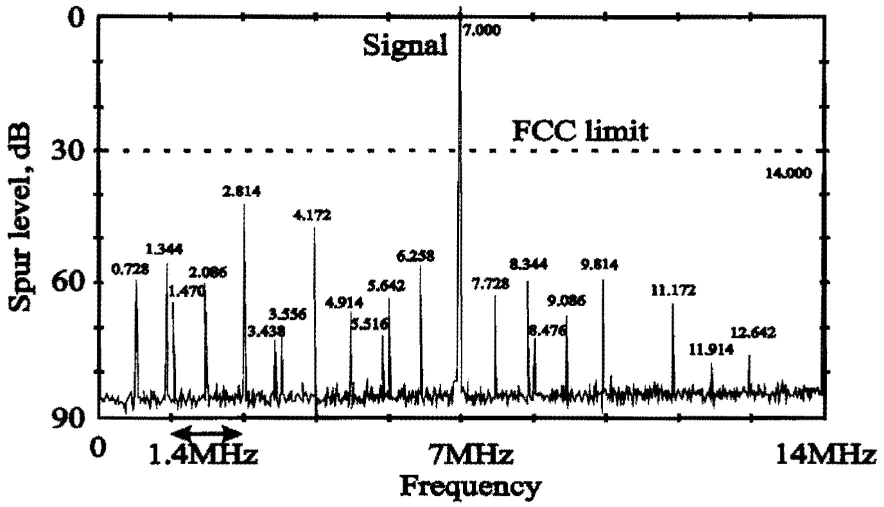


Figure 12.15. Frequency components for the NorCal 40A. The vertical axis shows the power on a dB scale, at 10 dB per division, and the horizontal axis shows the frequency, with a scale of 1.4 MHz per division. The different frequency components appear as lines on the plot, and the frequency is noted at each component.