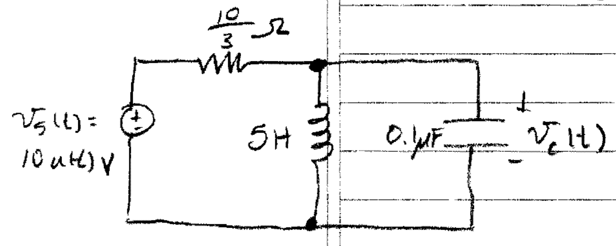
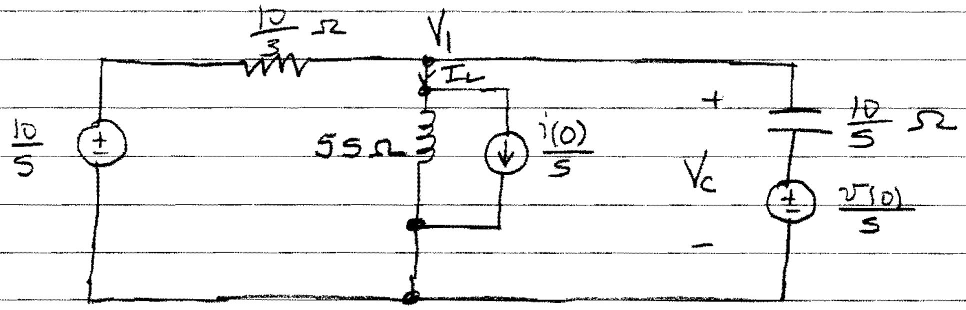
Example 16.4 Calculate $v_c(t)$ in the circuit below if at $t=0$, $-1\,A$ flows through the inductor an $+5V$ exists across the capacitor.



The first step is to construct the equivalent s-domain electrical ckt noting that both L & C have initial conditions. use Figs. 16.1 & 16.2:



Note that V_c is the voltage across C + $\frac{v_0}{s}$. $V_c = V_1$

using KCL at the V_1 node.

$$\frac{V_1 - \frac{10}{s}}{10/3} + \frac{V_1}{5s} + \frac{i(0)}{s} + \frac{V_1 - \frac{v(0)}{s}}{10/s} = 0 \qquad (16.4.1)$$

w/ $i(0) = -1$ & $v(0) = 5 \Rightarrow$

$$\frac{3}{10}V_1 - \frac{3}{s} + \frac{V_1}{5s} - \frac{1}{s} + \frac{s}{10}V_1 - \frac{5}{10} = 0$$

$$\left(\frac{3}{10} + \frac{1}{5s} + \frac{s}{10}\right)V_1 = \frac{3}{s} + \frac{1}{s} + \frac{1}{2}$$

$$\frac{1}{10}\left(s + 3 + \frac{2}{s}\right)V_1 = \frac{4}{s} + \frac{1}{2}$$

or $(s^2 + 3s + 2)V_1 = 40 + 5s$

$$V_1 = \frac{40+5s}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

P.F.E.

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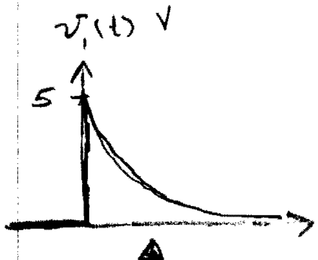
Solving the P.F.E. gives $A = \frac{40-5}{1} = 35$

$B = \frac{40-10}{-1} = -30$



$\therefore V_1 = \frac{35}{s+1} - \frac{30}{s+2}$

$\underline{\underline{v_1(t)}} = \mathcal{L}^{-1}\{V_1\} = \underline{\underline{35e^{-t}u(t) - 30e^{-2t}u(t)}}$

At $t=0$, $v_1(0) = 35 - 30 = 5V$ ✓

Referring to the original ckt, does this time domain response make sense?

Practice Problem 16.4 For the previous problem, determine the current through the inductor $\forall t > 0$.

We determined that $V_1 = \frac{35}{s+1} + \frac{30}{s+2}$ V.

Then, referring to the s-domain circuit : $i(0) = -1A$

Tricky:

$$I_L = \frac{V_1}{5s} + \frac{i(0)}{s} = \frac{35}{5s(s+1)} - \frac{30}{5s(s+2)} - \frac{1}{s}$$

initial condition.

From Table 15.1 $\frac{1}{s}F(s) \longleftrightarrow \int_0^t f(x)\,dx$

$$i_L(t) = \int_0^t 7e^{-\tau}\,d\tau - \int_0^t 6e^{-2\tau}\,d\tau - u(t)$$

$$= \frac{7}{(-1)}e^{-\tau}\Big|_0^t - \frac{6}{(-2)}e^{-2\tau}\Big|_0^t - u(t)$$

$$= -7(e^{-t}-1)^{u(t)} + 3(e^{-2t}-1)^{u(t)} - u(t)$$

$$= (-7e^{-t} + 7 + 3e^{-2t} - 3 - 1)\,u(t)$$

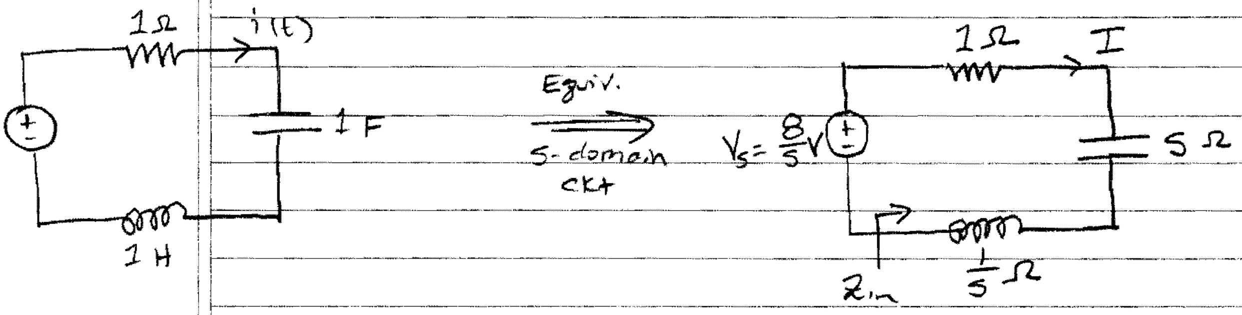
$\therefore \quad \underline{i_L(t) = [3 - 7e^{-t} + 3e^{-2t}]u(t) \ A}$

Check initial: find states

- at $t=0$, $i_L(t) = 3-7+3 = -1A$ ✓ $= i(0)$
- at $t=\infty$? $i_L(t) = 3A$. Make sense?

Check: $i_L = i_L = \frac{10}{10/3} = 3A$ ✓ Yes.

Problem 16.12 Determine $i(t)$ in the circuit shown below



$Z_{in} = 1 + s + \frac{1}{s}$ $\qquad (s+\frac{1}{2})^2$ $\quad s^2 +$

$\therefore \ I = \frac{V_s}{Z_{in}} = \frac{8/s}{1+s+\frac{1}{s}} = \frac{8}{s^2+s+1}$

Complete the square: $I = \frac{8}{(s+\frac{1}{2})^2 + \frac{3}{4}}$

$(s+\frac{1}{2})^2 = s^2 + \frac{s}{2} + \frac{s}{2} + \frac{1}{4}$

With an eye towards the Laplace transform pair

$$e^{-at}\sin\omega t \xleftrightarrow{\mathcal{L}} \frac{\omega}{(s+a)^2+\omega^2}$$

then $\omega^2 = \frac{3}{4}$

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$$I = 8\,\frac{1}{\frac{\sqrt{3}}{2}}\,\frac{\sqrt{3}/2}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

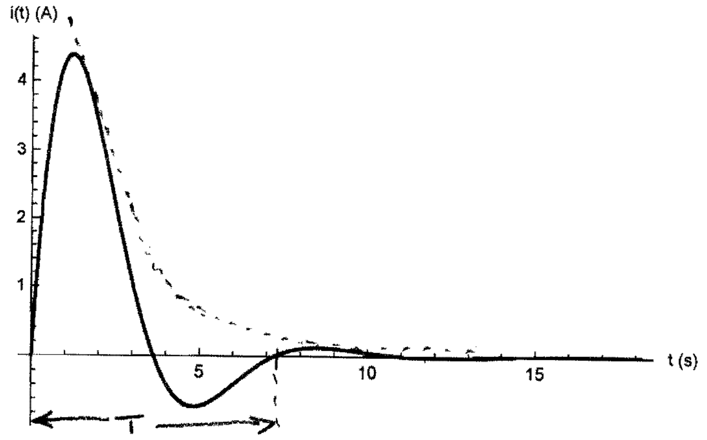
$\Rightarrow$

$$\underline{\underline{i(t)}} = \mathcal{L}^{-1}\{I\} = \frac{16}{\sqrt{3}}\,e^{-\frac{t}{2}}\,\sin\left(\frac{\sqrt{3}}{2}t\right)\,u(t)\ A$$

- Mathematica Plot
- Why oscillations?
- Why damping?
- B2Spice
- Diff eq.

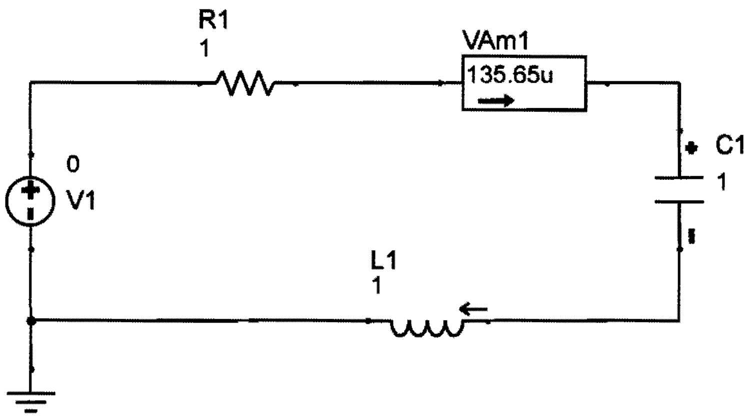
In[8]:=
```
Plot[16 / Sqrt[3] * Exp[-t / 2] * Sin[Sqrt[3] / 2 * t],
  {t, 0, 5 * 2 * Pi / Sqrt[3]}, PlotRange → All, AxesLabel → {"t (s)", "i(t) (A)"}]
```

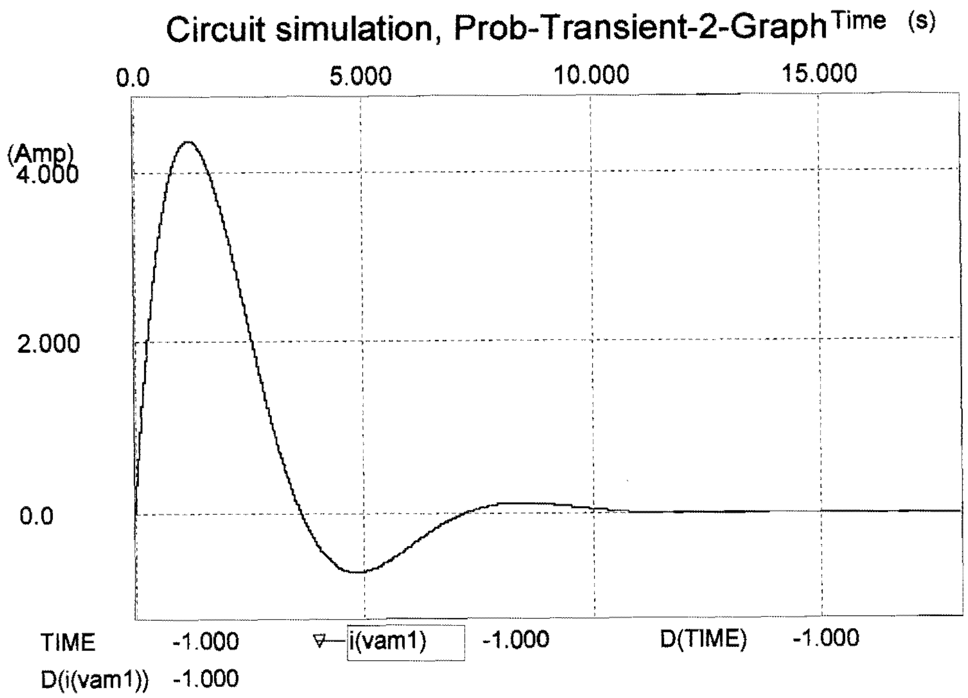
Out[8]=



$$\omega = \frac{\sqrt{3}}{2} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}/2} = \frac{4\pi}{\sqrt{3}} = 7.26\ s$$

B2 Spice simulation





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Chapter 6 of the text.

Differential eqn. approach. What if we didn't use Laplace x'form. How would we solve such a problem?

Form integro-differential eqn & solve.

For C: $$v_C(t) = \frac{1}{C}\int_{t_0}^{t} i_C(\tau)\,d\tau + v(t_0) \qquad (6.6)$$

For L: $$v_L(t) = L\,\frac{di_L(t)}{dt} \qquad (6.18)$$

By KVL:

$$v_S(t) = v_R(t) + v_C(t) + v_L(t)$$

For $t > 0$ & zero initial conditions

$$8 = 1 \cdot i(t) + \frac{1}{1}\int_0^t i(\tau)\,d\tau + 1 \cdot \frac{di(t)}{dt}$$

or $$\frac{di(t)}{dt} + \int_0^t i(\tau)\,d\tau + i(t) = 8.$$

This is an integro-differential eqn. Difficult to solve.