12.4 Balanced Wye-Delta Connection.

Fig 12.14.

There are two primary, but equivalent, approaches to solving such an arrangement.

In the previous section we determined the line voltages in terms of the phase voltages. It may not have been apparent why we did that then, but it should now:

$$\bar{V}_{ab} = \sqrt{3}\, V_p \angle 30^\circ$$
$$\bar{V}_{bc} = \sqrt{3}\, V_p \angle -90^\circ \qquad (12.20)$$
$$\bar{V}_{ca} = \sqrt{3}\, V_p \angle -210^\circ$$

In this Wye-Delta connection, in Fig 12.14, it is easy to see that

$$\bar{V}_{AB} = \bar{V}_{ab}, \quad \bar{V}_{BC} = \bar{V}_{bc}, \quad \& \quad \bar{V}_{CA} = \bar{V}_{ca} \qquad (12.20)$$

Consequently, the load phase currents are

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z_\Delta}, \quad \bar{I}_{BC} = \frac{\bar{V}_{BC}}{Z_\Delta}, \quad \& \quad \bar{I}_{CA} = \frac{\bar{V}_{CA}}{Z_\Delta} \qquad (12.21)$$

The line currents (I_a, I_b, & I_c) can be computed from these load phase currents using KCL at nodes A, B & C, respectively.

For example, at node A in Fig. 12.14,

$$\bar{I}_a = \bar{I}_{AB} - \bar{I}_{CA} \underset{(12.21)}{=} \frac{\bar{V}_{AB}}{Z_\Delta} - \frac{\bar{V}_{CA}}{Z_\Delta} \underset{(12.20)}{=} \frac{\bar{V}_{AB}}{Z_\Delta} \underbrace{\left(1 - 1\angle -240^\circ\right)}_{= \sqrt{3}\angle -30^\circ}$$

$$\therefore \quad \bar{I}_a = \bar{I}_{AB} \cdot \sqrt{3} \angle -30^\circ \qquad (12.24)$$

$\sqrt{3}$ larger than load phase current, & lags by 30°

In general, $|\bar{I}_L| = \sqrt{3}\,|\bar{I}_p|$ (12.25)

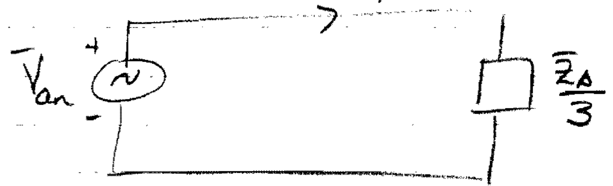
where $|\bar{I}_L| = |\bar{I}_a| = |\bar{I}_b| = |\bar{I}_c|$ (12.26)

& $|\bar{I}_p| = |\bar{I}_{AB}| = |\bar{I}_{BC}| = |\bar{I}_{CA}|$ (12.27)

Alternatively, could analyze this Wye-Delta connection by converting the Δ connected to a Y connected load noting that

$$\bar{Z}_Y = \frac{\bar{Z}_\Delta}{3} \qquad (12.28).$$

Then, analyze each phase separately using a single phase equivalent ckt. Fig 12.14



$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}_\Delta/3} = \frac{3\bar{V}_{an}}{\bar{Z}_\Delta}$$ ← same phase voltage.

but $\bar{V}_{an} = V_p \measuredangle 0^\circ$ (12.19)

so $$\bar{I}_a = \frac{3V_p \measuredangle 0^\circ}{\bar{Z}_\Delta} \qquad (1)$$

While from the direct analysis of the Y-Δ ckt (12.24)

$$\bar{I}_a = \bar{I}_{AB}\sqrt{3}\measuredangle -30^\circ \underset{\text{Fig 12.14}}{=} \frac{\bar{V}_{AB}}{\bar{Z}_\Delta}\sqrt{3}\measuredangle -30^\circ \underset{\text{Fig 12.14}}{=} \frac{\bar{V}_{ab}}{\bar{Z}_\Delta}\sqrt{3}\measuredangle -30^\circ \underset{(12.20)}{=} \frac{\sqrt{3}\,V_p\measuredangle 30^\circ}{\bar{Z}_\Delta}\cdot\sqrt{3}\measuredangle -30^\circ$$

$$\therefore\ \bar{I}_a = \frac{3\cdot V_p \measuredangle 0^\circ}{\bar{Z}_\Delta} \qquad (2)$$ ← same as (1)

Example 12.3

Balanced abc-sequence Y-connected source w/ $\bar{V}_{an} = 100\angle 10^\circ$ V is connected to Δ-connected balanced load w/ $8+j4$ Ω per phase. Calculate the phase & line currents.

$$\bar{Z}_\Delta = 8+j4 = 8.944\angle 26.57^\circ\ \Omega$$

With a phase voltage, $\bar{V}_{an} = 100\angle 10^\circ$ V, the line voltage $\bar{V}_{AB}$ is

$$\bar{V}_{AB} = \bar{V}_{ab} \underset{(12.20)}{=} \bar{V}_{an}\sqrt{3}\angle 30^\circ = 100\angle 10^\circ\sqrt{3}\angle 30^\circ\ \text{V} = 173.2\angle 40^\circ\ \text{V}$$

using (12.21) the load phase currents are

$$\underline{\underline{\bar{I}_{AB}}} = \frac{\bar{V}_{AB}}{\bar{Z}_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.570^\circ} = \underline{19.36\angle 13.43^\circ\ \text{A}}$$

Since positive abc sequence

$$\underline{\underline{\bar{I}_{BC}}} = \bar{I}_{AB}\cdot 1\angle -120^\circ = \underline{19.36\angle -106.57^\circ\ \text{A}}$$

&

$$\underline{\underline{\bar{I}_{CA}}} = \bar{I}_{AB}\cdot 1\angle -240^\circ = 19.36\angle -226.57^\circ\ \text{A}$$

The line currents can be calculated using (12.24)

$$\underline{\underline{\bar{I}_a}} = \bar{I}_{AB}\cdot\sqrt{3}\angle -30^\circ = (19.36\angle 13.43)\cdot(\sqrt{3}\angle -30^\circ)$$

$$= \underline{33.53\angle -16.57^\circ\ \text{A}}$$

& since a positive abc sequence

$$\underline{\underline{\bar{I}_b}} = \bar{I}_a\cdot 1\angle -120^\circ = \underline{\underline{33.53\angle -136.57^\circ\ \text{A}}}$$

$$\underline{\underline{\bar{I}_c}} = \bar{I}_a\cdot 1\angle -240^\circ = \underline{\underline{33.53\angle -256.57^\circ\ \text{A}}}$$

Alternatively, could convert the Δ connected load to Wye, then use direct circuit analysis for the single phase equivalent ckt of Fig. 12.16 to find the line currents. From those, calculate load currents.

12.5 Balanced Δ-Δ connection

Fig 12.17.

Our goal, as usual, is to solve for the phase and line currents.

For a positive abc phase sequence, the phase voltages are

$$\bar{V}_{ab} = V_p \angle 0^\circ, \quad \bar{V}_{bc} = V_p \angle -120^\circ, \; \& \; V_{ca} = V_p \angle -240^\circ \qquad (12.29)$$

As we can see from the figure, the phase & line voltages are the same:

$$\bar{V}_{AB} = \bar{V}_{ab}, \quad \bar{V}_{BC} = \bar{V}_{bc}, \; \& \; V_{CA} = V_{ca} \qquad (12.30)$$

It is also easily apparent from the circuit in Fig 12.17 that

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z_\Delta} = \frac{\bar{V}_{ab}}{Z_\Delta}, \quad \bar{I}_{BC} = \frac{\bar{V}_{BC}}{Z_\Delta} = \frac{\bar{V}_{bc}}{Z_\Delta}, \; \& \; \bar{I}_{CA} = \frac{\bar{V}_{CA}}{Z_\Delta} = \frac{\bar{V}_{ca}}{Z_\Delta} \qquad (12.31)$$

All of this is very similar to what we found in the previous section for the Y-Δ connection.

Because the load is Δ connected, as in the previous section, then the relationship between line & phase currents is identically the same. We don't need to rederive it. In particular,

$$\bar{I}_a = \bar{I}_{AB} \cdot \sqrt{3} \angle -30^\circ \text{ A}$$

$$\bar{I}_b = \bar{I}_a \angle -120^\circ = \bar{I}_{AB} \cdot \sqrt{3} \angle -150^\circ \text{ A} \qquad (3)$$

$$\bar{I}_c = \bar{I}_a \angle -240^\circ = \bar{I}_{AB} \cdot \sqrt{3} \angle -270^\circ \text{ A}$$

Example 12.4 A balanced Δ-connected load having an impedance of $20 - j15\,\Omega$ is connected to a Δ-connected balanced source w/ positive phase sequence having $\bar{V}_{ab} = 330\angle 0^\circ$ V. Calculate the load phase currents and the line currents.

$$\bar{Z}_\Delta = 20 - j15\,\Omega = 25.00\angle -36.87^\circ\,\Omega.$$

Referring to the circuit in Fig. 12.17,

$$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{Z_\Delta} = \frac{\bar{V}_{ab}}{Z_\Delta} = \frac{330\angle 0^\circ}{25\angle -36.87} = 13.2\angle 36.87^\circ\ A$$

Since the voltage source is phased positively,

$$\bar{I}_{BC} = \bar{I}_{AB} \cdot 1\angle -120^\circ = 13.2\angle -83.13^\circ\ A$$

$$\bar{I}_{CA} = \bar{I}_{AB} \cdot 1\angle -240^\circ = 13.2\angle -203.13^\circ\ A$$

For the line currents, using (3)

$$\bar{I}_a = \bar{I}_{AB} \cdot \sqrt{3}\angle -30^\circ = (13.2 \cdot \angle 36.87) \cdot (\sqrt{3}\angle -30^\circ) = 22.86\angle 6.87^\circ\ A$$

$$\bar{I}_b = \bar{I}_a \angle -120^\circ = 22.86\angle -113.13^\circ\ A$$

$$\bar{I}_c = \bar{I}_b \angle -120^\circ = 22.86\angle -233.13^\circ\ A$$