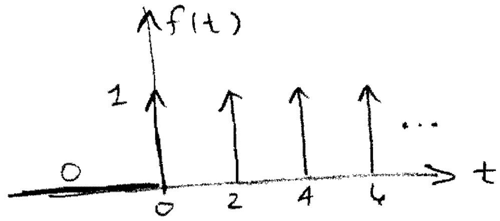
Whites

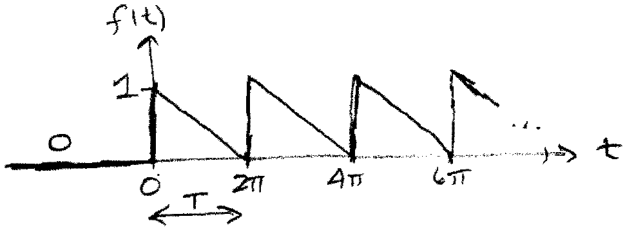
Solution 15.19

Time periodicity property of Laplace X'form, Table 15.1.

$$f(t) = f(t+nT) \xleftrightarrow{\mathcal{L}} \frac{F_1(s)}{1-e^{-sT}}$$ ← X'form over 1st period.



Since $\underbrace{L[\delta(t)] = 1}_{\text{X'form over 1st period}}$ and T = 2, $F(s) = \dfrac{1}{1-e^{-2s}}$



Solution 15.21

$T = 2\pi$

Let $f_1(t) = \left(1 - \frac{t}{2\pi}\right)[u(t) - u(t-2\pi)]$

$$f_1(t) = u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t-2\pi)u(t-2\pi)$$

$\mathcal{L}\{\}$

Table 15.2 & Table 15.1

$$F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-2\pi s}}{2\pi s^2} = \frac{2\pi s + \boxed{-1 + e^{-2\pi s}}}{2\pi s^2}$$

$$\underline{\underline{F(s)}} = \frac{F_1(s)}{1 - e^{-Ts}} = \underline{\underline{\mathbf{\frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2 (1 - e^{-2\pi s})}}}}$$

$T = 2\pi$

Time periodicity property of Laplace X'form, Table 15.1:

$$f(t) = f(t + nT) \overset{\mathcal{L}}{\longleftrightarrow} \frac{F_1(s)}{1 - e^{-sT}}$$

Where $F_1(s)$ is the Laplace xform over the 1st period.

From the figure: $f_1(t) = \left(1 - \frac{t}{2\pi}\right)[u(t) - u(t-2\pi)]$

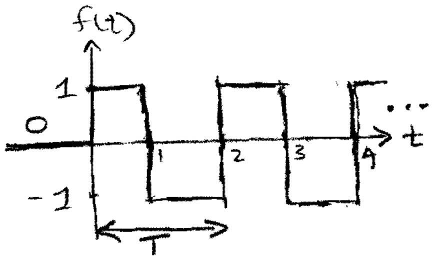
$$= u(t) - u(t-2\pi) - \frac{t}{2\pi}u(t) + \frac{t}{2\pi}u(t-2\pi)$$

$$= u(t) - \frac{t}{2\pi}u(t) - u(t-2\pi) + \frac{t}{2\pi}(t-2\pi)$$

$$= u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t-2\pi)u(t-2\pi)$$

- $\mathcal{L}\{u(t)\} = \frac{1}{s}$
- $\mathcal{L}\left\{\frac{t}{2\pi}u(t)\right\} = \frac{1}{2\pi s^2}$
- $\mathcal{L}\left\{\frac{1}{2\pi}(t-2\pi)u(t-2\pi)\right\} = \frac{1}{2\pi}e^{-2\pi s} \cdot \mathcal{L}\{t\} = \frac{e^{-2\pi s}}{2\pi s^2}$

whites



Solution 15.23

(a) Let $f_1(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases}$

Time periodicity property of Laplace transform, Table 15.1:

$f(t) = f(t+nT) \xleftrightarrow{\mathcal{L}} \dfrac{F_1(s)}{1-e^{-sT}}$

Where $F_1(s)$ is the Laplace x'form over the first period.

$$f_1(t) = \left[u(t) - u(t-1)\right] - \left[u(t-1) - u(t-2)\right]$$

$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$\mathcal{L}\{\}$ Table 15.2

$$F_1(s) = \frac{1}{s}(1 - 2e^{-s} + e^{-2s}) = \frac{1}{s}(1-e^{-s})^2$$

$$F(s) = \frac{F_1(s)}{(1-e^{-sT})}, \quad T = 2$$

$$\underline{\underline{F(s) = \frac{(1-e^{-s})^2}{s(1-e^{-2s})}}}$$

Whites

Solution 15.25

Let $F(s) = \dfrac{18(s+1)}{(s+2)(s+3)}$.

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.
(b) Verify your answer in part (a) by finding $f(t)$ using partial fractions.

Solution

$\rightarrow 18\frac{s^2}{s^2} = 18$

Table 15.1.

(a) $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \dfrac{18s(s+1)}{(s+2)(s+3)} = \mathbf{18.}$

$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \dfrac{18s(s+1)}{(s+2)(s+3)} = \mathbf{0.}$

$\rightarrow \lim_{s\to 0} \dfrac{18s(1)}{(2)\cdot(3)} = 0$

PFE

(b) $f(s) = \dfrac{18(s+1)}{(s+2)(s+3)} = \dfrac{A}{s+2} + \dfrac{B}{s+3}$ where A = 18(–2+1)/(–2+3) = –18 and
B = 18(–3+1)/(–3+2) = 36 therefore f(t) = $[-18e^{-2t} + 36e^{-3t}]u(t)$ which gives us f(0) = 18 and f(∞) = 0 and our answers check!

$F(s) = \dfrac{-18}{s+2} + \dfrac{36}{s+3}$

$f(t) = -18e^{-2t} + 36e^{-3t}$

- $f(0) = -18\cdot 1 + 36\cdot 1 = 18$ ✓
- $f(\infty) = -18\cdot 0 + 36\cdot 0 = 0$ ✓

Solution 15.27

(a) $\mathbf{f(t) = u(t) + 2e^{-t}u(t)}$ using Table 15.2.

long division

(b) $G(s) = \dfrac{3(s+4)-11}{s+4} = 3 - \dfrac{11}{s+4}$ G(s) is improper [i.e, order N(s) = order D(s)]

$\mathbf{g(t) = 3\delta(t) - 11e^{-4t}u(t)}$ using Table 15.2

P.F.E

(c) $H(s) = \dfrac{4}{(s+1)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$ $A = \dfrac{4}{-1+3} = 2$, $B = \dfrac{4}{-3+1} = -2$

$A = 2,\quad B = -2$

$H(s) = \dfrac{2}{s+1} - \dfrac{2}{s+3}$

$\mathbf{h(t) = [2e^{-t} - 2e^{-3t}]u(t)}$ using Table 15.2

(d) $J(s) = \dfrac{12}{(s+2)^2(s+4)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+4}$ $B = \dfrac{12}{-2+4} = 6$, $C = \dfrac{12}{(-4+2)^2} = 3$

$B = \dfrac{12}{2} = 6,\quad C = \dfrac{12}{(-2)^2} = 3$

$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$ ⟵ multiply out

$= A(s^2+6s+8) + B(s+4) + C(s^2+4s+4)$

Equating coefficients :

s^2: $0 = A + C \longrightarrow A = -C = -3$

s^1: $0 = 6A + B + 4C = 2A + B \longrightarrow B = -2A = 6$ already solved for this

s^0: $12 = 8A + 4B + 4C = -24 + 24 + 12 = 12$ ✓

$J(s) = \dfrac{-3}{s+2} + \dfrac{6}{(s+2)^2} + \dfrac{3}{s+4}$

Table 15.2

$\mathbf{j(t) = [3e^{-4t} - 3e^{-2t} + 6te^{-2t}]u(t)}$

Solution 15.30

(a) $F_1(s) = \dfrac{6s^2+8s+3}{s(s^2+2s+5)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+2s+5}$ $A = \dfrac{0+0+3}{0+0+5} = \dfrac{3}{5}$

multiply out: $6s^2+8s+3 = A(s^2+2s+5)+Bs^2+Cs = (A+B)s^2 + (2A+C)s + 5A$

We equate coefficients.

s^2 : $6 = A + B$ $B = 6 - A = 6 - \frac{3}{5} = \frac{30-3}{5} = \frac{27}{5}$

s: $8 = 2A + C$ $C = 8 - 2A = 8 - \frac{6}{5} = \frac{40-6}{5} = \frac{34}{5}$

constant: $3=5A$ or $A=3/5$

$B=6-A = 27/5$, $C=8-2A = 34/5$

$\therefore$ $F_1(s) = \dfrac{3/5}{s} + \dfrac{27s/5+34/5}{s^2+2s+5} = \dfrac{3/5}{s} + \dfrac{27(s+1)/5+7/5}{(s+1)^2+2^2} = \dfrac{3/5}{s} + \dfrac{\frac{27}{5}(s+1)}{(s+1)^2+2^2} + \dfrac{7/5}{(s+1)^2+2^2}$

$= \dfrac{3/5}{s} + \dfrac{27}{5}\dfrac{s+1}{(s+1)^2+2^2} + \dfrac{7}{5}\cdot\dfrac{1}{2}\dfrac{2}{(s+1)^2+2^2}$

Table 15.2

$$\underline{f_1(t)} = \underline{\left[\frac{3}{5} + \frac{27}{5}e^{-t}\cos 2t + \frac{7}{10}e^{-t}\sin 2t\right]u(t)}$$

(b) $F_2(s) = \dfrac{s^2+5s+6}{(s+1)^2(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C}{s+4}$ $B = \dfrac{(-1)^2-5+6}{-1+4} = \dfrac{2}{3}$, $C = \dfrac{(-4)^2-20+6}{(-3)^2} = \dfrac{2}{9}$

$s^2+5s+6 = A(s+1)(s+4) + B(s+4) + C(s+1)^2$

Equating coefficients, $= A(s^2+5s+4) + B(s+4) + C(s^2+2s+1)$

s^2 : $1=A+C$ $\Rightarrow$ $A = 1 - C = 1 - \frac{2}{9} = \frac{7}{9}$

s: $5=5A+B+2C$

constant: $6=4A+4B+C$

Solving these gives

$A=7/9$, $B= 2/3$, $C=2/9$

$\therefore$ $F_2(s) = \dfrac{7/9}{s+1} + \dfrac{2/3}{(s+1)^2} + \dfrac{2/9}{s+4}$

Table 15.2

$$\underline{f_2(t)} = \underline{\left[\frac{7}{9}e^{-t} + \frac{2}{3}te^{-t} + \frac{2}{9}e^{-4t}\right]u(t)}$$

Solution 15.34

(a) $F(s) = 10 + \frac{s^2+4-3}{s^2+4} = 11 - \frac{3}{s^2+4}$

$F(s) = 10 + \underbrace{\frac{s^2+1}{s^2+4}}_{\text{improper}} = 10 + \frac{(s^2+4)-3}{s^2+4}$

$= 10 + 1 - \frac{3}{s^2+4}$

$= 11 - \frac{3}{s^2+4} = 11 - 3 \cdot \frac{1}{2}\frac{2}{s^2+4}$

Table 15.2

$\underline{\mathbf{f(t) = 11\delta(t) - 1.5\sin(2t)}\ u(t)}$

(b) $G(s) = \frac{e^{-s}+4e^{-2s}}{(s+2)(s+4)}$

$G(s) = \frac{e^{-s}+4e^{-2s}}{s^2+6s+8} = \frac{e^{-s}+4e^{-2s}}{(s+2)(s+4)}$

Let $\frac{1}{(s+2)(s+4)} \overset{\text{PFE}}{=} \frac{A}{s+2} + \frac{B}{s+4}$

$A = \frac{1}{-2+4} = \frac{1}{2}$, $B = \frac{1}{-4+2} = -\frac{1}{2}$

$A = 1/2 \qquad B = -1/2$

$G(s) = \frac{e^{-s}}{2}\left(\frac{1}{s+2} - \frac{1}{s+4}\right) + 2e^{-2s}\left(\frac{1}{s+2} - \frac{1}{s+4}\right)$

Time shift property of Laplace x'form:

$f(t-a)\,u(t-a) = e^{-as}F(s)$

and

$e^{-at} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}$

$\underline{\mathbf{g(t) = 0.5\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1) + 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)}}$

$\therefore G(s) = \frac{1}{2}\left[e^{-2t}\Big|_{t\to t-1} - e^{-4t}\Big|_{t\to t-1}\right]u(t-1)$

$+ 2\left[e^{-2t}\Big|_{t\to t-2} - e^{-4t}\Big|_{t\to t-2}\right]u(t-2)$

$\therefore G(s) = \frac{1}{2}\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1)$

$+ 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)$

Solution 15.36

notice that the "3"'s cancel.

$B = \frac{2}{2\cdot 3} = \frac{1}{6}$, $C = \frac{1}{4\cdot 1} = \frac{1}{4}$

$D = \frac{1}{9\cdot(-1)} = -\frac{1}{9}$

(a) $$X(s) = 3\frac{1}{s^2(s+2)(s+3)} \overset{\text{PFE}}{=} 3\left\{\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{D}{s+3}\right\}$$

B = 1/6, C = 1/4, D = -1/9

Expand out

$$1 = A(s^3 + 5s^2 + 6s) + B(s^2 + 5s + 6) + C(s^3 + 3s^2) + D(s^3 + 2s^2)$$

Equating coefficients :

s^3: $0 = A + C + D \Longrightarrow A = -C - D = -\frac{1}{4} + \frac{1}{9} = \frac{-9}{36} + \frac{4}{36} = -\frac{5}{36}$

s^2: $0 = 5A + B + 3C + 2D = 3A + B + C$

s^1: $0 = 6A + 5B$

s^0: $1 = 6B \longrightarrow B = 1/6$ ✓

$A = -5/6B = -5/36$ ✓

$$X(s) = 3\left(\frac{-5/36}{s} + \frac{1/6}{s^2} + \frac{1/4}{s+2} - \frac{1/9}{s+3}\right)$$

Table 15.2

$$\mathbf{x(t) = \left(\frac{-5}{12}u(t) + \frac{1}{2}t + \frac{3}{4}e^{-2t} - \frac{1}{3}e^{-3t}\right)u(t)}$$

$$Z(s) = \frac{s}{s(s+1)(s^2+6s+10)} = 5\left(\frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+6s+10}\right)$$

(c) $Z(s) = 5\left(\frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+6s+10}\right)$ PFE

Notice that the "s"'s cancel! $A = \frac{1}{(0+1)(0+0+10)} = \frac{1}{10}$, $B = \frac{1}{(-1)(1-6+10)} = -\frac{1}{5}$

A = 1/10,   B = -1/5

$1 = A(s+1)(s^2+6s+10) + B(s^3+6s^2+10s) + (Cs+D)(s^2+s)$
$= A(s^3+6s^2+10s+s^2+6s+10) + B(s^3+6s^2+10s)$
$+ C(s^3+s^2) + D(s^2+s)$

multiply out:

$1 = A(s^3 + 7s^2 + 16s + 10) + B(s^3 + 6s^2 + 10s) + C(s^3 + s^2) + D(s^2 + s)$

Equating coefficients :

s^3: $0 = A + B + C$ $\Rightarrow$ $C = -A - B = -\frac{1}{10} + \frac{1}{5} = -\frac{1}{10} + \frac{2}{10} = \frac{1}{10}$

s^2: $\underline{0 = 7A + 6B + C + D} = 6A + 5B + D$ $\Rightarrow$ $D = -(7A + 6B + C) = -\left(\frac{7}{10} - \frac{6}{5} + \frac{1}{10}\right)$

s^1: $0 = 16A + 10B + D = 10A + 5B \longrightarrow B = -2A$ $\quad = -\left(\frac{7}{10} - \frac{12}{10} + \frac{1}{10}\right) = \frac{4}{10}$

s^0: $1 = 10A \longrightarrow A = 1/10$

A = 1/10, ✗   B = -2A = -1/5, ✗   C = A = 1/10, $D = 4A = \frac{4}{10}$ ✗

$$Z(s) = 5\left(\frac{1/10}{s} - \frac{1/5}{s+1} + \frac{1/10\, s + 4/10}{s^2+6s+10}\right)$$

$$\frac{10}{5}Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+4}{s^2+6s+10}$$

$$2Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+3}{(s+3)^2+1} + \frac{1}{(s+3)^2+1}$$

Table 15.2

$$\underline{\mathbf{z(t) = 0.5\left[1 - 2e^{-t} + e^{-3t}\cos(t) + e^{-3t}\sin(t)\right] u(t)}}$$

$$\frac{s+4}{s^2+6s+10} = \frac{s+3+1}{(s+3)^2+1} =$$

$$= \underbrace{\frac{s+3}{(s+3)^2+1}}_{e^{-at}\cos(\omega t)} + \underbrace{\frac{1}{(s+3)^2+1}}_{e^{-at}\sin(\omega t)}$$