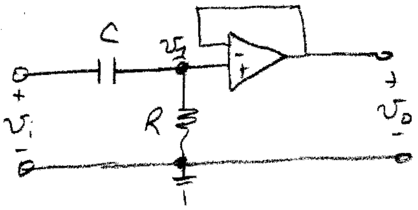


$v_- = v_o$, $v_+ = v_1$ $\Big\}$ $v_+ = v_- \Rightarrow v_o = v_1$

With input impedance to the op amp = ∞, then by voltage division

$$v_1 = \frac{R}{R + \frac{1}{j\omega C}} v_i$$

$$v_o = \frac{j\omega CR}{1 + j\omega CR} v_i$$

$$\Rightarrow H(\omega) = \frac{v_o(\omega)}{v_i(\omega)} = \frac{j\omega CR}{1 + j\omega CR}$$

Solution 14.62

This is a highpass filter.

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{1}{1 - j/\omega RC}$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j\omega_c/\omega}, \qquad \omega_c = \frac{1}{RC} = 2\pi(1000)$$

$$\mathbf{H}(\omega) = \frac{1}{1 - jf_c/f} = \frac{1}{1 - j1000/f}$$

As $\omega \to 0$, $H(0) = 0$
$\omega \to \infty$, $H(\infty) = 1$
∴ HPF

$|\bar{V}_i| = 120$ mV

$|\bar{V}_o| = |\bar{H}(f)|\,|\bar{V}_i|$

(a) $\mathbf{H}(f = 200\text{ Hz}) = \dfrac{1}{1 - j5} = \dfrac{\mathbf{V}_o}{\mathbf{V}_i}$

$f_c/f = \frac{1000}{200}$

$$|\mathbf{V}_o| = \frac{120\text{ mV}}{|1 - j5|} = \mathbf{23.53\ mV}$$

(b) $\mathbf{H}(f = 2\text{ kHz}) = \dfrac{1}{1 - j0.5} = \dfrac{\mathbf{V}_o}{\mathbf{V}_i}$

$f_c/f = \frac{1000}{2000}$

$$|\mathbf{V}_o| = \frac{120\text{ mV}}{|1 - j0.5|} = \mathbf{107.3\ mV}$$

(c) $\mathbf{H}(f = 10\text{ kHz}) = \dfrac{1}{1 - j0.1} = \dfrac{\mathbf{V}_o}{\mathbf{V}_i}$

$f_c/f = \frac{1000}{10{,}000}$

$$|\mathbf{V}_o| = \frac{120\text{ mV}}{|1 - j0.1|} = \mathbf{119.4\ mV}$$

increasing $|V_o|$ w/ increasing f. makes sense to HPF

Whites

Solution 14.63

For an active highpass filter,

$$\bar{H}(s) = -\frac{sC_iR_f}{1+sC_iR_i} \overset{s=j\omega}{=} -\frac{j\omega C_i R_f}{1+j\omega C_i R_i} \qquad (1) \quad (14.63)$$

But given

$$\bar{H}(s) = -\frac{10s}{1+s/10} = -\frac{j\omega 10}{1+\frac{j\omega}{10}} \qquad (2)$$

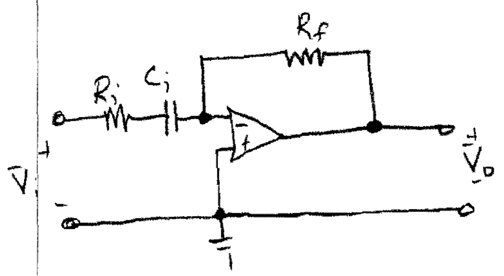
Comparing (1) and (2) leads to:

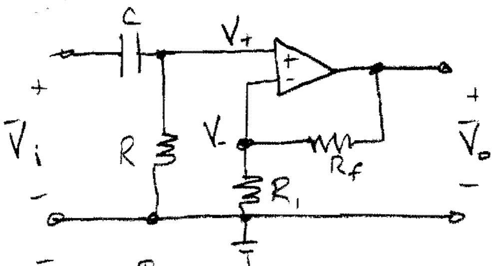
$$C_iR_f = 10 \xrightarrow{C_i = 1\mu F} \underline{\underline{R_f}} = \frac{10}{C_i} = \underline{\underline{10M\Omega}}$$ ← Huge!

$$C_iR_i = 0.1 \xrightarrow{C_i = 1\mu F} \underline{\underline{R_i}} = \frac{0.1}{C_i} = \underline{\underline{100k\Omega}}$$

Fig. 14.43:

HPF





$$\bar{V}_- = \frac{R_i}{R_i+R_f}\,\bar{V}_o \quad \& \quad \bar{V}_- = \bar{V}_+$$

$$\bar{V}_+ = \frac{R}{R+\frac{1}{j\omega C}}\,\bar{V}_i$$

$$\therefore\ \bar{V}_o = \left(1+\frac{R_f}{R_i}\right)\cdot\frac{j\omega RC}{1+j\omega RC}\cdot\bar{V}_i$$

$$\bar{H}(\omega) = \frac{\bar{V}_o(\omega)}{\bar{V}_i(\omega)} = \left(1+\frac{R_f}{R_i}\right)\left(\frac{j\omega RC}{1+j\omega RC}\right)$$

$$= \left(1+\frac{R_f}{R_i}\right)\left(\frac{j\,\omega/\omega_c}{1+j\,\omega/\omega_c}\right) \quad \text{w/ } \omega_c = \frac{1}{RC}$$

Solution 14.65

$$\mathbf{V}_+ = \frac{R}{R+1/j\omega C}\mathbf{V}_i = \frac{j\omega RC}{1+j\omega RC}\mathbf{V}_i$$

$$\mathbf{V}_- = \frac{R_i}{R_i+R_f}\mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{R_i}{R_i+R_f}\mathbf{V}_o = \frac{j\omega RC}{1+j\omega RC}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(\mathbf{1}+\frac{\mathbf{R}_f}{\mathbf{R}_i}\right)\left(\frac{\mathbf{j\omega RC}}{\mathbf{1}+\mathbf{j\omega RC}}\right)$$

It is evident that as $\omega \to \infty$, the gain is $\mathbf{1}+\dfrac{\mathbf{R}_f}{\mathbf{R}_i}$ and that the corner frequency is $\dfrac{\mathbf{1}}{\mathbf{RC}}$.

Solution 14.67

given

$$\text{DC gain} = \frac{R_f}{R_i} = \frac{1}{4} \longrightarrow R_i = 4R_f$$

given

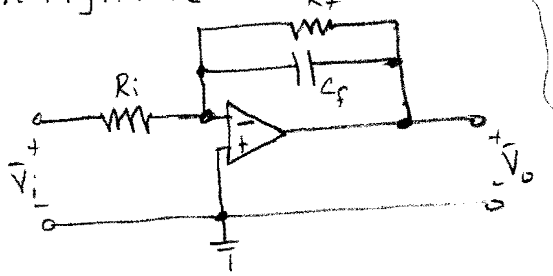
$$\text{Corner frequency} = \omega_c = \frac{1}{R_f C_f} = 2\pi(500) \text{ rad/s}$$

If we select $R_f = 20 \text{ k}\Omega$, then $\underline{\underline{R_i = 80 \text{ k}\Omega}}$ and

$$\underline{\underline{C}} = \frac{1}{(2\pi)(500)(20 \times 10^3)} = \underline{\underline{15.915 \text{ nF}}}$$

Therefore, if $R_f = \mathbf{20\ k\Omega}$, then $R_i = \mathbf{80\ k\Omega}$ and $C = \mathbf{15.915\ nF}$

From Fig. 14.42



From (14.60):

$$\bar{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f}$$

$$= -\frac{R_f}{R_i}\frac{1}{1 + j\frac{\omega}{\omega_c}}$$

$$\omega_c = \frac{1}{C_f R_f}$$

Whites

Solution 14.69

This is a first-order active high pass filter of Fig. 14.43. Where $R_i = R$ & $C_i = C$.

This is a highpass filter with $f_c = 2$ kHz.

From (14.64): $\omega_c = 2\pi f_c = \dfrac{1}{RC}$

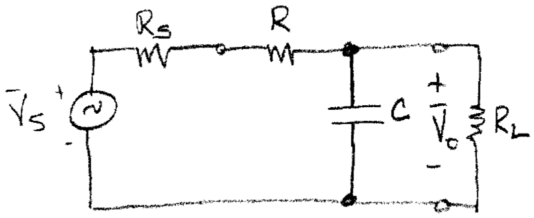
$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

10^8 Hz may be regarded as high frequency relative to $f_c = 2$ kHz. Hence the high-frequency gain is

$$\frac{-R_f}{R} = \frac{-10}{4} \qquad \text{or} \qquad R_f = 2.5R$$

If we let $R = \mathbf{10\ k\Omega}$, then $R_f = \mathbf{25\ k\Omega}$, and $C = \dfrac{1}{4000\pi \times 10^4} = \mathbf{7.96\ nF}$.

Whites



Solution 14.102 : $R = 4\,k\Omega$; $C = 40\,nF$

(a) When $R_s = 0$ and $R_L = \infty$, we have a low-pass filter.

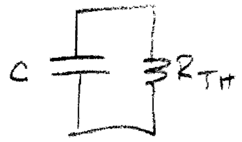
$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(4\times10^3)(40\times10^{-9})} = \mathbf{994.7\ Hz}$$

(b) We obtain R_{Th} across the capacitor. - as seen by the capacitor.

$R_{Th} = R_L \,\|\, (R + R_s)$ $\quad R_s = 1\,k\Omega,\ R_L = 5\,k\Omega \rightarrow$

$R_{Th} = 5 \,\|\, (4+1) = 2.5\ k\Omega$



$$f_c = \frac{1}{2\pi R_{Th} C} = \frac{1}{(2\pi)(2.5\times10^3)(40\times10^{-9})}$$

$f_c = \mathbf{1.59\ kHz}$

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Solution 15.3

From Tables 15.1 & 15.2 :

(a) $L\left[e^{-2t}\cos(3t)u(t)\right] = \dfrac{\mathbf{s+2}}{\mathbf{(s+2)^2+9}}$: $\mathcal{L}\left\{e^{-at}\cos(\omega t)\,u(t)\right\} = \dfrac{s+a}{(s+a)^2+\omega^2}$

(b) $L\left[e^{-2t}\sin(4t)u(t)\right] = \dfrac{\mathbf{4}}{\mathbf{(s+2)^2+16}}$: $\mathcal{L}\left\{e^{-at}\sin(\omega t)u(t)\right\} = \dfrac{\omega}{(s+a)^2+\omega^2}$

(c) Since $L\left[\cosh(at)\right] = \dfrac{s}{s^2-a^2}$: $\cosh(u) = \cos(ju) \Rightarrow \mathcal{L}\left\{\cosh(at)\right\} = \mathcal{L}\left\{\cos(jat)\right\} = \dfrac{s}{s^2-a^2}$

$L\left[e^{-3t}\cosh(2t)u(t)\right] = \dfrac{\mathbf{s+3}}{\mathbf{(s+3)^2-4}}$ Frequency Shift : $e^{-at}f(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s+a)$, $a=3$

(d) Since $L\left[\sinh(at)\right] = \dfrac{a}{s^2-a^2}$: $\sinh(u) = -j\sin(ju) \Rightarrow \mathcal{L}\left\{\sinh(at)\right\} = \mathcal{L}\left\{\sinh(at)\right\} = \dfrac{-j\cdot ja}{s^2-a^2}$ $+1$

$L\left[e^{-4t}\sinh(t)u(t)\right] = \dfrac{\mathbf{1}}{\mathbf{(s+4)^2-1}}$ Frequency shift : $e^{-at}f(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s+a)$, $a=4$

(e) $L\left[e^{-t}\sin(2t)\right] = \dfrac{2}{(s+1)^2+4}$ ← First step.

If $f(t) \longleftrightarrow F(s)$

$tf(t) \longleftrightarrow \dfrac{-d}{ds}F(s)$: Table 15.1

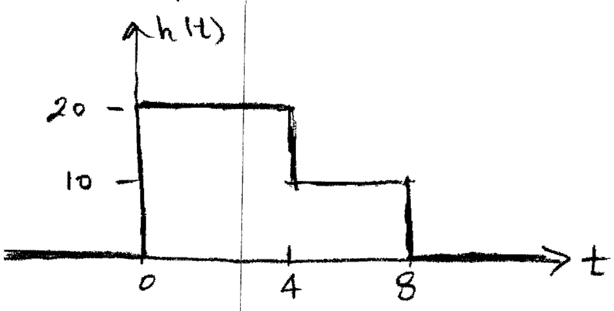
Thus, $L\left[te^{-t}\sin(2t)\right] = \dfrac{-d}{ds}\left[2\left((s+1)^2+4\right)^{-1}\right]$ $= -\dfrac{d}{ds}\left[\dfrac{2}{(s+1)^2+4}\right] = -2\left\{\dfrac{0-2(s+1)\cdot 1}{\left[(s+1)^2+4\right]^2}\right\}$

$= \dfrac{2}{((s+1)^2+4)^2}\cdot 2(s+1)$ $= \dfrac{4(s+1)}{\left[(s+1)^2+4\right]^2}$ ✓

$L\left[te^{-t}\sin(2t)\right] = \dfrac{\mathbf{4(s+1)}}{\mathbf{((s+1)^2+4)^2}}$

Practice Problem 15.5

Determine the Laplace transform of the function $h(t)$:



We can write $h(t)$ as

$$h(t) = 20\left[u(t) - u(t-4)\right] + 10\left[u(t-4) - u(t-8)\right]$$

Since $\mathcal{L}\left[u(t)\right] = \frac{1}{s}$ &

& $\mathcal{L}\left[f(t-a)\,u(t-a)\right] = e^{-as} F(s)$

then

$$H(s) = 20\left[\frac{1}{s} - \frac{e^{-4s}}{s}\right] + 10\left[\frac{e^{-4s}}{s} - \frac{e^{-8s}}{s}\right]$$

or $H(s) = \frac{10}{s}\left[2 - 2e^{-4s} + e^{-4s} - e^{-8s}\right]$

$$\underline{\underline{H(s) = \frac{10}{s}\left[2 - e^{-4s} - e^{-8s}\right]}}$$

Solution 15.16

Obtain the Laplace transform of $f(t)$ in Fig. 15.28.

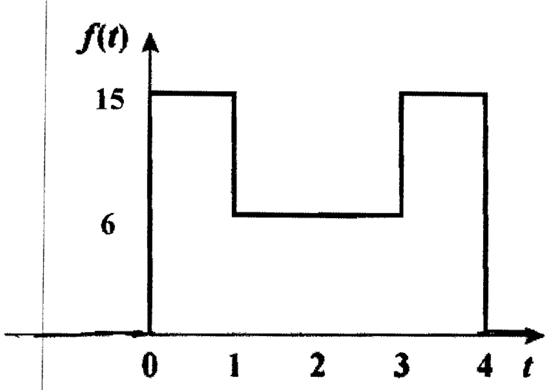


Figure 15.28
For Prob. 15.16.

Solution From the figure above, we can express f(t) directly as

$f(t) = 15u(t) - 9u(t-1) + 9u(t-3) - 15u(t-4)$

$F(s) = \mathbf{[15-9e^{-s}+9e^{-3s}-15e^{-4s}]/s}$.

$\mathcal{L}[u(t)] = \frac{1}{s}$

$\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s)$

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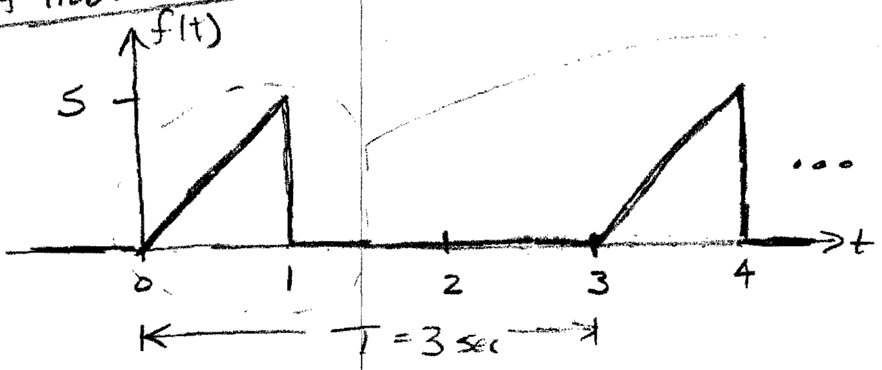
Text Problem 15.15



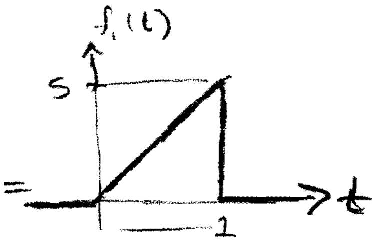
$f_1(t) = \frac{\text{rise}}{\text{run}} \cdot t = \frac{5}{1} \cdot t$

From Table 15.1 for a periodic function

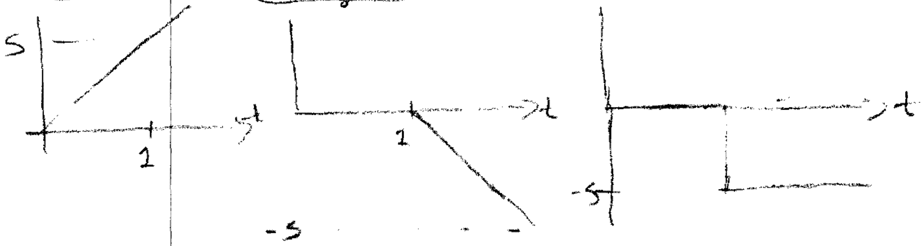
$$\mathcal{L}\{f(t) = f(t+nT)\} = \frac{F_1(s)}{1-e^{-sT}}$$

Laplace x'form over first period.

$$\therefore \; F(s) = \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-3s}} \cdot \mathcal{L}\{f_1(t)\}$$



$$f_1(t) = 5t\,u(t) - 5(t-1)u(t-1) - 5u(t-1) =$$



$$\therefore \; F(s) = \frac{1}{1-e^{-3s}}\left[\mathcal{L}\{5t\,u(t)\} - \mathcal{L}\{5(t-1)u(t-1)\} - \mathcal{L}\{5u(t-1)\}\right]$$

$$= \frac{5}{1-e^{-3s}}\left[\frac{1}{s^2} - \frac{e^{-1\cdot s}}{s^2} - \frac{e^{-1\cdot s}}{s}\right]$$

$$\therefore \; F(s) = \frac{5}{s^2(1-e^{-3s})}\cdot\left(1-e^{-s}-se^{-s}\right)$$

Practice Problem 15.7

Obtain the initial and final values of

$$G(s) = \frac{3s^3+2s+6}{s(s+1)^2(s+1.5)} = \frac{3s^3+2s+6}{s(s^2+2s+1)(s+1.5)}$$

- Using the initial value theorem from Table 15.1:

$$\underline{\underline{g(0)}} = \lim_{s\to\infty} sG(s) = \lim_{s\to\infty}\left[\frac{3s^3+2s+6}{(s^2+2s+1)(s+1.5)}\right] \to \frac{3s^3}{s^3} \to \underline{\underline{3}}$$

- Using the final value theorem from Table 15.1:

$$\underline{\underline{g(\infty)}} = \lim_{s\to 0} sG(s) = \lim_{s\to 0}\left[\frac{3s^3+2s+6}{(s^2+2s+1)(s+1.5)}\right] \to \frac{6}{1.5} = \underline{\underline{4}}$$