Whites EE 221 - Circuits II 1/1

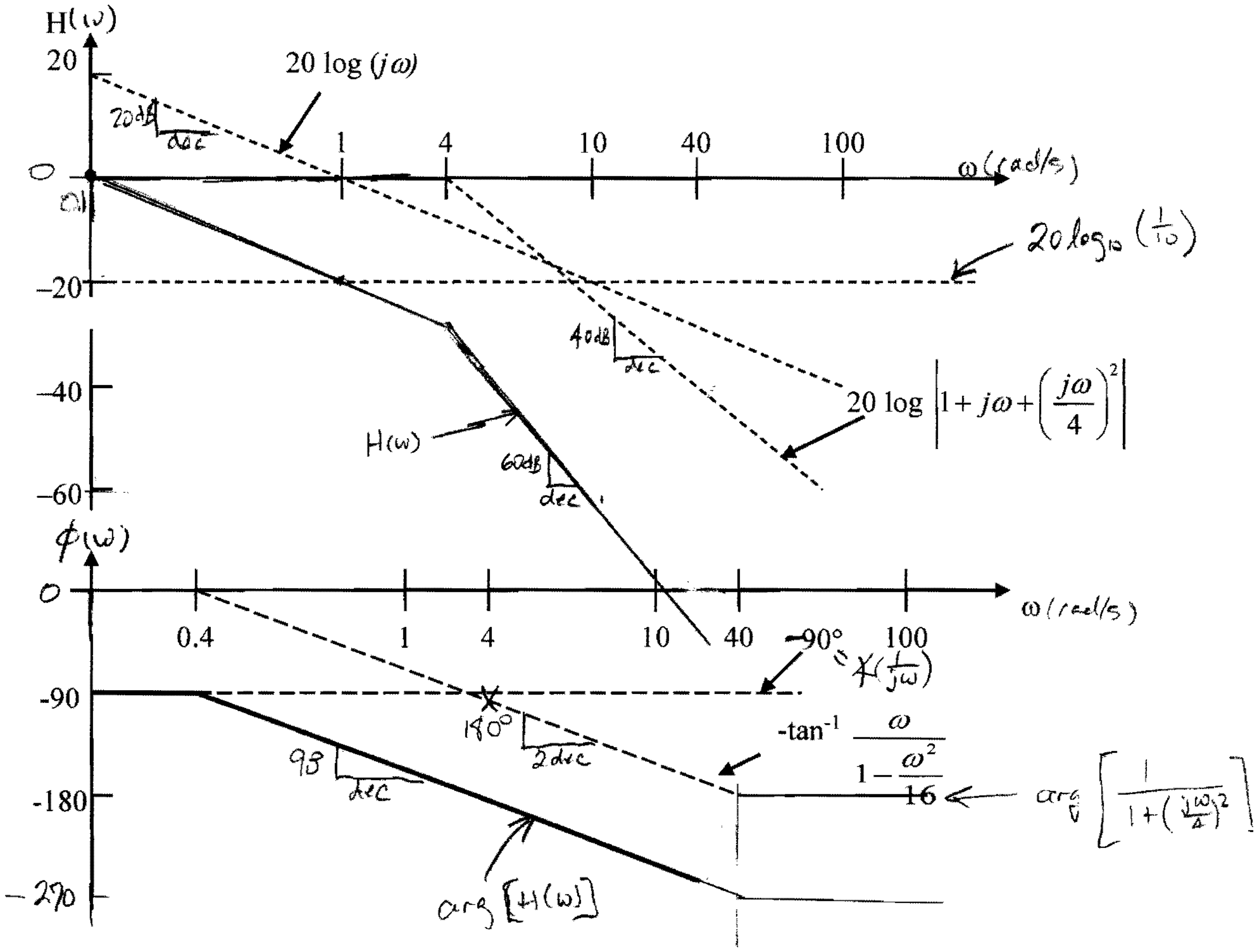
$$H(s) = \frac{1.6}{s(s^2+s+16)}, \quad s = j\omega$$

Solution 14.16 Factor out 16 and express in standard form:

$$H(\omega) = \frac{\frac{1.6}{16}}{j\omega\left[1+j\omega+\left(\frac{j\omega}{4}\right)^2\right]} = \frac{0.1}{j\omega\left[1+j\omega+\left(\frac{j\omega}{4}\right)^2\right]}$$

$H_{db} = 20\log_{10}|0.1| - 20\text{olg}_{10}|j\omega| - 20\log_{10}|1+\cancel{j\omega}+(j\omega/4)^2|$ ignore

The magnitude and phase plots are shown below.



Solution 14.22

Find the transfer function **H**(ω) with the Bode magnitude plot shown in Fig. 14.74.

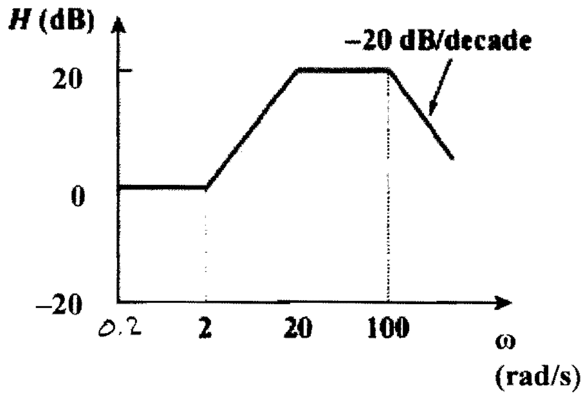


Figure 14.74
For Prob. 14.22.

Solution

$0 = 20\log_{10} k \longrightarrow k = 1$

A zero of slope $+20$ dB/dec at $\omega = 2 \longrightarrow 1 + j\omega/2$

A pole of slope -20 dB/dec at $\omega = 20 \longrightarrow \dfrac{1}{1 + j\omega/20}$

A pole of slope -20 dB/dec at $\omega = 100 \longrightarrow \dfrac{1}{1 + j\omega/100}$

Hence,

$$\mathbf{H}(\omega) = \frac{\pm\ 1(1 + j\omega/2)}{(1 + j\omega/20)(1 + j\omega/100)} = \frac{\pm\ 1{,}000(2 + j\omega)}{(20 + j\omega)(100 + j\omega)}$$

Solution 14.23

The Bode magnitude plot of **H**(ω) is shown in Fig. 14.75. Find **H**(ω).

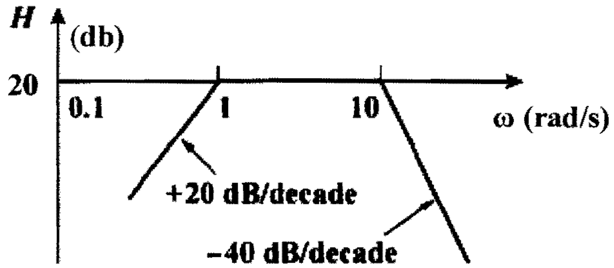


Figure 14.75
For Prob. 14.23.

Solution

The initial slope indicates we have jω in the numerator. Our approach to plotting requires the plot of jω to cross 0db at ω = 1 rad/s. Since it crosses at 20db, that indicates that the overall gain is 20db or,

$20 = 20\log_{10}|\underset{=K}{\text{gain}}|$ the gain has to be 10. = K

A zero of slope $+20$ dB/dec at the origin $\longrightarrow$ $j\omega$

A pole of slope -20 dB/dec at $\omega = 1$ $\longrightarrow$ $\dfrac{1}{1 + j\omega/1}$

A pole of slope -40 dB/dec at $\omega = 10$ $\longrightarrow$ $\dfrac{1}{(1 + j\omega/10)^2}$

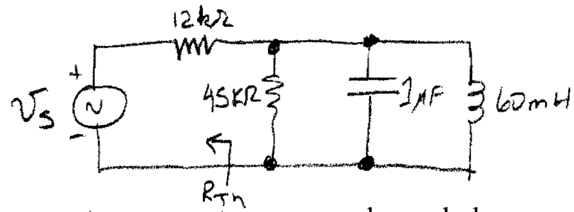
Hence,

$$\mathbf{H}(\omega) = \frac{\pm\ 10 j\omega}{(1 + j\omega)(1 + j\omega/10)^2}$$

$$\mathbf{H}(\omega) = \frac{\pm\ 1{,}000 j\omega}{(1 + j\omega)(10 + j\omega)^2}$$

(It should be noted that this function could also have a minus sign out in front and still be correct. The magnitude plot does not contain this information. It can only be obtained from the phase plot.)

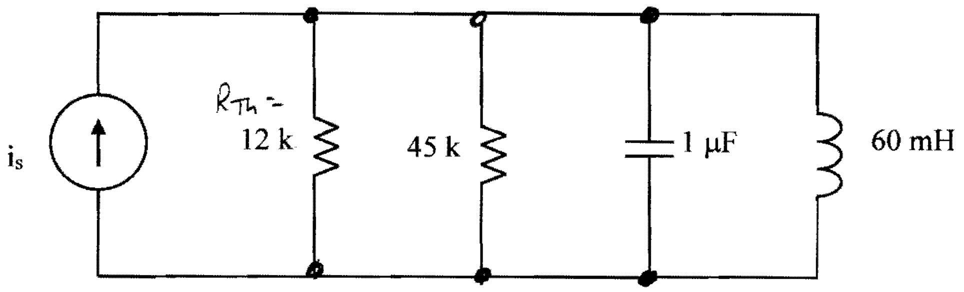
Whites



Solution 14.29

We convert the voltage source to a current source as shown below.

Parallel RLC network



not needed

$i_s = \frac{20}{12}\cos \omega t$, R_{eff}= 12//45= 12x45/57 = 9.4737 kΩ

(14.44): $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60x10^{-3}x1x10^{-6}}} = \underline{4.082\text{ krad/s}}$ = **4.082 krad/s** ⇒ f_o = 649.7 Hz

(14.46): $B = \frac{1}{R_{eff}C} = \frac{1}{9.4737x10^3 x10^{-6}} = \underline{105.55\text{ rad/s}}$ = **105.55 rad/s** Δf = 16.80 Hz

(14.47): $Q = \frac{\omega_o}{B} = \frac{4082}{105.55} = \underline{38.674}$ = **38.67**

4.082 krads/s, 105.55 rad/s, 38.67

$$f_o = \frac{1}{2\pi\sqrt{LC}} \Rightarrow f_o^2 = \frac{1}{4\pi^2 LC}$$
$$\therefore C = \frac{1}{4\pi^2 f_o^2 L}$$
$$\text{or } C = [4\pi^2(15\times10^3)^2 \cdot 10\times10^{-3}]^{-1}$$
$$C = 11.257\text{ nF}$$

Solution 14.30

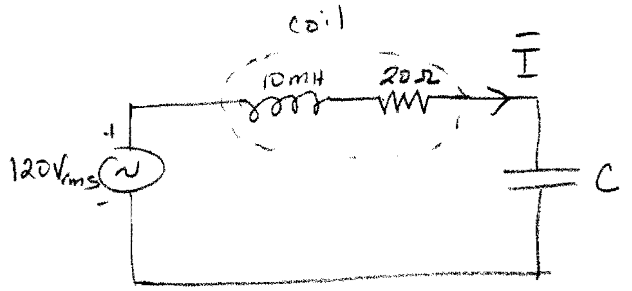
(a) f_o = 15,000 Hz leads to $\omega_o = 2\pi f_o$ = 94.25 krad/s = $1/(LC)^{0.5}$ or

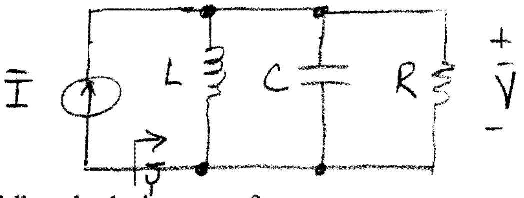
LC = $1/8.883\text{x}10^9$ or C = $1/(8.883\text{x}10^9\text{x}10^{-2}) = 11.257\text{x}10^{-9}$ F = **11.257 ~~pF~~.** C = 11.257 nF

(b) since the capacitive reactance cancels out the inductive reactance at resonance, the current through the series circuit is given by

$|\bar{I}|$ = 120/20 = **6 Arms**

(14.38): (c) Q = ω_oL/R = $94.25\text{x}10^3(0.01)/20$ = **47.12.**





Solution 14.36

It is expected that a parallel *RLC* resonant circuit has a midband admittance of 25×10^{-3} S, quality factor of 120, and a resonant frequency of 200 krad/s. Calculate the values of *R*, *L*, and *C*. Find the bandwidth and the half-power frequencies.

Solution

At resonance, $Z_L \| Z_C \to \infty \Rightarrow \bar{Y} = \frac{1}{R}$

$$\mathrm{Y} = \frac{1}{\mathrm{R}} \longrightarrow \mathrm{R} = \frac{1}{\mathrm{Y}} = \frac{1}{25 \times 10^{-3}} = \mathbf{40\ \Omega}$$

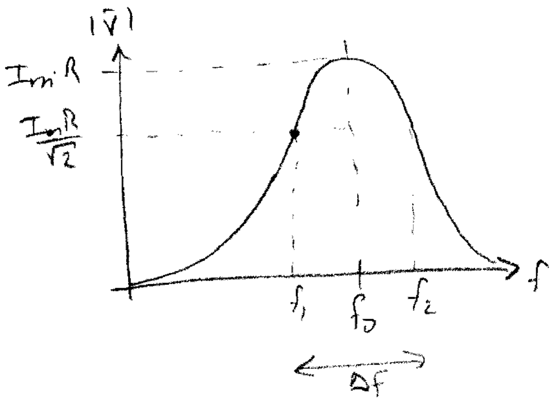
(14.47): $$Q = \omega_0 RC \longrightarrow C = \frac{Q}{\omega_0 R} = \frac{120}{(200 \times 10^3)(40)} = \mathbf{15\ \mu F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(4 \times 10^{10})(15 \times 10^{-6})} = \mathbf{1.6667\ \mu H}$$

$$B = \frac{\omega_0}{Q} = \frac{200 \times 10^3}{120} = \mathbf{1.6667\ krad/s}$$ $\left(\Delta f = \frac{f_0}{Q} = \frac{31{,}831}{120} = 265.3\ \text{Hz}\right)$

$$\omega_1 \approx \omega_0 - \frac{B}{2} = 200 - 0.8333 = \mathbf{199.167\ krad/s}$$ $\left(f_1 \approx f_0 - \frac{\Delta f}{2} = 31{,}698.4\ \text{Hz}\right)$

$$\omega_2 \approx \omega_0 + \frac{B}{2} = 200 + 0.8333 = \mathbf{200.833\ krad/s}$$ $\left(f_2 \approx f_0 + \frac{\Delta f}{2} = 31{,}963.7\ \text{Hz}\right)$



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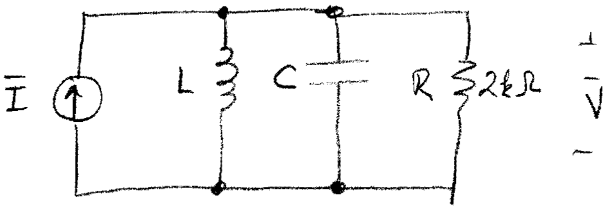


Solution 14.40

(a) $B = \omega_2 - \omega_1 = 2\pi\underbrace{(f_2 - f_1)}_{=\Delta f} = 2\pi(90-86)x10^3 = 8\pi krad/s$

$$\omega_o \approx \frac{1}{2}(\omega_1 + \omega_2) = 2\pi(88)x10^3 = 176\pi X10^3$$

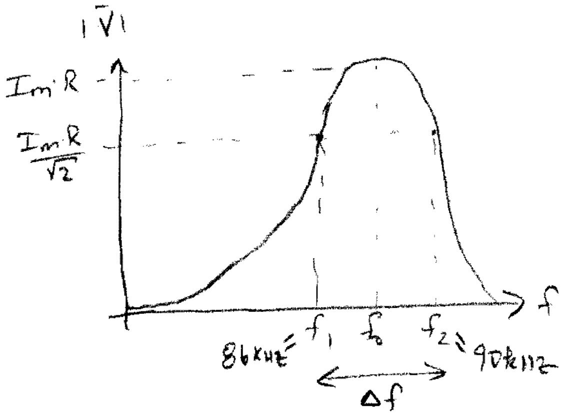
(14.46): $\Delta\omega = B = \frac{1}{RC} \longrightarrow C = \frac{1}{BR} = \frac{1}{8\pi x10^3 x2x10^3} = \underline{19.89nF}$

(14.44): (b) $\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega^2_o C} = \frac{1}{(176\pi X10^3)^2 x19.89x10^{-9}} =$ **164.45 μH**

(c) $\omega_o = 176\pi = \underline{552.9krad/s}$ $\Rightarrow f_o = 88$ kHz   Check, $\frac{1}{2\pi\sqrt{LC}} = 88$kHz ✓

(d) $B = 8\pi = \underline{25.13krad/s}$ ← from (a)

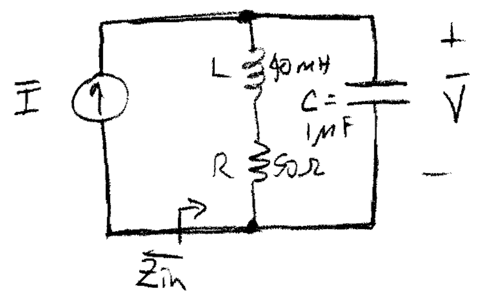
(e) $Q = \frac{\omega_o}{B} = \frac{176\pi}{8\pi} = \underline{22}$



$\Delta f = (90 - 86)$ kHz = 4 kHz

$f_o \approx \frac{1}{2}(f_1 + f_2) = 88$ kHz

$\underline{Q} = \frac{f_o}{\Delta f} = \underline{22}$



Solution 14.39

$$\overline{Y}_{in} = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\operatorname{Im}(Y_{in}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40\times10^{-3})(1\times10^{-6})} - \left(\frac{50}{40\times10^{-3}}\right)^2}$$

$\omega_0 =$ **4.841 krad/s**

OR, alternatively, at resonance $\operatorname{Im}(\overline{Z}_{in}) = 0$.

$$\overline{Z}_{in} = (R + j\omega L) \,\|\, \frac{1}{j\omega C} = \frac{(R + j\omega L)\cdot \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC} \cdot \frac{1 - \omega^2 LC - j\omega RC}{1 - \omega^2 LC - j\omega RC}$$

$$\therefore \overline{Z}_{in} = \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + \omega^2 RC} = \frac{R(1 - \omega^2 LC) - j\omega R^2 C + j\omega L(1 - \omega^2 LC) + \omega^2 LRC}{\text{DEN}}$$

Den is now a real #

For resonance, $\operatorname{Im}(\overline{Z}_{in}) = 0 \Rightarrow$

$$-\omega_0 R^2 C + \omega_0 L(1 - \omega_0^2 LC) = 0$$

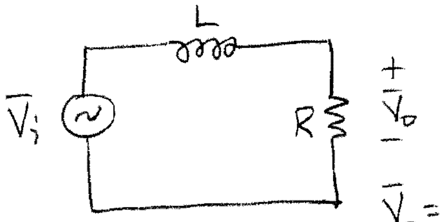
$$L - \omega_0^2 L^2 C = R^2 C$$

or $\omega_0^2 L^2 C = L - R^2 C$

$$\Rightarrow \omega_0^2 = \frac{L}{L^2 C} - \frac{R^2 C}{L^2 C} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \underline{\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} = 4{,}841.2 \ \frac{\text{rad}}{\text{s}}$$

(or $f_0 = 770.5$ Hz)



Solution 14.47

$$\bar{V}_o = \frac{R}{R+j\omega L} \cdot V_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R} = \frac{1}{1 + j\frac{\omega}{\omega_c}} \quad \text{where } \omega_c = \frac{R}{L}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that this circuit is a lowpass filter.

Low pass filter form.

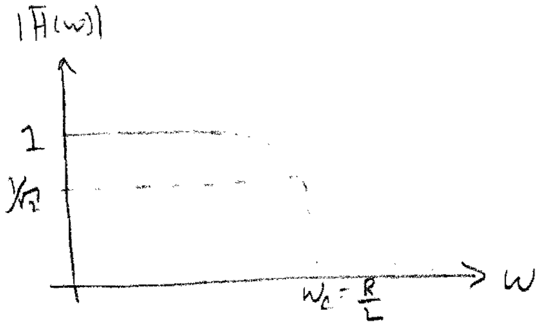
At the corner frequency, $\left|H(\omega_c)\right| = \frac{1}{\sqrt{2}}$, i.e.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega_c L}{R}\right)^2}} \longrightarrow 1 = \frac{\omega_c L}{R} \qquad \text{or} \qquad \omega_c = \frac{R}{L}$$

Hence,

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{10 \times 10^3}{2 \times 10^{-3}} = \mathbf{796\ kHz}$$



Solution 14.50

Determine what type of filter is in Fig. 14.87. Calculate the corner frequency f_c.

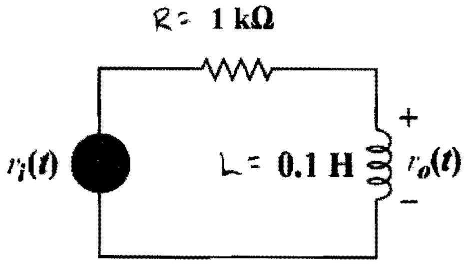


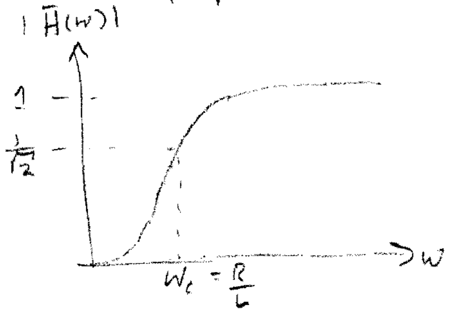
Figure 14.87
For Prob. 14.50.

Solution By voltage division $\bar{V}_o = \dfrac{j\omega L}{j\omega L + R}\bar{V}_i$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L} = \frac{1}{1 + \frac{R}{j\omega L}} = \frac{1}{1 - j\frac{\omega_c}{\omega}}$$ where $\omega_c = \dfrac{R}{L}$

H(0) = 0 and H(∞) = 1 showing that **this circuit is a highpass filter**.

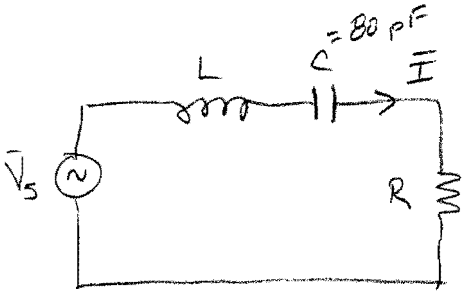
high pass response



$$|\mathbf{H}(\omega_c)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega_c L}\right)^2}} \longrightarrow 1 = \frac{R}{\omega_c L}$$

or $\quad \omega_c = \dfrac{R}{L} = 2\pi f_c$

$$f_c = \frac{1}{2\pi}\cdot\frac{R}{L} = \frac{1}{2\pi}\cdot\frac{1{,}000}{0.1} = \mathbf{1.5915\ kHz.}$$



Solution 14.53

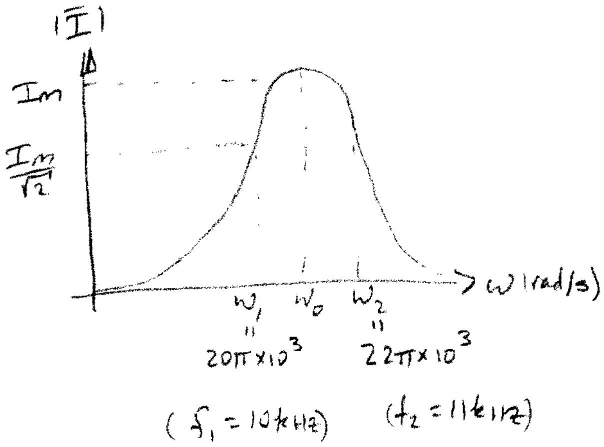
$\omega_1 = 2\pi f_1 = 20\pi \times 10^3$ rad/s

$\omega_2 = 2\pi f_2 = 22\pi \times 10^3$ rad/s

$B = \omega_2 - \omega_1 = 2\pi \times 10^3$ rad/s

$\omega_0 \approx \dfrac{\omega_2 + \omega_1}{2} = 21\pi \times 10^3$ rad/s

$Q = \dfrac{\omega_0}{B} = \dfrac{21\pi}{2\pi} = \mathbf{10.5}$



$\omega_0 = \dfrac{1}{\sqrt{LC}} \longrightarrow L = \dfrac{1}{\omega_0^2 C}$

$L = \dfrac{1}{(21\pi \times 10^3)^2 (80 \times 10^{-12})} = \mathbf{2.872\ H}$

huge!!

(14.39): $B = \dfrac{R}{L} \longrightarrow R = BL$

$R = (2\pi \times 10^3)(2.872) = \mathbf{18.045\ k\Omega}$

unusual for a series RLC circuit to have such a large resistance.