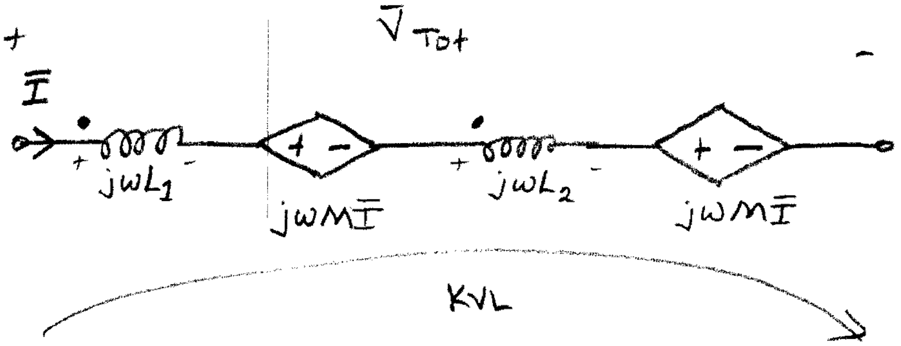
Prob. 5.1 Derive (13.18) in the text.

Substitute equivalent circuit model for the mutual coupling.



By KVL:

$$\bar{V}_{Tot} = j\omega L_1 \bar{I} + j\omega M\bar{I} + j\omega L_2 \bar{I} + j\omega M \bar{I}$$

$$= j\omega(\underbrace{L_1 + M + L_2 + M}_{\text{an effective inductance}})\bar{I}$$

$\therefore$

$$L_{eff} = L_1 + L_2 + 2M$$

Solution 13.3

$L_1 + L_2 + 2M = 500$ mH (1) , (13.18)

$L_1 + L_2 - 2M = 300$ mH (2) , (13.19)

Adding (1) and (2),

$2L_1 + 2L_2 = 800$ mH (3)

But, $L_1 = 3L_2$,; or $8L_2 = 800$, and L_2 = **100 mH**
(3)

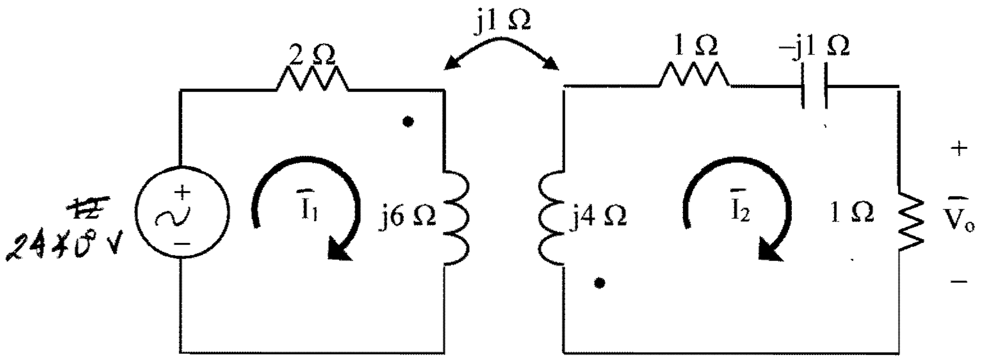
$L_1 = 3L_2$ = **300 mH**

From (2), ~~150~~ 300 + ~~50~~ 100 – 2M = ~~150~~ 300 leads to M = **50 mH**

(13.36): $k = M/\sqrt{L_1L_2} = 50/\sqrt{100x300}$ = **0.2887**

Solution 13.7 . Determine $\bar{V}_o$

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$(2+j6)\bar{I}_1 + j\bar{I}_2 = 24$

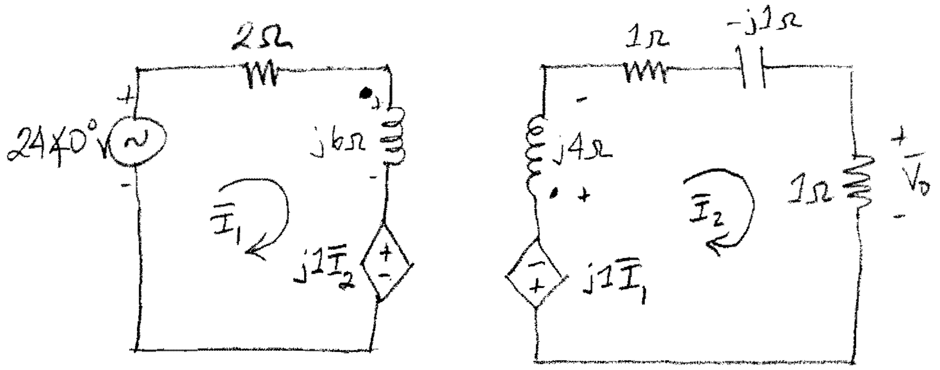
For mesh 2,

$j\bar{I}_1 + (2-j+j4)\bar{I}_2 = j\bar{I}_1 + (2+j3)\bar{I}_2 = 0$ or $\bar{I}_1 = (-3+j2)\bar{I}_2$

Substituting into the first equation results in $\bar{I}_2 = (-0.8762+j0.6328)$ A.

$$\bar{V}_o = \bar{I}_2 x 1 = \mathbf{1.081\angle 144.16^\circ\ V.}$$

Equivalent circuit:



Solution 13.8

Find v(t) for the circuit in Fig. 13.77.

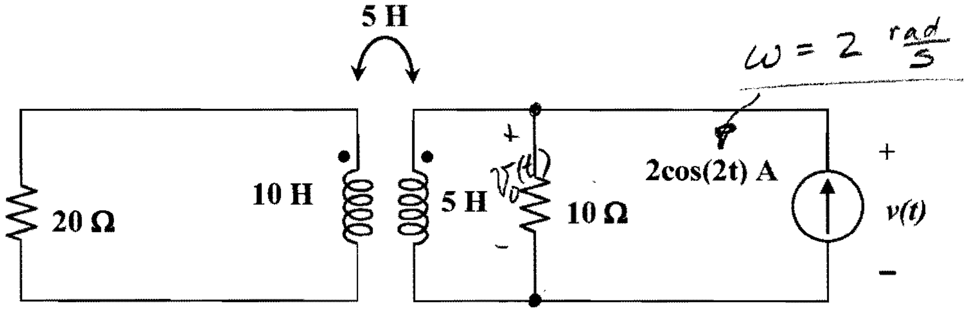
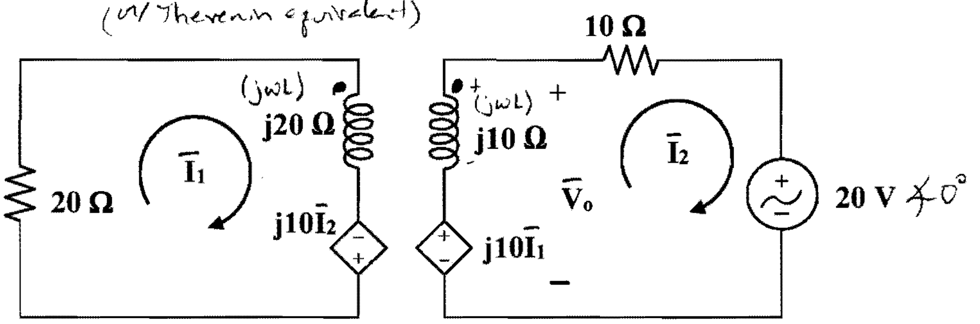


Figure 13.77
For Prob. 13.8.

Solution

Step 1. We ~~need to~~ transform the circuit into the frequency domain, then ~~and~~ replace the coupled inductors with the dependent source model. In addition, we ~~need to~~ replace the current source in parallel with the resistor with the equivalent voltage source in series with the resistor (source transformation). (w/ Thevenin equivalent)



The 10 H inductor becomes j2x10 = j20 Ω and the 5 H mutual coupling and 5 H inductor becomes j2x5 = j10 Ω. The source transformation converts the current source resistor combination of 10 Ω in parallel with the 2 A source with a 10 Ω resistor in series with a 20 V source.

Loop 1, (1) $20\mathbf{I}_1 + j20\mathbf{I}_1 - j10\mathbf{I}_2 = 0$ and loop 2, (2) $-j10\mathbf{I}_1 + j10\mathbf{I}_2 + 10\mathbf{I}_2 + 20 = 0$. Finally (4) $\mathbf{V}_o = -j10\mathbf{I}_2 + j10\mathbf{I}_1$. Note we are representing the frequency domain value of v(t) by $\mathbf{V}_o$.

Step 2. From the first loop equation we ~~get~~ find $(20+j20)\mathbf{I}_1 = j10\mathbf{I}_2$ or (3) $\mathbf{I}_1 = (0.25+j0.25)\mathbf{I}_2$.

sub (2): This leads to $-j10(0.25+j0.25)\mathbf{I}_2 + (10+j10)\mathbf{I}_2 = -20 = (12.5+j7.5)\mathbf{I}_2$ or

$\mathbf{I}_2 = 20\angle180°/(14.5774\angle30.964°) = 1.37199\angle149.036°$ and

sub (3): $\mathbf{I}_1 = (0.35355\angle45°)(1.37199\angle149.039°) = 0.48507\angle-165.961°$. Finally,

sub (4): $\mathbf{V}_o = -j10(-1.17647 + j0.70589) + j10(-0.47058-j0.117669)$

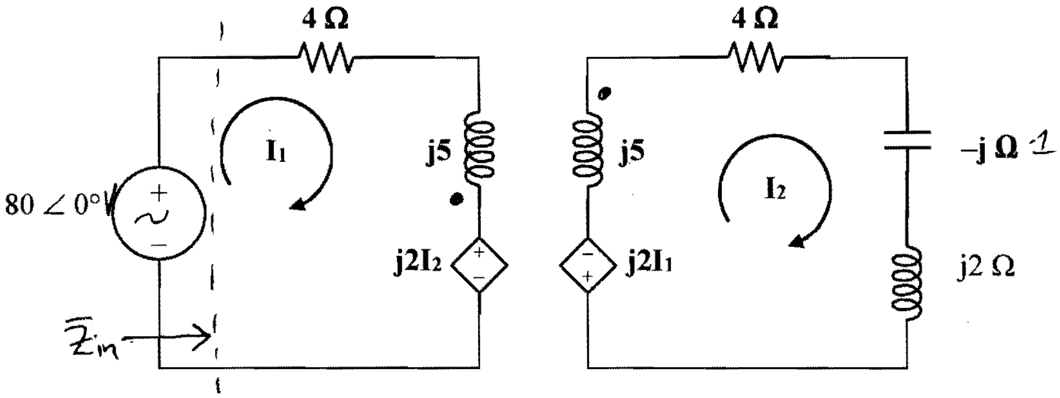
$= 8.2356+j7.0589 = 10.847\angle40.6°$ V or

v(t) = **10.847 cos(10t + 40.6°) A.**

Whites

Solution 13.13 Determine impedance seen by source.

Equivalent Circuit:



mesh 1: $-80 + (4+j5)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$ or $(4+j5)\mathbf{I}_1 + j2\mathbf{I}_2 = 80$ (1)

mesh 2: $j2\mathbf{I}_1 + (4+j6)\mathbf{I}_2 = 0$ or $\mathbf{I}_2 = [-j2/(7.2111\angle 56.31°)]\mathbf{I}_1 = (0.27735\angle -146.31°)\mathbf{I}_1$ (2)

Sub into (1): $[4+j5 + j2(-0.230769-j0.153846)]\bar{I}_1 = [4+j5+0.307692-j0.461538]\bar{I}_1 = 80$

$[4.307692+j4.538462]\bar{I}_1 = 80$ or $\bar{I}_1 = 80/(6.2573\angle 46.494°)$
$= 12.78507\angle -46.494°$ A.

$\bar{Z}_{in} = \frac{\bar{V}_s}{\bar{I}_1}$

$\bar{Z}_{in} = 80/\bar{I}_1 = 6.2573\angle 46.494°\ \Omega =$ **(4.308 + j4.538) Ω**

An alternate approach would be to use the equation,

$$\mathbf{Z}_{in} = 4 + j(5) + \frac{4}{j5+4-j+j2} = 4 + j5 + \frac{4}{7.2111\angle 56.31°}$$
$= 4+j5+0.5547\angle -56.31° = 4+0.30769+j(5-0.46154)$

$=$ **[4.308+j4.538] Ω**.

Solution 13.24

(a) $k = M/\sqrt{L_1 L_2} = 1/\sqrt{4x2} = \mathbf{0.3535}$

(b) $\omega = 4$ convert circuit to the frequency domain

C: 1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

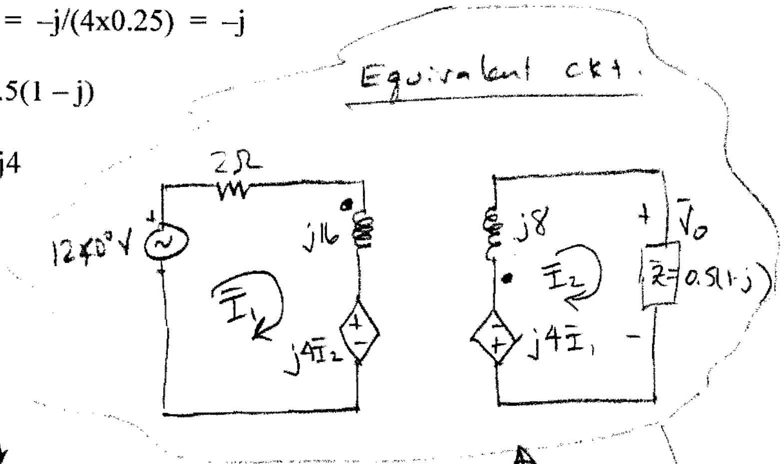
R||C: $1||(-j) = -j/(1-j) = 0.5(1-j)$

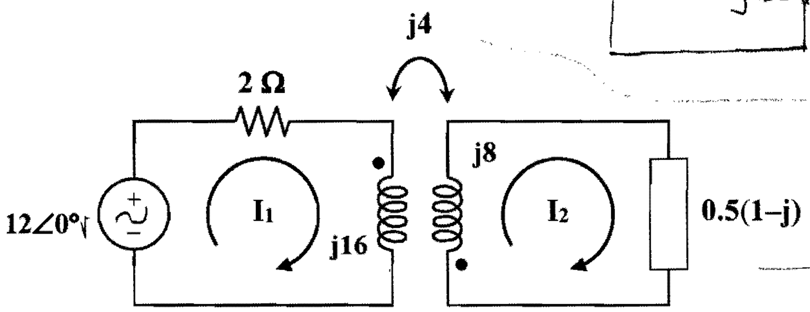
M: 1 H produces $j\omega M = j4$

L: 4 H produces j16

L: 2 H becomes j8

Equivalent ckt.





KVL, mesh 1: $12 = (2 + j16)\bar{I}_1 + j4\bar{I}_2$

or $6 = (1 + j8)\bar{I}_1 + j2\bar{I}_2$ (1)

KVL, mesh 2: $0 = (j8 + 0.5 - j0.5)\bar{I}_2 + j4\bar{I}_1$ or $\bar{I}_1 = (0.5 + j7.5)\bar{I}_2/(-j4)$ (2)

$= (-1.875 + j0.125)\bar{I}_2$

Substituting (2) into (1),

$6 = (1+j8)(-1.875+j0.125)\bar{I}_2 + j2\bar{I}_2$

or, $24 = (-11.5 - j51.5)\bar{I}_2$ or $\bar{I}_2 = -24/(11.5 + j51.5) = -0.455\angle -77.41° = 0.455\angle 102.59$ A

$\bar{V}_o = \bar{I}_2(0.5)(1 - j) = 0.3217\angle 57.59°$ V

$v_o(t) = \mathrm{Re}\{\bar{V}_o e^{j\omega t}\}$

$v_o(t) = \mathbf{321.7\cos(4t + 57.6°)\ mV}$

(c) From (2), $\bar{I}_1 = (0.5 + j7.5)\bar{I}_2/(-j4) = 0.855\angle{-81.21°}$ A → $i_1(t) = Re\{\bar{I}_1 e^{j\omega t}\}$

$i_1(t) = 0.885\cos(4t - 81.21°)$ A, $i_2(t) = +0.455\cos(4t + ~~77.41~~ 102.59°)$ A

At t = 2s,

$$4t = 8 \text{ rad} = 98.37°$$ ← $\frac{180°}{\pi \text{ rad}}$, then $-360°$

$$i_1\big|_{t=2s} = 0.885\cos(98.37° - 81.21°) = 0.~~8169~~ 8456 \text{ A}$$

$$i_2\big|_{t=2s} = +0.455\cos(98.37° + ~~77.41~~ 102.59°) = -0.4249$$

Eqn (13.32) w/ '+' since both currents assumed entering dotted terminals:

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(4)(0.~~8169~~ 8456)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = ~~1.168~~ \ 1.531 \text{ J}$$

Solution 13.36

$N_2 = n$
$N_1 = 1$
$\frac{N_2}{N_1} = n$

Following the two rules in section 13.5, we obtain the following:

(a) $\bar{V}_2/\bar{V}_1 = \mathbf{-n}$, $\bar{I}_2/\bar{I}_1 = \mathbf{-1/n}$

(b) $\bar{V}_2/\bar{V}_1 = \mathbf{-n}$, $\bar{I}_2/\bar{I}_1 = \mathbf{-1/n}$

(c) $\bar{V}_2/\bar{V}_1 = \mathbf{n}$, $\bar{I}_2/\bar{I}_1 = \mathbf{1/n}$

(d) $\bar{V}_2/\bar{V}_1 = \mathbf{n}$, $\bar{I}_2/\bar{I}_1 = \mathbf{-1/n}$

Solution 13.37

A 240/2400 V (rms) step-up ideal transformer delivers 50 kW to a resistive load. Calculate: (a) the turns ratio, (b) the primary current, (c) the secondary current.

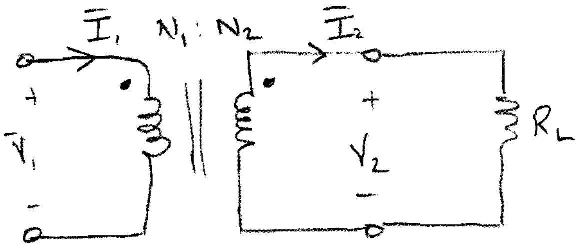
Solution

(a) $n = \dfrac{V_2}{V_1} = \dfrac{2400}{240} = \mathbf{10}$

(b) $\mathbf{S_1} = \mathbf{V_1}(\mathbf{I_1})^* = \mathbf{S_2} = \mathbf{V_2}(\mathbf{I_2})^* = 50,~~000~~$ kVA which leads to

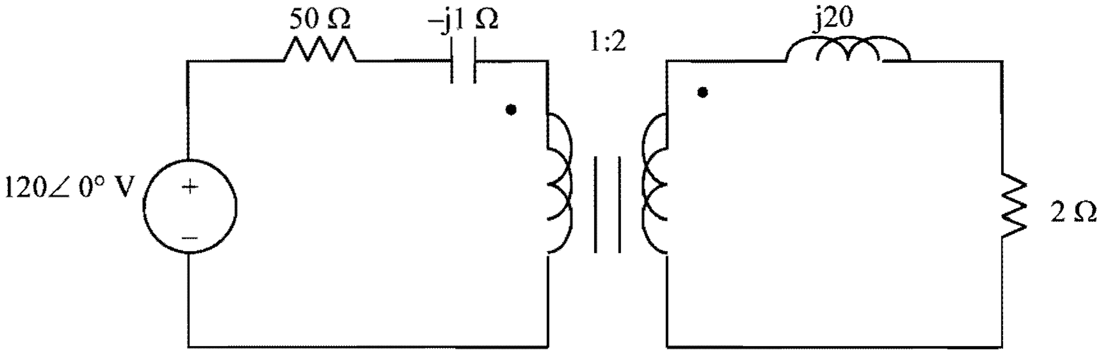
$\mathbf{I_1} = 50{,}000/240 = \mathbf{208.3\ A.}$

(c) $\mathbf{I_2} = 50{,}000/2{,}400 = \mathbf{20.83\ A.}$



Solution 13.42

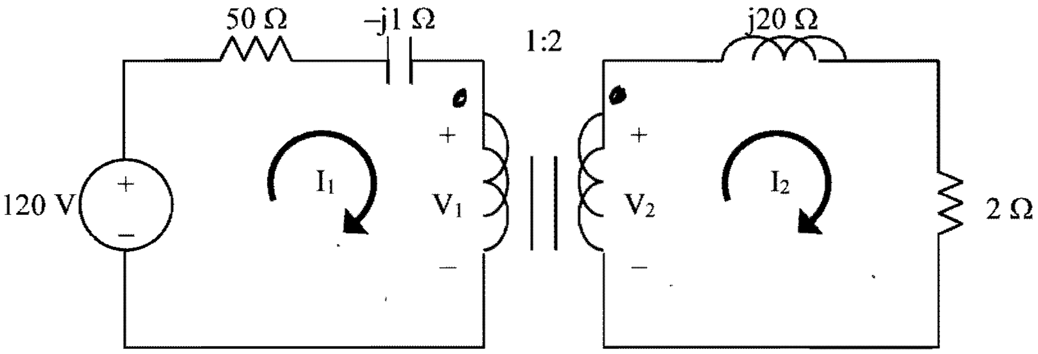
For the circuit in Fig. 13.106, determine the power absorbed by the 2 Ω resistor. Assume the 120 V source is an rms value.



Ideal
Figure 13.106
For Prob. 13.42.

Solution

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-120 + (50-j)\bar{I}_1 + \bar{V}_1 = 0 \qquad (1)$$

For mesh 2,

$$-\bar{V}_2 + (2+j20)\bar{I}_2 = 0 \qquad (2)$$

At the transformer terminals,

$$\bar{V}_2 = 2\bar{V}_1 \text{ or } 2\bar{V}_1 - \bar{V}_2 = 0 \qquad (3)$$

$$\bar{I}_1 = 2\bar{I}_2 \text{ or } \bar{I}_1 - 2\bar{I}_2 = 0 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} 50-j & 0 & 1 & 0 \\ 0 & 2+j20 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB,

```
>> A = [(50-j) 0 1 0;0 (2+20j) 0 -1;0 0 2 -1;1 -2 0 0]

A =

  Columns 1 through 3

  50.0000 - 1.0000i        0              1.0000
       0           2.0000 +20.0000i        0
       0                0           2.0000
   1.0000          -2.0000                0

  Column 4

       0
  -1.0000
  -1.0000
       0

>> B = [120;0;0;0]

B =

   120
    0
    0
    0

>> C = inv(A)*B

C =

   2.3614 – 0.8170i            (Ī1)
   1.1806 – 0.0934i            (Ī2)
   2.1159 + 11.7136i           (V̄1)
   4.2318 + 23.4272i           (V̄2)
```

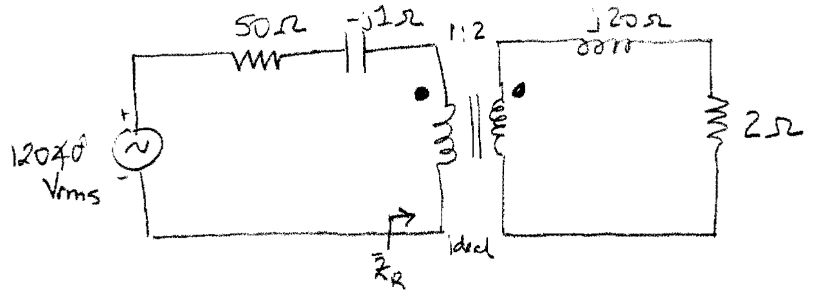
$$\bar{I}_2 = (1.1806 - j0.0934) \text{ A or } 1.1843\angle{-4.52^\circ} \text{ A}$$

The power absorbed by the 2-Ω resistor is

Rms

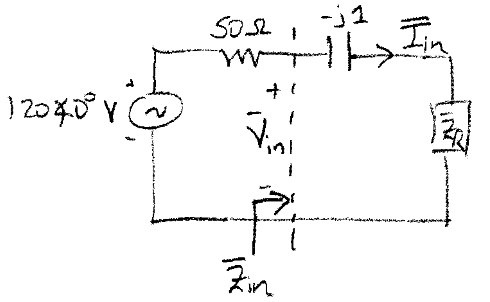
$\underline{\underline{P}} = |\bar{I}_2|^2 R = (1.1843)^2 2 = \underline{\underline{\mathbf{2.805\ W.}}}$

OK.. that was a long, but rigorous, sol'n. Here's the quick way to solve these types of problems.



From (13.60), $\bar{Z}_R = \frac{\bar{Z}_L}{n^2} = \frac{2+j20}{(2/1)^2} = 0.5+j5\ \Omega$

Equivalent circuit for the primary side:



$\bar{Z}_{in} = \bar{Z}_R - j1 = 0.5+j5-j1 = 0.5+j4\ \Omega$

By voltage division,

$\bar{V}_{in} = \frac{\bar{Z}_{in}}{\bar{Z}_{in}+50} \cdot V_s = \frac{0.5+j4}{0.5+j4+50} \cdot 120°$

$= 9.549\angle 78.35°\ V$

The time-average power delivered to the $\bar{V}_{in}$ port is (w/ $\bar{V}_{in}$ in rms)

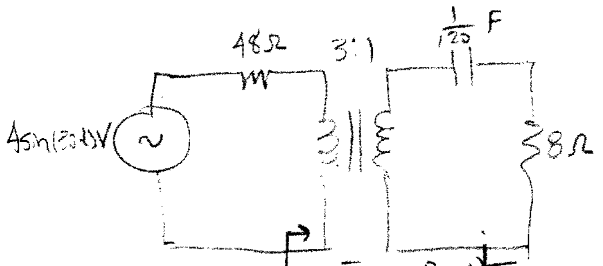
$P_{in} = Re(\bar{V}_{in}\bar{I}_{in}^*) = Re\left\{\frac{|\bar{V}_{in}|^2}{\bar{Z}_{in}^*}\right\} = |\bar{V}_{in}|^2 Re\left\{\frac{1}{\bar{Z}_{in}^*}\right\} = 9.549^2\, Re\left\{\frac{1}{0.5-j4}\right\}$

$= 91.184\, Re\{0.03077+j0.2462\} = 91.184 \cdot 0.03077 = 2.806\ W$

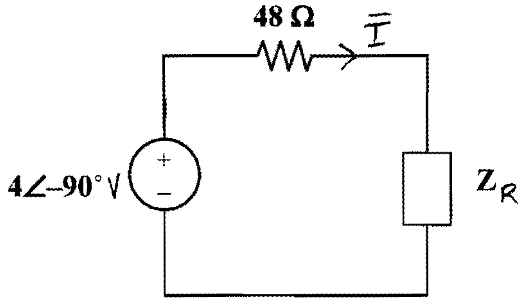
Because the components from the input port V_{in} to the 2 Ω load is comprised of lossless components, all of this time average power is ultimately delivered to the 2 Ω load.

$\therefore\ \underline{\underline{P_{2\Omega} = P_{in} = 2.806\ W}}$

Solution 13.45



$\bar{Z}_R = \frac{\bar{Z}_L}{n^2} = \frac{8 - \frac{j}{\omega C}}{(1/3)^2}$
$= 72 - \frac{j9}{30/120}$
$= 72 - j36\ \Omega$



$$\bar{Z}_L = 8 - \frac{j}{\omega C} = 8 - j4,\ n = 1/3$$

$$\bar{Z}_R = \frac{Z_L}{n^2} = 9Z_L = 72 - j36\ \Omega$$

$$\bar{I} = \frac{4\angle -90^\circ}{48 + 72 - j36} = \frac{4\angle -90^\circ}{125.28\angle -16.7^\circ} = 0.03193\angle -73.3^\circ\ A$$

$48 + \bar{Z}_R$

We now have some choices, we can go ahead and calculate the current in the second loop and calculate the power delivered to the 8-ohm resistor directly, or we can merely say that the time-average power delivered to the equivalent resistor in the primary side must be the same as the time-average power delivered to the 8-ohm resistor. Therefore,

$$P_{8\Omega} = \left|\frac{\bar{I}^2}{2}\right| 72 = 0.5098 \times 10^{-3}\, 72 = \mathbf{36.71\ mW}$$

The student is encouraged to calculate the current in the secondary and calculate the power delivered to the 8-ohm resistor to verify that the above is correct.

Solution 13.55

For the circuit in Fig. 13.119, calculate the equivalent resistance.

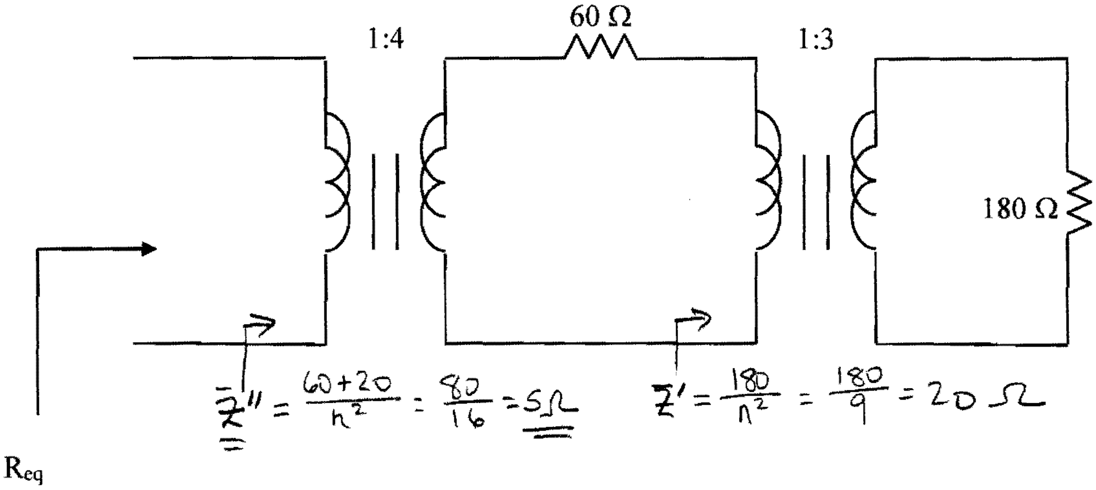


Figure 13.119
For Prob. 13.55.

Solution

We first reflect the 80-Ω resistance to the middle circuit.

$$Z' = 60 + [180/(3)^2] = 60 + 20 = 80\ \Omega$$

We now reflect this to the primary side.

$$R_{eq} = Z'/(4)^2 = \underline{5\ \Omega}.$$