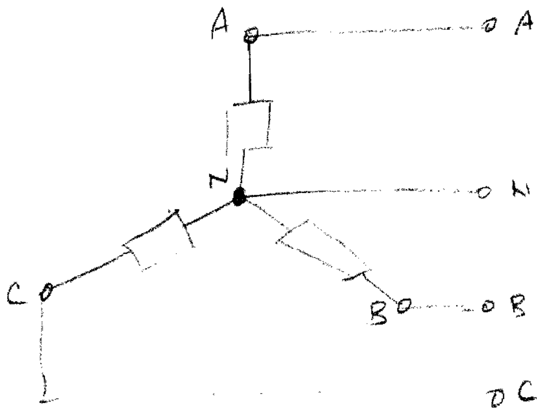
Solution 12.5 positive abc phase sequence. From Table 12.1:

$\mathbf{V}_{AB}$ = 1.7321x$\mathbf{V}_{AN}\angle$+30° = 207.8$\angle$(32°+30°) = 207.8$\angle$62° V or

$\bar{V}_{AB}$ = 207.8cos(ωt+62°) V $-120°$
which also leads to,
$\bar{V}_{BC}$ = 207.8cos(ωt–58°) V $-120°$

and
$\bar{V}_{CA}$ = 207.8cos(ωt+182°) V

207.8cos(ωt+62°) V, 207.8cos(ωt–58°) V, 207.8cos(ωt+182°) V

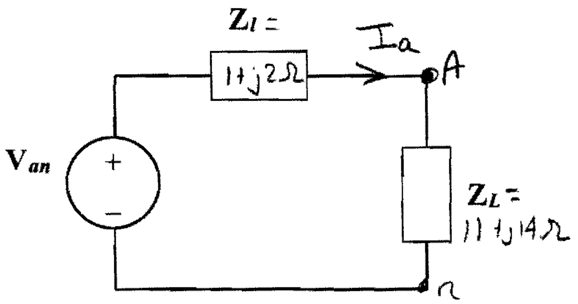


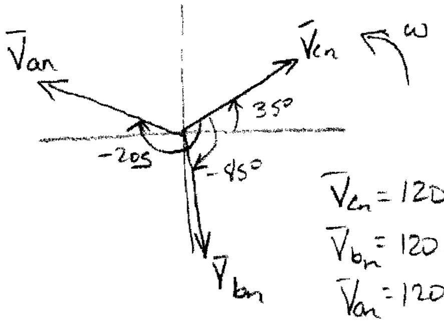
Solution 12.8

In a balanced three-phase wye-wye system, the source is an acb-sequence of voltages and $\mathbf{V}_{cn} = 120\angle 35^\circ$ V rms. The line impedance per phase is $(1+j2)\Omega$, while the per phase impedance of the load is $(11+j14)\ \Omega$. Calculate the line currents and the load voltages.

Solution Refer to Fig. 12.10 for the Y-Y connection, Can analyze each phase separately.

Consider the per phase equivalent circuit shown below.





$\bar{V}_{cn} = 120\angle 35^\circ$ Vrms
$\bar{V}_{bn} = 120\angle -85^\circ$ Vrms
$\bar{V}_{an} = 120\angle -205^\circ$ Vrms

Since the sequence is acb and $\mathbf{V}_{cn} = 120\angle 35^\circ$ V_{rms}, then $\mathbf{V}_{an} = 120\angle 155^\circ$ V_{rms} and $\mathbf{V}_{bn} = 120\angle -85^\circ$ V_{rms}

$\mathbf{I}_a = \mathbf{V}_{an}/(\mathbf{Z}_l + \mathbf{Z}_L) = (120\angle 155^\circ)/(12+j16) = (120\angle 155^\circ)/(20\angle 53.13^\circ)$

$= \mathbf{6\angle 101.87^\circ}$ ~~amps.~~ Arms

$= \angle -138.13^\circ$

$\mathbf{I}_b = \mathbf{I}_a \angle 120^\circ = \mathbf{6\angle 221.87^\circ}$ **amps.** −120°

$\mathbf{I}_c = \mathbf{I}_a \angle -120^\circ = \mathbf{6\angle -18.13^\circ}$ ~~amps.~~ Arms

−120° acb sequence

$\bar{V}_{AN} = \mathbf{I}_a\mathbf{Z}_L = (6\angle 101.87^\circ)(11+j14) = (6\angle 101.87^\circ)(17.8045\angle 51.843^\circ)$

$= \mathbf{106.83\angle 153.71^\circ}$ ~~volts.~~ Vrms

$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle 120^\circ = \mathbf{106.83\angle -86.29^\circ}$ ~~volts.~~ Vrms −120°

$\mathbf{V}_{CN} = \mathbf{V}_{AN}\angle -120^\circ = \mathbf{106.83\angle 33.71^\circ}$ **volts.** Vrms

−120° acb sequence

Solution 12.9

Refer to Fig. 12.10 for the Y-Y connection. Can analyze each phase separately.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{120\angle 0°}{20 + j15} = \mathbf{4.8\angle -36.87°\ A}_{rms}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{4.8\angle -156.87°\ A}_{rms}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{4.8\angle 83.13°\ A}_{rms}$$

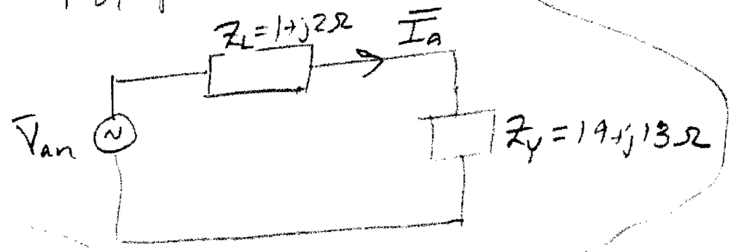
-120°

As a balanced system, $\mathbf{I}_n = \mathbf{0\ A}$

With $\bar{V}_{an} = 120\angle 0°$ Vrms
$\bar{V}_{bn} = 120\angle -120°$ Vrms
$\bar{V}_{cn} = 120\angle 120°$ Vrms

We recognize a positive abc sequencing.

For phase a:



✓

To check: $\bar{I}_a + \bar{I}_b + \bar{I}_c = 4.8\angle -36.87° + 4.8\angle -156.87° + 4.8\angle 83.13°$
$= 0$ ✓

Problem 4.4

Problem

Solve for the line currents in the **Y-Δ** circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45°\Omega$.

recognize a positive abc phase sequencing:

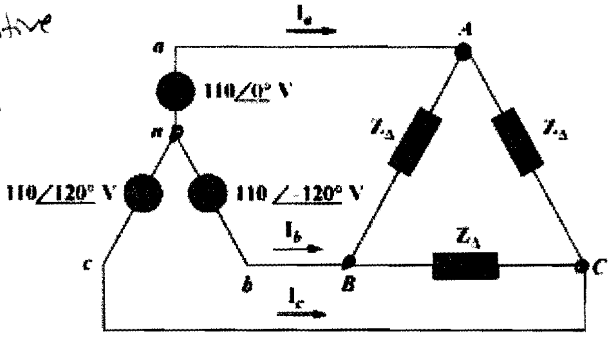
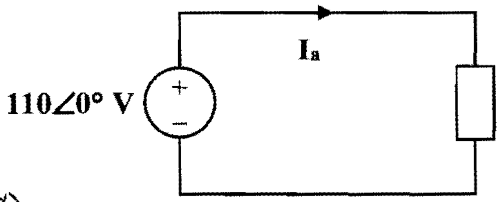


Figure 12.45

Solution

Convert the delta-load to a wye-load and apply per-phase analysis.



$\mathbf{Z}_Y = \frac{Z_\Delta}{3} = \frac{60\angle 45°\,\Omega}{3} = 20\angle 45°\,\Omega$

$$\bar{I}_a = \frac{V_{an}}{Z_Y} = \frac{110\angle 0°\ V_{rms}}{20\angle 45°\,\Omega} = 5.5\angle -45°\ A_{rms}$$

(12.28)

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\ \Omega$$

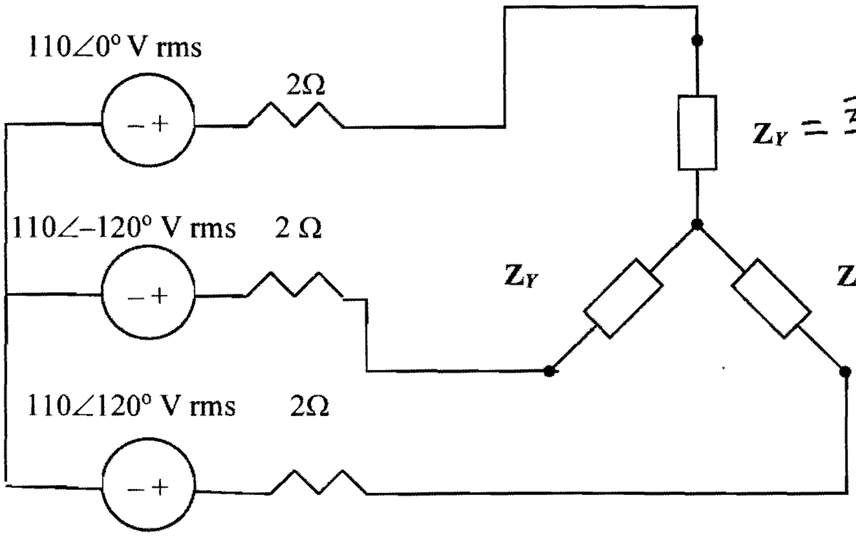
$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \mathbf{5.5\angle -45°\ A}_{rms}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{5.5\angle -165°\ A}_{rms} \quad -120°$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{5.5\angle 75°\ A}_{rms} \quad -120°$$

Solution 12.13

Convert the delta load to wye as shown below.

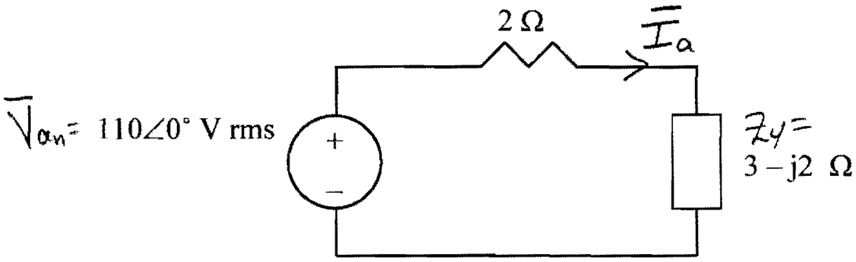


$Z_Y = \frac{Z_\Delta}{3} = \frac{9-j6}{3} = 3-j2\ \Omega$

eqn (12.28):

$$Z_Y = \frac{1}{3} Z_\Delta = 3 - j2\ \Omega$$

We consider the single phase equivalent shown below.



$\bar{I}_a = \frac{\bar{V}_{an}}{2+\bar{Z}_Y} = \frac{110\angle 0°}{2+3-j2}$
$= 20.43 \angle 21.8°$ Arms

$\bar{S} = \bar{V}\bar{I}^*$
$= \bar{I}\cdot\bar{Z}_Y\cdot\bar{I}^*$
$= |\bar{I}|^2\cdot\bar{Z}_Y$

$\mathbf{I_a} = 110/(2 + 3 - j2) = 20.43\angle 21.8°$ Arms

$I_L = |\mathbf{I_a}| = \mathbf{20.43}$ **Arms** ← same for every phase.

$S = 3\underbrace{|\mathbf{I}_a|^2 Z_Y}_{\text{per phase}} = 3(20.43)^2(3-j2) = 4514\angle{-33.96°} = \cancel{3744}\ 3756 - j\cancel{2522}\ 2{,}504$ VA

all 3 phases

$P = \text{Re}(S) = \mathbf{3.7\cancel{44}\,56}$ **kW.**

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Solution 12.19

For the Δ-Δ circuit of Fig. 12.50, calculate the phase and line currents.

See that there is abc positive phase sequencing!

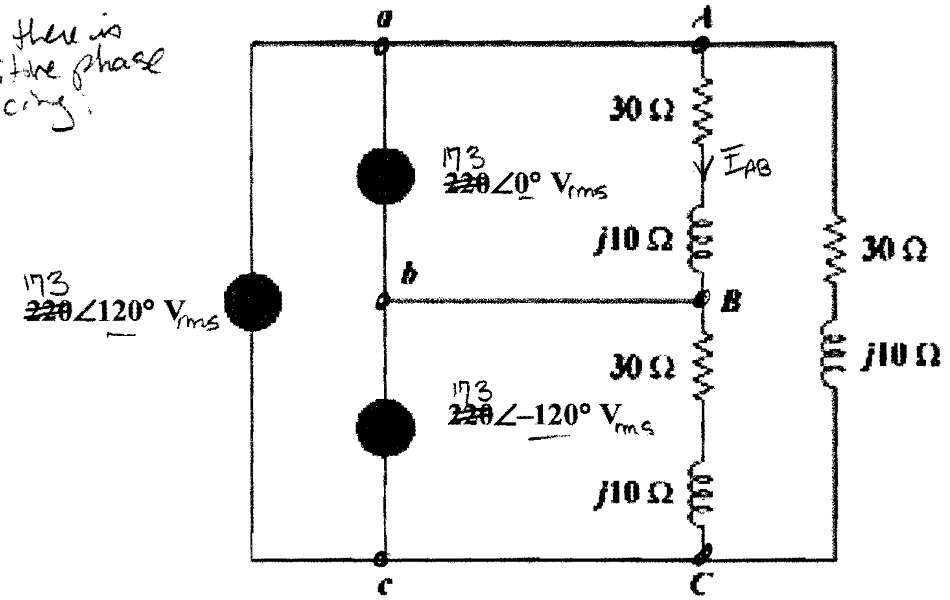


Figure 12.50
For Prob. 12.19.

Solution

$\mathbf{Z}_\Delta = 30 + j10 = 31.62\angle 18.43°\ \Omega$

From directly observing this ckt, we see that The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\overset{173}{\cancel{440}}\angle 0°}{31.62\angle 18.43°} = \overset{5.471}{\cancel{13.915}}\angle -18.43°\ \mathbf{A}_{rms}$$

($\mathbf{V}_{ab} = V_{AB}$)

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \overset{5.471}{\cancel{13.915}}\angle -138.43°\ \mathbf{A}_{rms}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \overset{5.471}{\cancel{13.915}}\angle 101.57°\ \mathbf{A}_{rms}$$

$-120°$ $-120°$

The line currents are From Table 12.1 $I_a = I_{AB}\sqrt{3}\angle -30°$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

$$\mathbf{I}_a = 13.915\sqrt{3}\angle -48.43° = \overset{9.476}{\cancel{24.1}}\angle -48.43°\ \mathbf{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \overset{9.476}{\cancel{24.1}}\angle -168.43°\ \mathbf{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \overset{9.476}{\cancel{24.1}}\angle 71.57°\ \mathbf{A}$$

$-120°$ $-120°$

Solution 12.21

Three 440-volt generators, form a delta connected source which is connected to a balanced delta connected load of $\mathbf{Z}_L = (8.66 + j5)\ \Omega$ per phase as shown in Fig. 12.52. Determine the value of $\mathbf{I}_{BC}$ and $\mathbf{I}_{aA}$. What is the pf of the load?

positive abc phase sequencing:

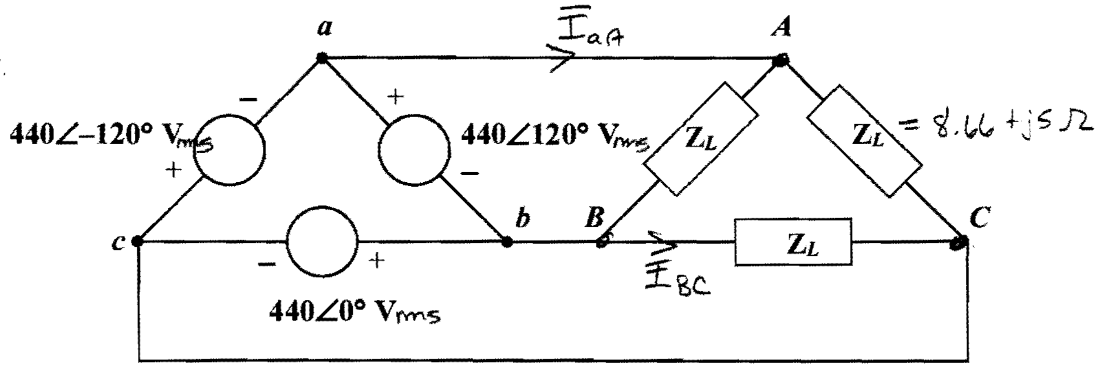


Figure 12.52
For Prob. 12.21.

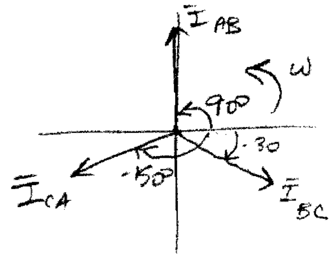
Solution Directly from the circuit we observe that $I_{BC} = \frac{V_{BC}}{Z_L} = \frac{V_{bc}}{Z_L}$;

$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_L = 440\angle 0^\circ/(10\angle 30^\circ) = \mathbf{44\angle{-30^\circ}}$ Arms.

$\mathbf{I}_{aA} = \mathbf{I}_{AC} + \mathbf{I}_{AB} = [440\angle 60^\circ/(10\angle 30^\circ)] + [440\angle 120^\circ/(10\angle 30^\circ)]$
$= [44\angle 30^\circ] + [44\angle 90^\circ] = 38.105 + j22 + j44 = 38.105 + j66 = \mathbf{76.21\angle 60^\circ}$ **A.**

pf = 8.66/10 = **0.866.**

positive abc sequence



$I_{CA} = 44 \angle -150^\circ$ Arms
$I_{AB} = 44 \angle -270^\circ$ Arms

From Table 12.1

$I_a = I_{AB}\sqrt{3}\angle -30^\circ = 44\cdot\sqrt{3}\angle(-270-30^\circ)$

$I_a = 76.21 \angle -300^\circ = 76.21 \angle 60^\circ$ Arms

$S = V\cdot I^* = 3\cdot V_{BC}\cdot I_{BC}^* = 3(440\angle 0^\circ)(44\angle +30^\circ) = 150{,}896 + j\,87{,}120$

$pf = \cos\theta = \frac{P}{S} = \frac{150{,}890}{174{,}235} = 0.866$

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Solution 12.25 positive abc sequencing. Δ-Y connection. Convert to a Y-Y connection using

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent. Or, from Table 12.1 $\bar{I}_a = \frac{\bar{V}_p \angle -30^\circ}{\sqrt{3}\,\bar{Z}_Y}$

$$\mathbf{I}_a = \frac{440\angle(10^\circ - 30^\circ)}{\sqrt{3}\,\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.318\angle -24.78^\circ\ \Omega$

$$\mathbf{I}_a = \frac{440\angle -20^\circ}{\sqrt{3}(14.318\angle -24.78^\circ)} = \mathbf{17.742\angle 4.78^\circ}\ \text{Arms}\ \cancel{\textbf{amps}}.$$

-120°

$$\mathbf{I_b = I_a\angle{-120^\circ} = 17.742\angle{-115.22^\circ}}\ \text{Arms}\ \cancel{\textbf{amps}}.$$

-120°

$$\mathbf{I_c = I_a\angle{+120^\circ} = 17.742\angle 124.78^\circ}\ \text{Arms}\ \cancel{\textbf{amps}}.$$

Solution 12.27 See Fig. 12.18 for the circuit. $\bar{V}_{ab} = 208 \angle 0° \text{ V}_{rms}$

Since $\bar{Z}_L$ and $\bar{Z}_\ell$ are in series, we can lump them together so that $\bar{Z}_Y = \bar{Z}_L + \bar{Z}_\ell$ or

$$\bar{Z}_Y = (2+j) + (6+j4) = 8 + j5 \ \Omega$$

$$\bar{I}_a = \frac{\frac{V_P}{\sqrt{3}} < -30^o}{\bar{Z}_Y} = \frac{208 < -30^o}{\sqrt{3}(8+j5)} = 12.729 \angle -62.01° \text{ A}_{rms}$$

$$\bar{V}_L = (6+j4)\bar{I}_a = \frac{208(0.866 - j0.5)(6+j4)}{\sqrt{3}(8+j5)} = 80.81 - j43.54 = 91.79 \angle -28.32° \text{ V}_{rms}$$

$\bar{V}_L = \bar{I}_a \cdot \bar{Z}_\ell$ $(= \bar{V}_{AN})$

$$\underline{|V_L| = \mathbf{91.79 \ V}}$$

From Table 12.1, $\bar{I}_a = \dfrac{V_P \angle -30°}{\sqrt{3}\,\bar{Z}_Y}$ $\quad$ V_P in this case is the source voltage $\bar{V}_{ab} = V_P \angle 0°$.

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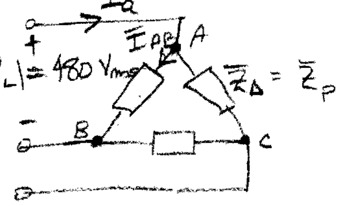

Solution 12.31

A balanced delta-connected load is supplied by a 60-Hz three-phase source with a line voltage of 480V. Each load phase draws $\underbrace{24\text{ kW}}_{=P_p}$ at a lagging power factor of $\underbrace{0.8}_{\cos\theta = 0.8}$. Find:

(a) the load impedance per phase
(b) the line current magnitude
(c) the value of capacitance needed to be connected in parallel with each load phase to minimize the current from the source.

Solution

S_p, Q_p, θ, P_p

(a) per phase:

$$P_p = 24{,}000, \qquad \cos\theta = 0.8, \Rightarrow S_p = \frac{P_P}{\cos\theta} = 24/0.8 = 30\text{kVA} \ \text{ and } \theta = 36.87°$$

$\therefore \quad Q_p = S_P \sin\theta = 18\text{ kVAR}$ "p": phase

$\therefore \quad \bar{S} = 3\bar{S}_p = 3(24 + j18) = 72 + j54\text{ kVA} = 90 \angle 36.87°\text{ kVA}$

For delta-connected load, $V_p = V_L = 480$ V(rms). But $\bar{S} = 3\bar{S}_p = 3\bar{V}_p \cdot \bar{I}_p^* = 3\bar{V}_p \cdot \frac{\bar{V}_p^*}{\bar{Z}_p^*} = \frac{3V_p^2}{\bar{Z}_p^*}$ eqn (12.52)

$$\bar{S} = \frac{3V^2_p}{\bar{Z}^*_p} \longrightarrow \bar{Z}^*_p = \frac{3V^2_p}{\bar{S}} = \frac{3(480)^2}{(72 + j54)x10^3}, \qquad \underline{\bar{Z}_P = [6.144 + j4.608]\Omega} = \bar{Z}_\Delta$$

eqn (12.50):

(b) $$P_p = \frac{\sqrt{3}V_L I_L \cos\theta}{3} \longrightarrow I_L = \frac{3 \cdot 24{,}000}{\sqrt{3}x480x0.8} = \cancel{\mathbf{36.08}}\ 108.24\ \mathbf{A}\text{ rms}$$

check

(c) We find C to bring the power factor to unity

$$Q_c = Q_p = 18\text{ kVA} \longrightarrow C = \frac{Q_c}{\omega V^2_{rms}} = \frac{\overset{=Q_p}{18{,}000}}{2\pi x 60x480^2} = \mathbf{207.2\ \mu F.}$$

$\bar{I}_{AB} = \frac{\bar{V}_{AB}}{\bar{Z}_\Delta}$

From Table 12.1, $\bar{I}_a = \bar{I}_{AB}\sqrt{3}\angle{-30°} \Rightarrow |\bar{I}_a| = \sqrt{3}\,|\bar{I}_{AB}| = \sqrt{3}\left|\frac{480}{\bar{Z}_p}\right|$

$= \sqrt{3} \cdot \frac{480}{|6.144 + j4.608|}$

$= 108.25\ A_{rms}$ ✓

Solution 12.49

Each phase load consists of a 20-ohm resistor and a 10-ohm inductive reactance. With a line voltage of 480 V rms, calculate the average power taken by the load if:
(a) the three phase loads are delta-connected,
(b) the loads are wye-connected.

Solution

phase and line voltages are equal for Δ connected load.

(a) For the delta-connected load, $Z_p = 20 + j10\Omega$, $V_p = V_L = 480$ V(rms),

(12.52)

$$\bar{S} = \frac{3V^2_p}{\bar{Z}^*_p} = \frac{3x480^2}{(20 - j10)} = \frac{(13{,}824 + j6{,}912)k}{500} = (\underbrace{27.648}_{=P} + j\underbrace{13.824}_{=Q})kVA$$

P = **27.65 kW**

(b) For the wye-connected load, $Z_p = 20 + j10\Omega$, $V_p = V_L / \sqrt{3}$, ← See Table 12.1 or (12.12)

$$\bar{S} = \frac{3V^2_p}{\bar{Z}^*_p} = \frac{3x480^2}{3(20 - j10)} = (\underbrace{9.216}_{=P} + j\underbrace{4.608}_{=Q})kVA$$

P = **9.216 kW**