

**Solution 11.46**

$$(a) \quad S = VI^* = (220\angle 30^\circ)(0.5\angle -60^\circ) = 110\angle -30^\circ$$

$$V = \text{rms}, \quad I = \text{rms}$$

$$S = \underbrace{[95.26]}_P - \underbrace{j55}_Q \text{ VA}$$

$$\text{Apparent power} = 110 \text{ VA} = |S|$$

$$\text{Real power} = 95.26 \text{ W} = P$$

$$\text{Reactive power} = 55 \text{ VAR} = -Q$$

pf is leading because current leads voltage

$$(b) \quad S = VI^* = (250\angle -10^\circ)(6.2\angle 25^\circ) = 1550\angle 15^\circ$$

$$S = \underbrace{[1497.2]}_P + \underbrace{j401.2}_Q \text{ VA}$$

$$\text{Apparent power} = 1550 \text{ VA} = |S|$$

$$\text{Real power} = 1497.2 \text{ W} = P$$

$$\text{Reactive power} = 401.2 \text{ VAR} = +Q$$

pf is lagging because current lags voltage

**Solution 11.48**

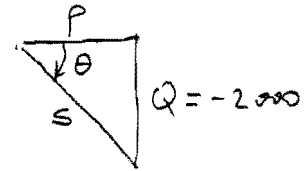
(a)  $S = P - jQ = [269 - j150] VA$

(b)  $pf = \cos\theta = 0.9 \longrightarrow \theta = 25.84^\circ$

$$Q = S \sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{2000}{\sin(25.84^\circ)} = 4588.31$$

$$P = S \cos\theta = 4129.48$$

$$S = [4.129 - j2] kVA$$



$$S \cdot \sin\theta = Q$$

$$S \cdot \cos\theta = P$$

**Solution 11.51**

For the entire circuit in Fig. 11.70, calculate:

- the power factor
- the average power delivered by the source
- the reactive power
- the apparent power
- the complex power

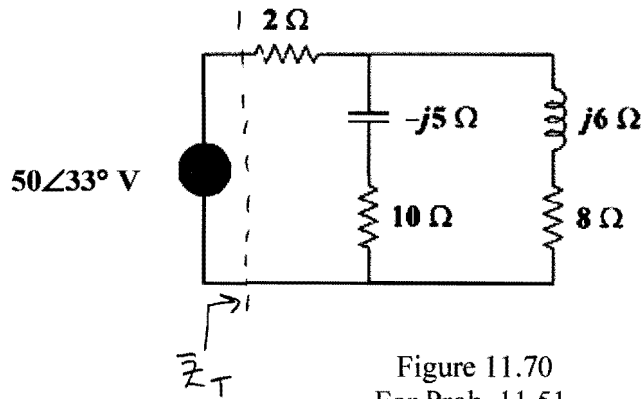


Figure 11.70  
For Prob. 11.51.

**Solution**

$$(a) \quad Z_T = 2 + (10 - j5) \parallel (8 + j6)$$

$$Z_T = 2 + \frac{(10 - j5)(8 + j6)}{18 + j} = 2 + \frac{110 + j20}{18 + j}$$

$$Z_T = 8.152 + j0.768 = 8.188 \angle 5.382^\circ$$

$$pf = \cos \theta = \cos 5.382^\circ = 0.9956 \quad (\text{lagging})$$

$$(b) \quad S = \frac{1}{2} \mathbf{V} \mathbf{I}^* = \frac{|\mathbf{V}|^2}{2(\mathbf{Z}_T)^*} = \frac{(50)^2}{2(8.188 \angle -5.382^\circ)}$$

$$S = \frac{305.325 \angle 5.382^\circ}{152.66} \text{ VA} = \underbrace{151.99}_{P} + j14.32 \text{ VA}$$

$$P = S \cos \theta = \frac{304}{151.99} \text{ W}$$

$$\mathbf{I}^* = \frac{\mathbf{V}^*}{\mathbf{Z}_T^*}$$

$$(c) \quad Q = S \sin \theta = \frac{28.64}{14.32} \text{ VAR}$$

$$(d) \quad S = |S| = \frac{305.3}{152.66} \text{ VA}$$

$$(e) \quad \mathbf{S} = \frac{305.325 \angle 5.382^\circ}{152.66} = \frac{304 + j28.64}{152.66} \text{ VA}$$

See (b)

**Solution 11.39**

An ac motor with impedance  $Z_L = (2 + j1.2) \Omega$  is supplied by a 220-V, 60-Hz source. (a) Find pf, P, and Q. (b) Determine the capacitor required to be connected in parallel with the motor so that the power factor is corrected to unity.

**Solution**

(a)  $Z_L = 2 + j1.2 = 2.3324 \angle 30.964^\circ$

pf =  $\cos(30.964) = 0.8575 = \frac{1}{\angle Z_L}$  (lagging)

$S = V^2 / (Z_L)^* = 48,400 / (2.3324 \angle -30.964^\circ) = 20,751 \angle 30.964^\circ$   
 $= \underbrace{17.794 \text{ kW}}_P + j \underbrace{10.676 \text{ kVAR}}_Q$

P = 17.794 kW

Q = 10.676 kVAR (lagging)

(b)  $X_C = V^2 / Q_C = 48,400 / 10,676 = 4.5335 = 1 / (377C)$  or  
 C = 585.1  $\mu\text{F}$ .

$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{Q}{2\pi f (220)^2}$

*reduce all reactive power*

**{It is important to note that this capacitor will see a peak voltage of  $220\sqrt{2} = 311.08\text{V}$ , this means that the specifications on the capacitor must be at least this or greater!}**

**Solution 11.69**

Refer to the circuit shown in Fig. 11.88.

- What is the power factor?
- What is the average power dissipated?
- What is the value of the capacitance that will give a unity power factor when connected to the load?

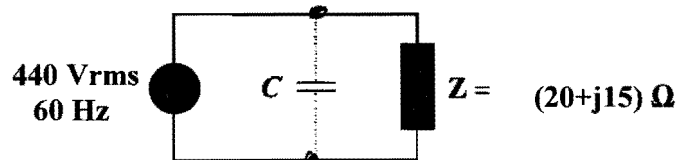
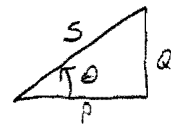


Figure 11.88  
For Prob. 11.69.

**Solution**

- (a) Given that  $Z = 20 + j15 = 25 \angle 36.87^\circ$      $\theta = \angle Z$   
 $\tan \theta = \frac{15}{20} \longrightarrow \theta = 36.87^\circ$   
 $\text{pf} = \cos \theta = 0.8$

- (b)  $S = \frac{|V_{rms}|^2}{Z^*} = \frac{(440)^2}{25 \angle -36.87^\circ} = \underbrace{6,195.2}_P + j \underbrace{4,646.4}_Q \text{ VA}$   
 The average power absorbed =  $P = \text{Re}(S) = 6.195 \text{ kW}$



- (c) For unity power factor,  $\theta_1 = 0^\circ$ , which implies that the reactive power due to the capacitor is  $Q_C = 4.6464 \text{ kVAR}$

$$\text{But } Q_c = \frac{V^2}{X_c} = \omega C V^2$$

$$C = \frac{Q_c}{\omega V^2} = \frac{(4,646.4)}{(2\pi)(60)(440)^2} = 63.66 \mu\text{F}$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{Q}{2\pi f (440)^2}$$

*reduce all reactive power*

## Solution 11.73

$$(a) \quad \mathbf{S} = 10 - j15 + j22 = 10 + j7 \text{ kVA}$$

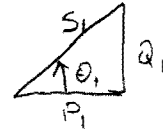
$$S = |\mathbf{S}| = \sqrt{10^2 + 7^2} = 12.21 \text{ kVA}$$

$$(b) \quad \mathbf{S} = \mathbf{V}\mathbf{I}^* \longrightarrow \mathbf{I}^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{10,000 + j7,000}{240} = 41.667 + j29.167 \text{ Arms}$$

$$\mathbf{I} = 41.667 - j29.167 = 50.86 \angle -35^\circ \text{ Arms}$$

$$(c) \quad \theta_1 = \tan^{-1}\left(\frac{7}{10}\right) = 35^\circ, \quad \theta_2 = \cos^{-1}(0.96) = 16.26^\circ$$

*requested*



$$Q_c = P_1 [\tan \theta_1 - \tan \theta_2] = 10 [\tan(35^\circ) - \tan(16.26^\circ)]$$

$$Q_c = 4.083 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{\text{rms}}^2} = \frac{4083}{(2\pi)(60)(240)^2} = 188.03 \mu\text{F}$$

$$(d) \quad \mathbf{S}_2 = P_2 + jQ_2, \quad P_2 = P_1 = 10 \text{ kW}$$

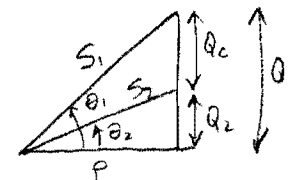
$$Q_2 = Q_1 - Q_c = 7 - 4.083 = 2.917 \text{ kVAR}$$

$$\mathbf{S}_2 = 10 + j2.917 \text{ kVA}$$

$$\text{But } \mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^*$$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}} = \frac{10,000 + j2917}{240} = 41.667 + j12.154 \text{ Arms}$$

$$\mathbf{I}_2 = 41.667 - j12.154 = 43.4 \angle -16.26^\circ \text{ Arms}$$



Note  $|\mathbf{I}_2| < |\mathbf{I}_1|$ , which is an important aspect of power factor correction.  $\approx 50.87 \text{ Arms}$ .

**Solution 11.91**

The nameplate of an electric motor has the following information:

Line voltage: 220 V rms  
 Line current: 15 A rms  
 Line frequency: 60 Hz  
 Power: 2700 W

Determine the power factor (lagging) of the motor. Find the value of the capacitance  $C$  that must be connected across the motor to raise the pf to unity.

**Solution**

$I = V/Z$  which leads to  $Z = [220/15] \angle \theta = 14.6667 \angle \theta$ ,  $S = (220)(15) \angle \theta = 3.3 \angle \theta$  kVA,  
 where  $\cos^{-1}(2700/3300) = \cos^{-1}(0.818182) = 35.097^\circ$ , and  ~~$X_L = 3300 \sin(35.097^\circ) =$~~   
 ~~$1897.38 = X_C$ . This leads to  $C = 1/[377(1897.38)] = 1.398 \mu\text{F}$ .~~

$$\text{pf} = 0.8182 \text{ (lagging)}$$

~~$$C = 1.398 \mu\text{F}$$~~

$$0.8182 \text{ (lagging), } 1.398 \mu\text{F}$$

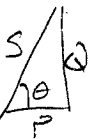
$$|Z| = \frac{|V|}{|I|} = \frac{220 \text{ V}_{\text{rms}}}{15 \text{ A}_{\text{rms}}} = 14.667 \Omega$$

$$\angle Z = \theta_v - \theta_i = \cos^{-1}(\text{pf})$$

$$S = V_{\text{rms}} \cdot I_{\text{rms}} = 220 \cdot 15 = 3300 \text{ VA}$$

$$P = 2700 \text{ W (given)}$$

$$P = S \cos \theta \Rightarrow \cos \theta = \frac{P}{S} = \frac{2700}{3300}$$



$$\therefore \text{pf} = \cos \theta = 0.8182 ; \theta = 35.10^\circ$$

(lagging)

$$C = \frac{Q_c}{\omega V_{\text{rms}}^2} = \frac{Q}{\omega V_{\text{rms}}^2} \leftarrow \text{reduce all reactive power}$$

$$Q = S \sin \theta = 3300 \cdot \sin(\theta) = 1897.52 \text{ VAR}$$

$$C = \frac{1897.52}{2\pi \cdot 60 \cdot (220)^2} = \underline{\underline{103.99 \mu\text{F}}}$$

## Solution 11.95

- (a) Source impedance  $Z_s = R_s - jX_c$   
 Load impedance  $Z_L = R_L + jX_L$

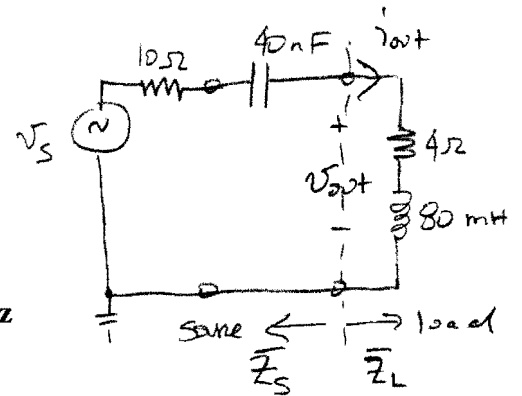
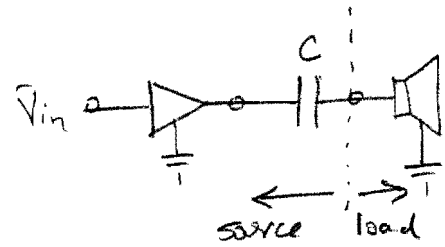
For maximum ~~load~~ <sup>power</sup> transfer to the load.

$$Z_L = Z_s^* \longrightarrow R_s = R_L, X_c = X_L$$

$$X_c = X_L \longrightarrow \frac{1}{\omega C} = \omega L \quad \text{Can't happen!}$$

$$\text{or } \omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(80 \times 10^{-3})(40 \times 10^{-9})}} = 2.814 \text{ kHz}$$



- (b)  $P = \left( \frac{V_s}{(10+4)} \right)^2 4 = \left( \frac{4.6}{14} \right)^2 4 = 431.8 \text{ mW}$  (since  $V_s$  is in rms)

$$\begin{aligned} \bar{I}_{out} &= \frac{\sqrt{V_s}}{\bar{Z}_L + \bar{Z}_s} = \frac{4.6V \cdot \sqrt{2}}{R_L + jX_L + R_s + jX_s} \uparrow \\ &= \frac{4.6\sqrt{2}}{4+10} = 0.4647 \text{ A } \neq 0^\circ \text{ at } f = 2.814 \text{ kHz.} \end{aligned}$$

$X_L = X_c$

$$\underline{P} = \frac{1}{2} |\bar{I}_{out}|^2 \cdot R_L = \frac{1}{2} (0.4647)^2 \cdot 4 = \underline{\underline{431.8 \text{ mW}}}$$



## Solution 12.1

For the abc sequence,

(a) If  $V_{ab} = 400$ , then

$$\begin{aligned} V_{an} &= \frac{400}{\sqrt{3}} \angle -30^\circ = 231 \angle -30^\circ \text{ V} \\ V_{bn} &= 231 \angle -150^\circ \text{ V} \\ V_{cn} &= 231 \angle -270^\circ \text{ V} \end{aligned}$$

$$\begin{aligned} \sqrt{3} V_p \angle 30^\circ & \quad ; \quad V_{an} = V_p \angle 0^\circ \\ \therefore V_{an} &= \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ = \frac{400}{\sqrt{3}} \angle -30^\circ \end{aligned}$$

(b) For the acb sequence,

$$V_{ab} = V_{an} - V_{bn} = V_p \angle 0^\circ - V_p \angle 120^\circ$$

$$V_{ab} = V_p \left( 1 + \frac{1}{2} - j \frac{\sqrt{3}}{2} \right) = V_p \sqrt{3} \angle -30^\circ$$

i.e. in the acb sequence,  $V_{ab}$  lags  $V_{an}$  by  $30^\circ$ .Hence, if  $V_{ab} = 400$ , then

$$V_{an} = \frac{400}{\sqrt{3}} \angle 30^\circ = 231 \angle 30^\circ \text{ V}$$

$$V_{bn} = 231 \angle 150^\circ \text{ V}$$

$$V_{cn} = 231 \angle -90^\circ \text{ V}$$

**Solution 12.3**

Given a balanced Y-connected three-phase generator with a line-to-line voltage of  $V_{ab} = 100\angle 45^\circ$  V and  $V_{bc} = 100\angle 165^\circ$  V, determine the phase sequence and the value of  $V_{ca}$ .

**Solution**

Since  $V_{bc}$  leads  $V_{ab}$  by  $120^\circ$  we have a **acb** sequence and  $V_{ca} = 100\angle -75^\circ$  V.

