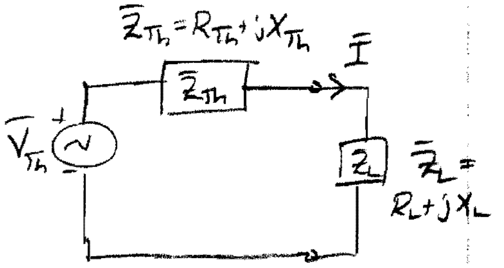
Problem 2.1



$$\bar{I} = \frac{\bar{V}_{Th}}{\bar{Z}_{Th} + \bar{Z}_L} = \frac{\bar{V}_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)} \qquad (1)$$

The time avg. power P delivered to the load will consumed entirely in R_L (none in X_L). Consequently,

$$P = \frac{1}{2} |\bar{I}|^2 \cdot R_L$$

$$|\bar{I}|^2 = \bar{I} \cdot \bar{I}^* \underset{(1)}{=} \frac{|\bar{V}_{Th}|^2}{\left[(R_{Th} + R_L) + j(X_{Th} + X_L)\right] \cdot \left[(R_{Th} + R_L) + j(X_{Th} + X_L)\right]^*}$$

$$= \frac{|\bar{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

$$\therefore P = \frac{1}{2} \frac{|\bar{V}_{Th}|^2 \cdot R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} \qquad (2) \longleftarrow (11.15)$$

To maximize this fct, we note it depends on two variables. So, we'll take partial derivatives wrt R_L & X_L setting each to zero separately:

$$\frac{\partial P}{\partial R_L} = 0 \quad \& \quad \frac{\partial P}{\partial X_L} = 0$$

From these two equations we'll find those R_L & X_L that produce maximum time avg power deposited on Z_L.

$$\frac{\partial P}{\partial R_L} \underset{(2)}{=} \frac{|V_{Th}|^2}{2} \frac{\partial (2)}{\partial R_L} = \frac{|V_{Th}|^2}{2} \left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right] \frac{\partial R_L}{\partial R_L} -$$

Quotient Rule:

$$= \frac{|V_{Th}|^2}{2} \frac{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right] \frac{\partial R_L}{\partial R_L} - R_L \cdot \frac{\partial}{\partial R_L} \left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]}{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2}$$

$$= \frac{|V_{Th}|^2}{2} \frac{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2 - R_L \cdot 2 \cdot (R_{Th} + R_L)}{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2} = 0$$

$$\Rightarrow \quad R_{Th}^2 + R_L^2 + \cancel{2R_{Th}R_L} - \cancel{2R_{Th}R_L} - 2R_L^2 + (X_{Th} + X_L)^2 = 0$$

or $$R_L^2 = R_{Th}^2 + (X_{Th} + X_L)^2$$

$$\therefore R_L = \pm \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2} \qquad (11.18)$$

Then for

$$\frac{\partial P}{\partial X_L} = \frac{|\bar{V}_{Th}|^2}{2}\frac{\partial(2)}{\partial X_L} \overset{\text{Quotient Rule}}{=} \frac{|\bar{V}_{Th}|^2}{2}\cdot\frac{0 - R_L\cdot 2(X_{Th}+X_L)}{[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2]^2} = 0$$

$$\Rightarrow \quad \underline{\underline{X_L = -X_{Th}}} \qquad (11.17).$$

Sub. (11.17) into (11.18) gives

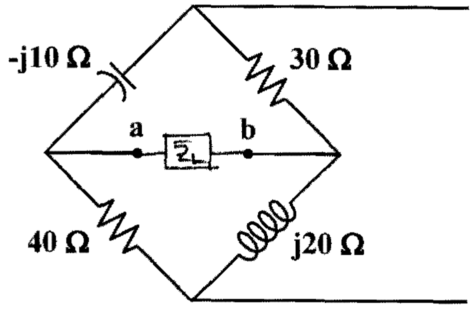
$$\underline{\underline{R_L}} = \pm\sqrt{R_{Th}^2} = \underline{\underline{R_{Th}}} \qquad (3)$$

Consequently, for max. power transfer from the thevenin circuit to the load

$$\underline{\underline{Z_L}} = R_L + jX_L = \underline{\underset{(3)}{R_{Th}} - j\underset{(11.17)}{X_{Th}}} = Z_{Th}^* \qquad (11.19)$$

Solution 11.17

We find Z_{eq} at terminals a-b following Fig. (a).

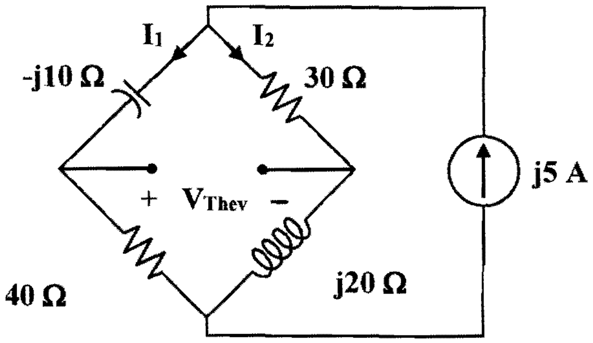


(a)

max power x^{fer}

$$\mathbf{Z}_{eq} = (-j10+30)\,\|\,(j20+40) = \frac{(30-j10)(40+j20)}{70+j10} = \mathbf{20\ \Omega} = \mathbf{Z_L^*}$$

We obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

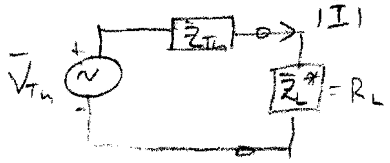
Using current division,

$$\mathbf{I}_1 = \frac{30+j20}{70+j10}(j5) = -1.1 + j2.3\ A$$

$$\mathbf{I}_2 = \frac{40-j10}{70+j10}(j5) = 1.1 + j2.7\ A$$

$$\mathbf{V}_{Th} = 30\mathbf{I}_2 + j10\mathbf{I}_1 = 10 + j70\ V$$

or $\bar{V}_{Th} = 40\bar{I}_1 - j20\cdot\bar{I}_2$
$= 10 + j70\ V$



$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{2(Z_{eq}+Z_L)^2} Z_L = \frac{5000}{(2)(2x20)^2} 20 = \mathbf{31.25\ W}$$

or, $P_{max} = \frac{1}{2}|\bar{I}|^2 \cdot R_L$
$= \frac{1}{2} \frac{|\bar{V}_{Th}|^2 \cdot R_L}{|\bar{Z}_{Th} + \bar{Z}_L|^2}$
$= \frac{1}{2} \frac{(70.71)^2 \cdot 20}{(20+20)^2}$
$= 31.25\ W$

Solution 11.19

The variable resistor R in the circuit of Fig. 11.50 is adjusted until it absorbs the maximum average power available amongst all possible R values. Find R and the ~~maximum~~ average power absorbed. for this R.

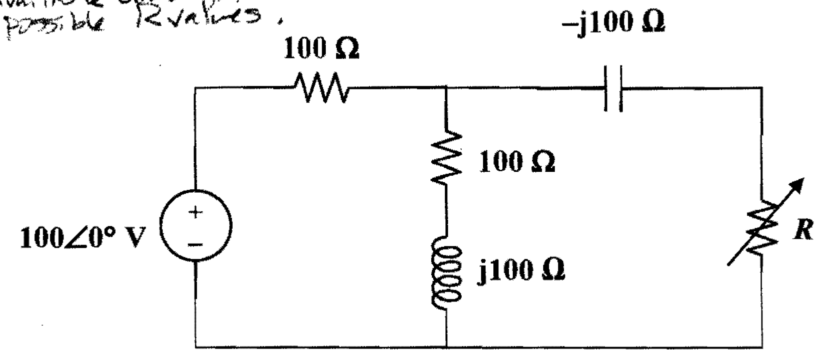
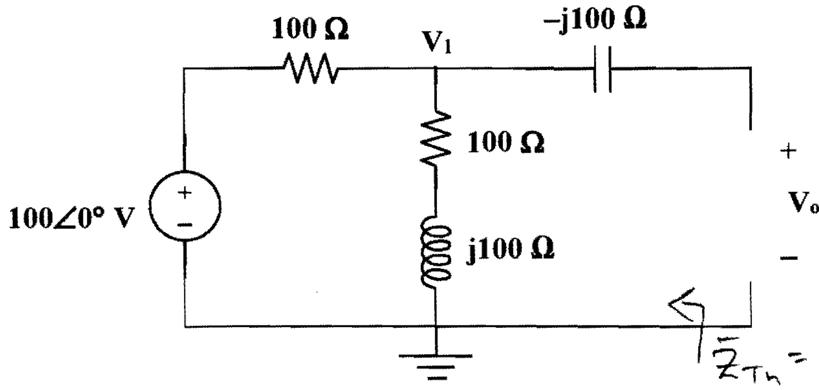


Figure 11.50
For Prob. 11.19.

Solution

Step 1. We first remove R from the circuit and then find the Thevenin equivalent circuit. Once we have $\mathbf{V}_{Thev}$ and $\mathbf{Z}_{eq}$ we then know that for maximum power transfer to the load, R must be equal to $|\mathbf{Z}_{eq}|$ and ~~$P_{avg} = |\mathbf{V}_{Thev}|^2/(8R)$~~. We now find $\mathbf{V}_{Thev}$ by writing and solving a nodal equation for the circuit shown below. To find $\mathbf{Z}_{eq}$, we just set the source to zero (a short) and determine the impedance looking in from the right. $\mathbf{Z}_{eq} = -j100 + 100(100+j100)/(100+100+j100)$.

Not true! Only true if $Z_L = Z_{Th}^*$



$V_{oc} = \bar{V}_{Th} = \frac{100+j100}{100+j100+100} \cdot 100\text{ V} = 60+j20\text{ V}$

$\bar{Z}_{Th} = \bar{Z}_{eq} = -j100 + [(100+j100) \| 100] = 60 - j80\ \Omega$

$[(\mathbf{V}_1-100)/100] + [(\mathbf{V}_1-0)/(100+j100)] + 0 = 0$ and $\mathbf{V}_{oc} = \mathbf{V}_{Thev} = \mathbf{V}_1$.

Step 2. $\mathbf{Z}_{eq} = -j100+100(1.4142\angle 45°)/(2.2361\angle 26.57°) = -j100+63.244\angle 18.43°$ $= -j100 + 60 + j20 = (60 - j80)\ \Omega = 100\angle -53.13°\ \Omega$.

The node equation becomes, $(0.01+0.005-j0.005)\mathbf{V}_1 = 1 = 0.0158114\angle -18.43°\mathbf{V}_1$ or $\mathbf{V}_1 = 63.246\angle 18.43°$. Thus,

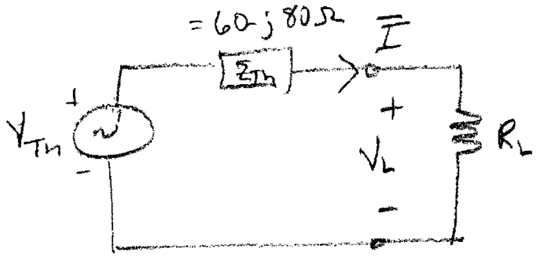
or use voltage division

From (11.18) w/ $X_L = 0 \Rightarrow R_L = |\bar{Z}_{Th}| = \underline{100\,\Omega}$ will draw the most power from Thevenin equiv. cct than all possible R values.

R = **100 Ω**

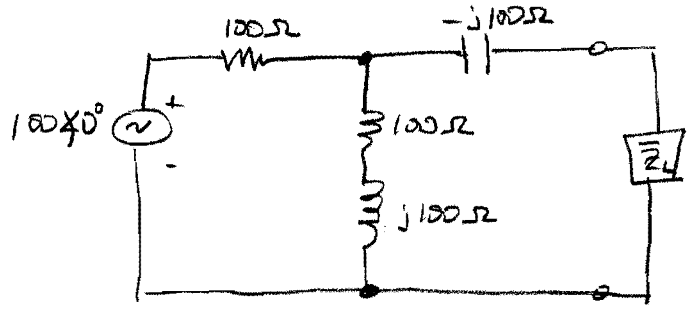
and |**I**| = 63.246/|60–j80+100| = 63.246/178.885 = 0.353557 A and

~~the maximum~~ $P_{avg} = [(0.353557)^2/2]100 = \mathbf{6.25\ W}$.

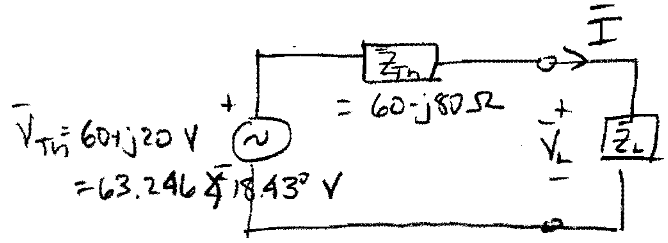


Prob. 2.4

Fig 11.50 but variable $R \rightarrow \bar{Z}_L$:



From previous problem, the Thevenin equiv. cct is



To extract maximum time avg. power from the circuit, select

$$\bar{Z}_L = \bar{Z}_{Th}^* = (60-j80)^* = 60+j80\ \Omega$$

$$\bar{I} = \frac{\bar{V}_{Th}}{\bar{Z}_{Th}+\bar{Z}_{Th}^*} = \frac{60+j20}{60\cancel{-j80}+60\cancel{+j80}} = 0.5270 \measuredangle 18.43° \text{ A.}$$

$$P_{\bar{Z}_L} = \frac{1}{2}\mathrm{Re}\left(\bar{V}_L \cdot \bar{I}^*\right) = \frac{1}{2}\mathrm{Re}\left(\bar{I}\cdot\bar{Z}_L\cdot\bar{I}^*\right) = \frac{1}{2}\mathrm{Re}\left(|\bar{I}|^2 \cdot \bar{Z}_L\right)$$

$$= \frac{1}{2}|\bar{I}|^2 \cdot \mathrm{Re}(\bar{Z}_L) = \frac{1}{2}|\bar{I}|^2 \cdot R_L$$

$$\therefore\ P_{\bar{Z}_L} = \frac{1}{2}\, 0.5270^2 \cdot 60 = \underline{8.33\text{ W}}$$

This is different from the previous problem, and greater. With a conjugate matched load, we have satisfied the maximum power transfer theorem. No load will extract more time avg. power from the circuit than this one.

Because we have conjugate matched, eqn. (11.20) is applicable:

$$P_{max} = \frac{|\bar{V}_{Th}|^2}{8R_{Th}} = \frac{63.246^2}{8\cdot 60} = \underline{8.33\text{W}}$$

Solution 11.23

Using Fig. 11.54, design a problem to help other students to better understand how to find the rms value of a waveshape.

Problem

Determine the rms value of the voltage shown in Fig. 11.54. Choose $V_p = 10\,V$ & $T = 3\,s$.

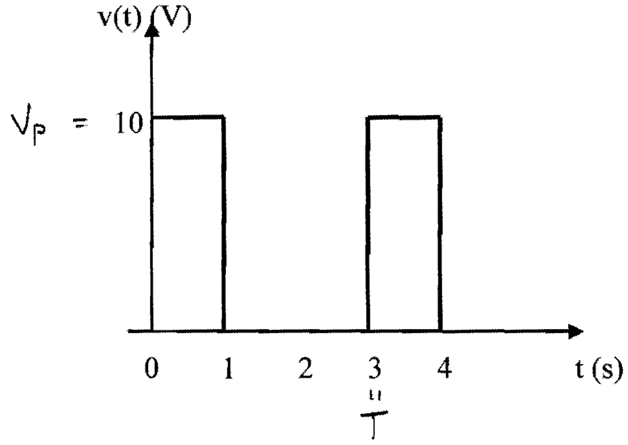


Figure 11.54 For Prob. 11.23.

By definition,

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

or $V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt$

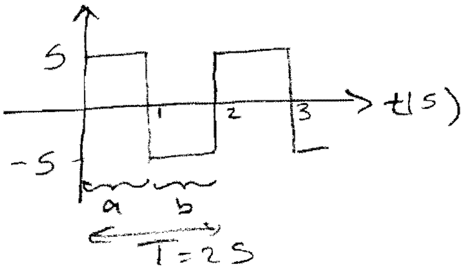
$\therefore V_{rms}^2 = \frac{1}{T}\int_0^{T/3} \underbrace{v^2(t)}_{=10^2}dt + \overbrace{\frac{1}{T}\int_{T/3}^{T} 0^2\,dt}^{\to 0}$

Solution

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{3}\int_0^1 10^2\,dt = \frac{100}{3}$$

$\underline{\mathbf{V_{rms} = 5.7735\ V}}$

Solution 11.24



$T = 2$, $\quad v(t) = \begin{cases} 5, & 0 < t < 1 \\ -5, & 1 < t < 2 \end{cases}$

$$V_{rms}^2 = \frac{1}{2}\left[\underbrace{\int_0^1 5^2\,dt}_{a} + \underbrace{\int_1^2 (-5)^2\,dt}_{b}\right] = \frac{25}{2}[1+1] = 25$$

$$\boxed{\mathbf{V_{rms} = 5\,V}}$$

By definition $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

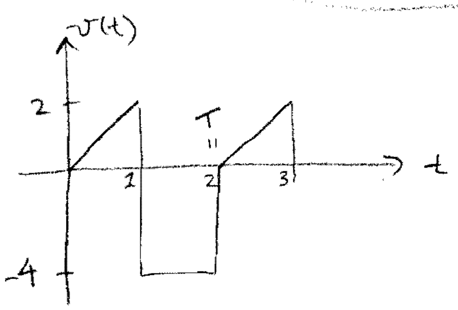
$$\therefore V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2}\int_0^{T/2} \underbrace{v^2(t)}_{=5^2}\,dt + \frac{1}{2}\int_{T/2}^{T} \underbrace{v^2(t)}_{(-5)^2}\,dt$$

Solution 11.31

$$V^2_{rms} = \frac{1}{2}\int_0^2 v(t)dt = \frac{1}{2}\left[\int_0^1 (2t)^2\,dt + \int_1^2 (-4)^2\,dt\right] = \frac{1}{2}\left[\frac{4}{3}+16\right] = 8.6667 \quad V^2$$

$$V_{rms} = \underline{\mathbf{2.944\ V}}$$

w/ T=2



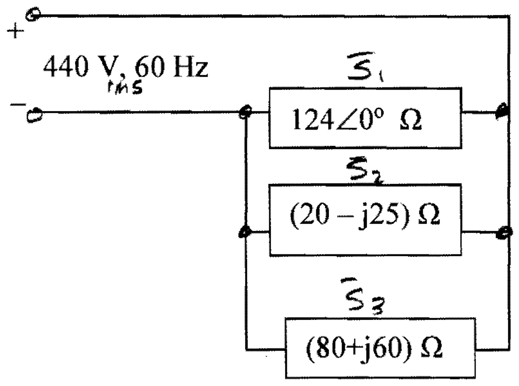
By definition,

$$V^2_{rms} = \frac{1}{T}\int_0^T v^2(t)\,dt$$

$$= \frac{1}{T}\int_0^{T/2} \underbrace{v^2(t)}_{(2t)^2}\,dt + \frac{1}{T}\int_{T/2}^{T} \underbrace{v^2(t)}_{(-4)^2}\,dt$$

Solution 11.38

For the power system in Fig. 11.67, find: (a) the average power, (b) the reactive power, (c) the power factor. Note that 440 V is an rms value.



In general

$$\bar{S} = \frac{1}{2}\bar{V}\cdot\bar{I}^* = \frac{\bar{V}}{\sqrt{2}}\cdot\frac{\bar{V}^*}{\sqrt{2}\cdot\bar{Z}^*}$$

$$= \frac{\bar{V}_{rms}\cdot\bar{V}_{rms}^*}{\bar{Z}^*} = \frac{|\bar{V}_{rms}|^2}{\bar{Z}^*} = \frac{V_{rms}^2}{\bar{Z}^*}$$

$$\therefore \bar{S} = \frac{V_{rms}^2}{\bar{Z}^*}$$

Figure 11.67
For Prob. 11.38.

Solution

$\mathbf{S_1} = V^2/(\mathbf{Z_1})^* = 1.56129$ kW. $\left(= \frac{(440 V_{rms})^2}{124} \right)$

$\mathbf{S_2} = V^2/(\mathbf{Z_2})^* = 193{,}600/(32.0156\angle 51.34°) = 6{,}047.05\angle{-51.34°}$ VA
$= 3.7776$ kW – j4.7219 kVAR

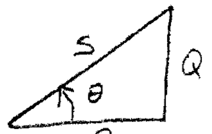
$\mathbf{S_3} = V^2/(\mathbf{Z_3})^* = 193{,}600/(100\angle{-36.87°}) = 1{,}936\angle 36.87°$ VA
$= 1.5488$ kW + j1.1616 lVAR.

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3} = (1.56129+3.7776+1.5488)$ kW + j(0–4.7219+1.1616) kVAR
$= 6.888$ kW – j3.56 kVAR.
Therefore,

(a) P = Re(**S**) = **6.888 kW**

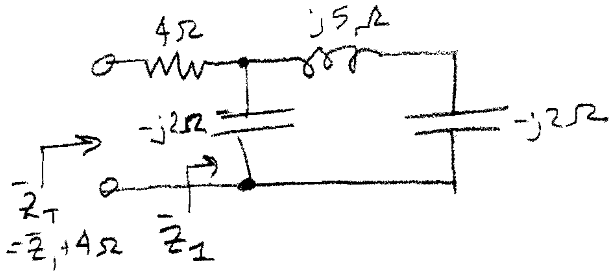
(b) Q = Im (**S**) = **–3.56 kVAR (leading)**

(c) pf = cos [$\tan^{-1}$([–3.56/6.888])] = cos {$\tan^{-1}$[–0.51684]}
= cos (–27.332°) = **0.89**



$\theta = \tan^{-1}\left(\frac{Q}{P}\right)$

Solution 11.41



Z_1

(a) $-j2 \,||\, (j5 - j2) = -j2 \,||\, -j3 = \dfrac{(-j2)(-j3)}{j} = -j6$

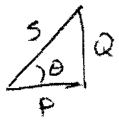
$\mathbf{Z}_T = 4 - j6 = 7.211\angle -56.31°$

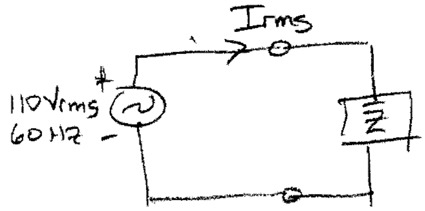
pf = cos(-56.31°) = **0.5747** **(leading)**

In general, $\bar{Z} = \dfrac{\bar{V}}{\bar{I}} = \dfrac{|\bar{V}|}{|\bar{I}|}\angle(\theta_V - \theta_I)$

$\therefore \measuredangle \bar{Z}_T = \measuredangle$ pf.

$\therefore$ pf $= \cos(\measuredangle \bar{Z}_T)$





Solution 11.42

(lagging) +

(a) S=120 VA $pf = 0.707 = \cos\theta \longrightarrow \theta = 45^o$

$\overline{S} = \underbrace{S\cos\theta}_{P} + j\underbrace{S\sin\theta}_{Q} = \underline{84.84 + j84.84 \text{ VA}}$

(b) $S = V_{rms} I_{rms} \longrightarrow I_{rms} = \frac{S}{V_{rms}} = \frac{120}{110} = \underline{1.091 \text{ A rms}}$

(c) $\overline{S} = I_{rms}^2 \overline{Z} \longrightarrow \overline{Z} = \frac{\overline{S}}{I_{rms}^2} = \underline{71.278 + j71.278\ \Omega}$

(d) If $\overline{Z} = R + j\varpi L$, then $\underline{R = \mathbf{71.278\ \Omega}}$

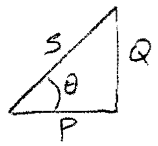
$\omega L = 2\pi f L = 71.278 \longrightarrow \underline{L} = \frac{71.278}{2\pi x 60} = \underline{0.1891\ \text{H}} = \underline{\mathbf{189.1\ mH.}}$

↑
60Hz

Solution 11.46

(a) $\mathbf{S} = \overset{\bar{V}_{rms}}{\mathbf{V}}\overset{\bar{I}^*_{rms}}{\mathbf{I}^*} = (220\angle 30°)(0.5\angle -60°) = \underbrace{110}_{|\bar{S}| = S}\angle -30°$

$\mathbf{S} = [\underbrace{95.26}_{P} - \underbrace{j55}_{Q}]\ VA$



Apparent power = **110 VA** $= |\bar{S}|$

Real power = **95.26 W**

Reactive power = **55 VAR**

pf is **leading** because current leads voltage

(b) $\mathbf{S} = \overset{\bar{V}_{rms}}{\mathbf{V}}\overset{\bar{I}^*_{rms}}{\mathbf{I}^*} = (250\angle -10°)(6.2\angle 25°) = \underbrace{1550}_{|\bar{S}| = S}\angle 15°$

$\mathbf{S} = [\underbrace{\cancel{497.2}\ 1497.2}_{P} + \underbrace{j401.2}_{Q}]\ VA$

Apparent power = **1550 VA** $= |\bar{S}|$

Real power = **1497.2 W**

Reactive power = **401.2 VAR**

pf is **lagging** because current lags voltage