Solution 18.36

The transfer function of a circuit is

$H(\omega) = 10/(j\omega+2)$

using Fourier Transform methods.

If the input signal to the circuit is $v_s(t) = e^{-4t}u(t)$ V, find the output signal. Assume all initial conditions are zero.

Solution

With x(t) as the input, y(t) as the output, and h(t) the unit impulse response ⇒

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \quad \text{or} \quad Y(\omega) = H(\omega)X(\omega) \qquad (1)$$

$$x(t) = v_s(t) = e^{-4t}u(t) \xrightarrow{\mathcal{F}} X(\omega) = \frac{1}{4+j\omega}$$

$$\therefore \text{ from (1): } Y(\omega) = \frac{10}{(j\omega+2)(4+j\omega)} \overset{s=j\omega}{=} \frac{10}{(s+2)(s+4)},\ s = j\omega$$

partial fraction expansion

$Y(s) = [A/(s+2)] + [B/(s+4)]$ where $A = 5$ and $B = -5$

$$A = \frac{10}{-2+4} = 5 \ ; \ B = \frac{10}{-4+2} = -5$$

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+4}$$

$$Y(s) = \frac{5}{s+2} - \frac{5}{s+4}$$

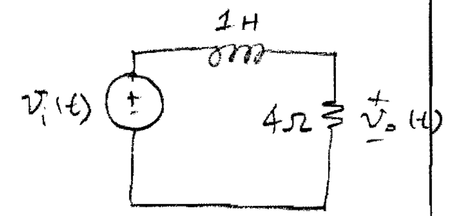
using Table 18.2 w/ $s=j\omega$

$$\underline{\underline{y(t) = 5[e^{-2t} - e^{-4t}]u(t)}}$$

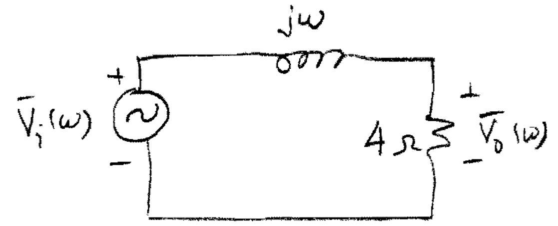
Practice Problem 18.7

Determine $v_o(t)$ using Fourier Transform methods if

$$v_i(t) = 5\,\mathrm{sgn}(t) = -5 + 10u(t) \text{ V}$$



The Fourier transformed ckt is



By voltage division, $\dfrac{\bar{V}_o(\omega)}{\bar{V}_i(\omega)} \equiv H(\omega) = \dfrac{4}{4+j\omega}$

$v_i = 5\,\mathrm{sgn}(t) \xrightarrow{\mathcal{F}\ \text{(Table 18.2)}} V_i(\omega) = 10/(j\omega)$

$H(\omega) = 4/(4 + j\omega)$

Partial fraction expansion

$$\therefore\ V_o(\omega) = H(\omega)V_i(\omega) = \frac{40}{j\omega(4+j\omega)} = \frac{A}{j\omega} + \frac{B}{4+j\omega}$$

$\therefore A = \frac{40}{4} = 10$; $B = \frac{40}{-4} = -10$

$$= \frac{10}{j\omega} - \frac{10}{4+j\omega}$$

Using Table 18.2 we can determine $v_o(t) = \mathcal{F}^{-1}\{\bar{V}_o(\omega)\}$

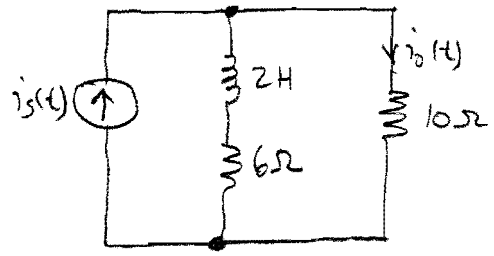
$v_o(t) = 5\,\mathrm{sgn}(t) - 10e^{-4t}u(t) = 5[-1 + u(t)] - 10e^{-4t}u(t)$

$= \underline{\mathbf{-5 + 10\,[1 - e^{-4t}]u(t)\ V}}$

Practice Problem 18.8

Using the Fourier Transform method, calculate the current $i_o(t)$ in the circuit shown given that

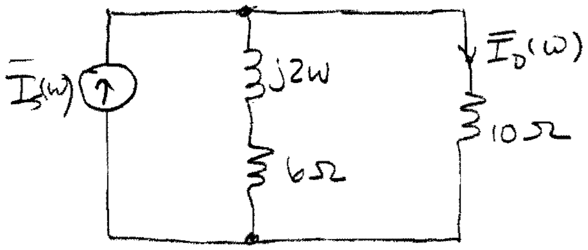
$$i_s(t) = 50\cos(4t)\ A$$



Using Table 18.2,

$$\bar{I}_S(\omega) = \mathcal{F}\{i_s(t)\} = 50\pi[\delta(\omega+4) - \delta(\omega-4)]$$

Frequency domain circuit:



By current division,

$$\bar{I}_o = \frac{6+j2\omega}{6+j2\omega+10}\bar{I}_S$$

$$\therefore \bar{H}(\omega) \equiv \frac{\bar{I}_o}{\bar{I}_S} = \frac{6+j2\omega}{16+j2\omega} = \frac{3+j\omega}{8+j\omega}$$

P.P.18.8 $I_s(\omega) = 50\pi[\delta(\omega + 4) + \delta(\omega - 4)]$

$$H(\omega) = \frac{6+j\omega 2}{10+6+j2\omega} = \frac{3+j\omega}{8+j\omega}$$

$$I_0(\omega) = H(\omega)I_s(\omega) = \left(\frac{3+j\omega}{8+j\omega}\right)(50\pi)[\delta(\omega+4)+\delta(\omega-4)]$$

$$i_o(t) = F^{-1}\{I_0(\omega)\} = \frac{50\pi}{2\pi}\int_{-\infty}^{\infty}\left(\frac{3+j\omega}{8+j\omega}\right)[\delta(\omega+4)+\delta(\omega-4)e^{j\omega t}d\omega]$$

$$= 25\left[\underbrace{\frac{3-j4}{8-j\omega}}_{= 0.559\angle -26.57^\circ}e^{-j4t} + \frac{3+j4}{8+j4}e^{j4t}\right]$$

But

$$\frac{3+j4}{8+j4} = \frac{5\angle 53.13^\circ}{\sqrt{80}\angle 26.56^\circ} = 0.559\angle 26.57^\circ$$

$$i_o(t) = 13.975\left(e^{-j(4t+26.57^\circ)} + e^{j(4t+26.57^\circ)}\right) \cdot \frac{2}{2}$$

$i_o(t)$ = 27.95cos (4t + 26.57°) A

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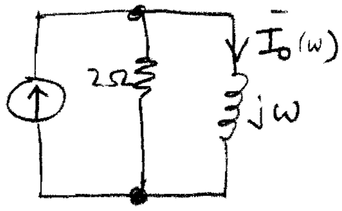
Frequency domain ckt:



Solution 18.42

By current division, $I_o = \overbrace{\dfrac{2}{2+j\omega}}^{H(\omega)} \cdot I(\omega)$ (1)

(a) For $i(t) = 5\,\text{sgn}(t)$, ← Table 18.2

$$I(\omega) = \frac{10}{j\omega}$$

$$I_o = \frac{2}{2+j\omega}\cdot\frac{10}{j\omega} = \frac{20}{j\omega(2+j\omega)}$$

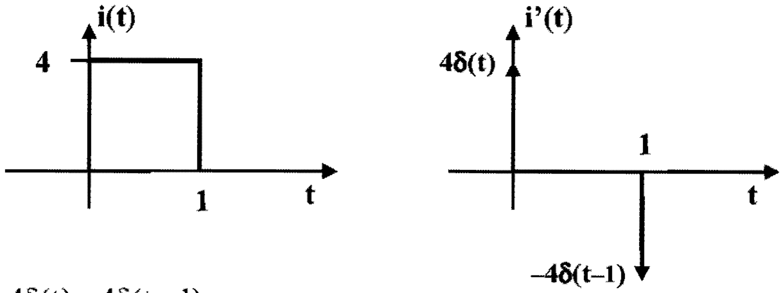
Let $I_o = \dfrac{20}{s(s+2)} \underset{\text{PFE}}{=} \dfrac{A}{s} + \dfrac{B}{s+2}$, $A = 10,\ B = -10$

$A = \frac{20}{2} = 10$, $B = \frac{20}{-2} = -10$

$\therefore$ $I_o(\omega) = \dfrac{10}{j\omega} - \dfrac{10}{2+j\omega}$ ← Table 18.2

$$i_o(t) = \boxed{\mathbf{5\,sgn(t) - 10e^{-2t}u(t)}}\ \mathbf{A}$$

(b)



$i'(t) = 4\delta(t) - 4\delta(t-1)$ ← Table 18.1

$j\omega I(\omega) = 4 - 4e^{-j\omega}$

$\therefore$ $I(\omega) = \dfrac{4\left(1-e^{-j\omega}\right)}{j\omega}$

Partial fraction expansion

$= \left(\dfrac{A}{j\omega} + \dfrac{B}{2+j\omega}\right)\left(1-e^{-j\omega}\right)$

$\therefore$ $A = \frac{8}{2} = 4$, $B = \frac{8}{-2} = -4$

From (1): $I_o = \dfrac{8\left(1-e^{-j\omega}\right)}{j\omega(2+j\omega)} = 4\left(\dfrac{1}{j\omega} - \dfrac{1}{2+j\omega}\right)\left(1-e^{-j\omega}\right)$

$$= \frac{4}{j\omega} - \frac{4}{2+j\omega} - \frac{4e^{-j\omega}}{j\omega} + \frac{4e^{-j\omega}}{2+j\omega}$$ ← Tables 18.2 & 18.1

$$i_o(t) = \boxed{\mathbf{2\,sgn(t) - 2\,sgn(t-1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t-1)}}\ \mathbf{A}$$

$$V_i(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow V_o(\omega)$$

Solution 18.59

$$v_i(t) = 2\delta(t) \xRightarrow{\text{Tab 18.2}} V_i(\omega) = 2$$

$$v_o(t) = \left[10e^{-2t} - 6e^{-4t}\right]u(t)\,\text{V}$$

$$\Rightarrow V_o(\omega) = \frac{10}{2+j\omega} - \frac{6}{4+j\omega}$$

$$H(\omega) \equiv \frac{V_o(\omega)}{V_i(\omega)} = \frac{\dfrac{10}{2+j\omega} - \dfrac{6}{4+j\omega}}{2} = \frac{5}{2+j\omega} - \frac{3}{4+j\omega}$$

Now, find $v_o(t)$ when $v_i(t)$ becomes $v_i(t) = 4e^{-t}u(t)$ V

Table 18.2 $\quad V_i(\omega) = \dfrac{4}{1+j\omega}$

$$V_o(\omega) = H(\omega)V_i(\omega) = \left(\frac{5}{2+j\omega} - \frac{3}{4+j\omega}\right)\frac{4}{1+j\omega}$$

$$= \frac{20}{(s+1)(s+2)} - \frac{12}{(s+1)(s+4)}, \qquad s = j\omega$$

Using partial fraction, expansion,

$$V_o(\omega) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+1} + \frac{D}{s+4} = \frac{A+C}{s+1} + \frac{B}{s+2} + \frac{D}{s+4}$$

Tricky!

- $A + C = \dfrac{20}{-1+2} - \dfrac{12}{-1+4} = 20 - 4 = 16$
- $B = \dfrac{20}{-2+1} - 0 = -20$
- $D = 0 - \dfrac{12}{-4+1} = 4$

Thus, $\quad \therefore V_o(\omega) = \dfrac{16}{j\omega+1} - \dfrac{20}{j\omega+2} + \dfrac{4}{j\omega+4}$

Table 18.2

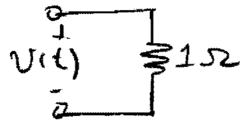
$$\underline{\underline{\mathbf{v_o(t) = \left(16e^{-t} - 20e^{-2t} + 4e^{-4t}\right)u(t)\,V}}}$$

Whites

$P = \dfrac{v^2(t)}{R}$

$v(t) = te^{-2t}u(t)\ V$

$= 0 - \dfrac{2}{(-4)^3}$



Solution 18.56

(a) $W = \int_{-\infty}^{\infty} v^2(t)dt = \int_0^{\infty} t^2 e^{-4t}dt = \dfrac{e^{-4t}}{(-4)^3}(16t^2+8t+2)\Big|_0^{\infty} = \dfrac{2}{64} = \underline{0.0313\ \text{J}}$

$\int x^2 e^{ax}dx = \dfrac{x^2e^{ax}}{a} - \dfrac{2}{a}\int xe^{ax}dx$

$= \dfrac{x^2e^{ax}}{a} - \dfrac{2}{a}\dfrac{e^{ax}}{a^2}(ax-1)$

$= \dfrac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$

$a \to -4,\ x \to t$

(b) In the frequency domain,

$V(\omega) = \dfrac{1}{(2+j\omega)^2}$

$n=1$

$|V(\omega)|^2 = V(\omega)V^*(\omega) = \dfrac{1}{(4+j\omega)^2}$

Parseval's Thm (18.63):

$W_o = \dfrac{1}{2\pi}\int_{-2}^{2}|V(\omega)|^2d\omega = \dfrac{2}{2\pi}\int_0^2 \dfrac{1}{(4+\omega^2)^2}d\omega$

$= \dfrac{1}{\pi}\dfrac{1}{2\cdot 4}\left(\dfrac{\omega}{\omega^2+4} + 0.5\tan^{-1}(0.5\omega)\right)\Big|_0^2 = \dfrac{1}{32\pi} + \dfrac{1}{64} = \underline{0.0256}$

$= \dfrac{1}{8\pi}\left[\dfrac{2}{4+4} + \dfrac{\tan^{-1}(1)}{2} - 0 - \dfrac{\tan^{-1}(0)}{2}\right]$

$= \dfrac{1}{8\pi}\left[\dfrac{1}{4} + \dfrac{\pi}{2\cdot 4} - 0\right]$

$= \dfrac{1}{32\pi} + \dfrac{1}{64}$

Fraction $= \dfrac{W_o}{W} = \dfrac{0.0256}{0.0313} = \underline{81.79\%}$

From Table 18.2,

$\mathcal{F}\{t^n e^{-at}u(t)\} = \dfrac{n!}{(a+j\omega)^{n+1}}$

$\int \dfrac{dx}{(a+bx^2)^2} = \dfrac{x}{2a(a+bx^2)} + \dfrac{1}{2a}\int\dfrac{dx}{a+bx^2}$

$= \dfrac{x}{2a(a+bx^2)} + \dfrac{1}{2a}\dfrac{1}{\sqrt{ab}}\tan^{-1}\left(\dfrac{x\sqrt{ab}}{a}\right)\ (ab>0)$

$a=4,\ b=1,\ x=\omega$