Solution 18.1

Obtain the Fourier transform of the function in Fig. 18.26.

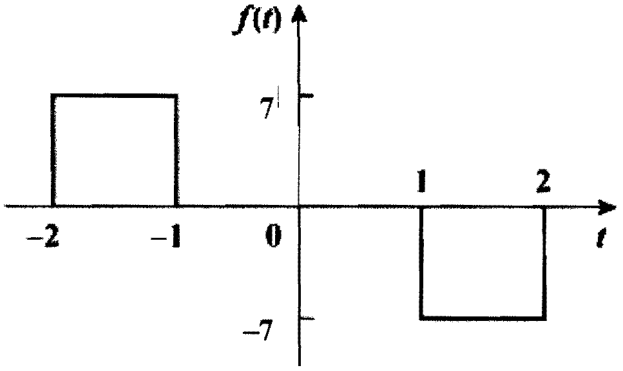


Figure 18.26
For Prob. 18.1.

Solution

$f(t) = 7u(t+2) - 7u(t+1) - 7u(t-1) + 7u(t-2)$ Take derivative wrt t.

F.T.

$f'(t) = 7\delta(t+2) - 7\delta(t+1) - 7\delta(t-1) + 7\delta(t-2)$

$j\omega F(\omega) = 7(e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j2\omega}) = 14\cos(2\omega) - 14\cos(\omega)$ or

$14\left[\frac{e^{j2\omega}+e^{-j2\omega}}{2} - \frac{e^{j\omega}+e^{-j\omega}}{2}\right]$

$\therefore$ $F(\omega) = \mathbf{14[\cos(2\omega) - \cos(\omega)]/(j\omega)}$

From Table 18.1: $\frac{df(t)}{dt} \Leftrightarrow j\omega F(\omega)$, $f(t-a) \Leftrightarrow e^{-j\omega a} F(\omega)$

From Table 18.2: $\delta(t) \overset{\mathcal{F}}{\Longleftrightarrow} 1$

Solution 18.10

Obtain the Fourier transforms of the signals shown in Fig. 18.35.

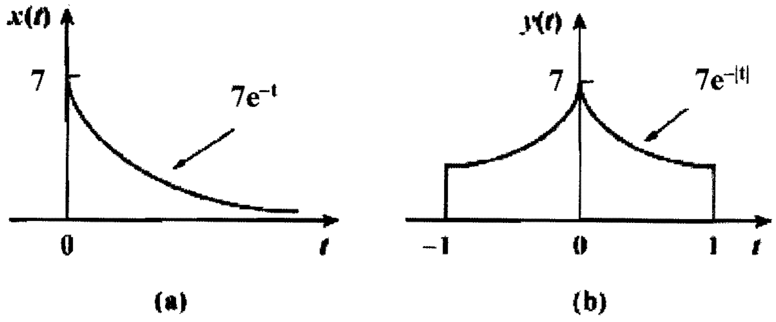


Figure 18.35
For Prob. 18.10.

Solution

(a) $x(t) = 7e^{-t}u(t)$ From Table 18.2: $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{a+j\omega}$

$\underline{X(\omega) = 7/(1 + j\omega)}$

(b) $y(t) = \begin{cases} 7e^{-t}, & t>0 \ (t<1) \\ 7e^{t}, & t<0 \ (t>-1) \\ 0 & \text{otherwise.} \end{cases}$ using (18.8) $Y(\omega) \equiv \mathcal{F}[y(t)] = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt$

$$Y(\omega) = \int_{-1}^{1} y(t)e^{-j\omega t}dt = \int_{-1}^{0} 7e^{t}e^{-j\omega t}dt + \int_{0}^{1} 7e^{-t}e^{-j\omega t}dt = 7\int_{-1}^{0} e^{(1-j\omega)t}dt + 7\int_{0}^{1} e^{-(1+j\omega)t}dt$$

$$= \frac{7e^{(1-j\omega)t}}{1-j\omega}\bigg|_{-1}^{0} + \frac{7e^{-(1+j\omega)t}}{-(1+j\omega)}\bigg|_{0}^{1} = \frac{7}{1-j\omega} - \frac{7e^{-(1-j\omega)}}{1-j\omega} + \frac{7e^{-(1+j\omega)}}{-(1+j\omega)} - \frac{7}{-(1+j\omega)}$$

$$= \frac{14}{1+\omega^2} - 7e^{-1}\left[\frac{\cos\omega + j\sin\omega}{1-j\omega} + \frac{\cos\omega - j\sin\omega}{1+j\omega}\right]$$

$\underline{Y(\omega) = [14/(1+\omega^2)][1 - e^{-1}\{\cos(\omega) - \omega\sin(\omega)\}]}$

$$Y(\omega) = \frac{7(1+j\omega)}{(1-j\omega)(1+j\omega)} + \frac{7(1-j\omega)}{(1+j\omega)(1-j\omega)} - 7e^{-1}\left[\frac{e^{j\omega}}{1-j\omega} + \frac{e^{-j\omega}}{1+j\omega}\right]$$

$$= \frac{14}{1+\omega^2} - 7e^{-1}\left[\frac{e^{j\omega}(1+j\omega)}{(1-j\omega)(1+j\omega)} + \frac{e^{-j\omega}(1-j\omega)}{(1+j\omega)(1-j\omega)}\right] = \frac{14}{1+\omega^2} - 7e^{-1}\left[\frac{e^{j\omega} + j\omega e^{j\omega} + e^{-j\omega} - j\omega e^{-j\omega}}{1+\omega^2}\right]$$

$$= \frac{14}{1+\omega^2}\left\{1 - e^{-1}\left[\frac{e^{j\omega} + e^{-j\omega}}{2} - \omega\frac{e^{j\omega} - e^{-j\omega}}{2j}\right]\right\}$$

Practice Problem 18.4

(a) $g(t) = u(t) - u(t-1)$. From Table 18.2 $u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \pi\delta(\omega) + \frac{1}{j\omega}$

From Table 18.1 $f(t-a) \overset{\mathcal{F}}{\Longleftrightarrow} e^{-j\omega a} F(\omega)$

(a) $g(t) = u(t) - u(t-1)$ or $G(\omega) = U(\omega) - e^{-j\omega}U(\omega) = (1 - e^{-j\omega})U(\omega)$
$= (1 - e^{-j\omega})[\pi\delta(\omega) + 1/(j\omega)]$. But $(1-e^{-j\omega})\pi\delta(\omega) = 0$ since $\delta(\omega)$ is only non-zero when $\omega = 0$ (DC). Thus,

$$\mathbf{G(\omega) = (1-e^{-j\omega})/(j\omega)}$$

(b)

Table 18.2 $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{a+j\omega}$

(b) $f(t) = 10te^{-2t}u(t)$

Let $g(t) = e^{-2t}u(t) \longrightarrow G(\omega) = 1/(2 + j\omega)$

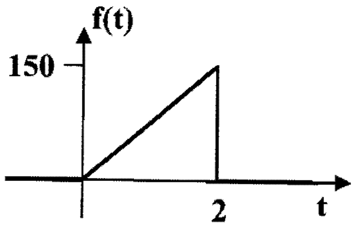
$f(t) = 10tg(t)$ which leads to $j\dfrac{dG}{d\omega} = j(-1)(2 + j\omega)^{-2}(j)$

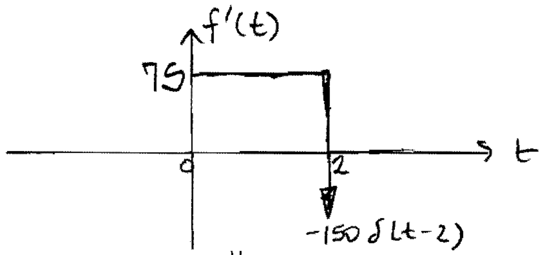
$F(\omega) = \mathbf{10/[(2+j\omega)^2]}$

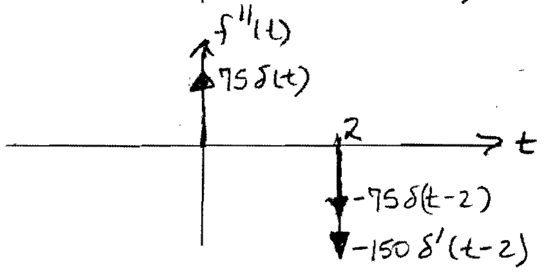
From Table 18.1 Frequency differentiation

$t^n f(n) \Longleftrightarrow j^n \frac{d^n}{d\omega^n} F(\omega)$

(c) (c) f(t) is sketched below. So, $f(t) = 75t[u(t) - u(t-2)]$ leads to







$f'(t) = 75[u(t)-u(t-2)] + 75t[\delta(t)-\delta(t-2)] = 75[u(t)-u(t-2)] - 150\delta(t-2)$
$f''(t) = 75\delta(t) - 75\delta(t-2) - 150\delta'(t-2) \;\; \mathcal{F}$
$(j\omega)^2F(\omega) = 75(1 - e^{-j\omega 2}) - 150j\omega e^{-j\omega 2}$

$$F(\omega) = \frac{75\left(e^{-j\omega 2} - 1\right)}{\omega^2} + \frac{150 j e^{-j\omega 2}}{\omega}$$

- $\mathcal{F}[\delta(t)] = 1$
- $f(t-a) \overset{\mathcal{F}}{\Longleftrightarrow} e^{-j\omega a} F(\omega)$
- $\frac{df}{dt} \overset{\mathcal{F}}{\Longleftrightarrow} j\omega F(\omega)$
- $\frac{d^2f}{dt} \overset{\mathcal{F}}{\Longleftrightarrow} (j\omega)^2 F(\omega)$

Alternatively, use the definition of the Fourier Transform:

(18.8) : $F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt.$

$$f(t) = \frac{150}{2} \cdot t \left[u(t) - u(t-2)\right]$$

$$\therefore \; F(\omega) = \int_0^2 75\, t\, e^{-j\omega t}\, dt = \frac{75}{\omega^2}\left[-1 + e^{-j2\omega}(1 + j2\omega)\right]$$

$$= \frac{75(e^{-j2\omega} - 1)}{\omega^2} + \frac{j\,150\, e^{-j2\omega}}{\omega}$$

integrate by parts, use tables, or Mathematica:

```
In[1]:= Integrate[75 * t * Exp[-ⅈ * ω * t], {t, 0, 2}]
```

Out[1]= $\frac{75\left(-1 + e^{-2 i \omega}(1 + 2 i \omega)\right)}{\omega^2}$

Solution 18.11

Find the Fourier transform of the "sine-wave pulse" shown in Fig. 18.36.

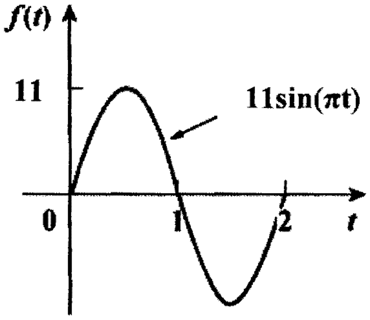


Figure 18.36
For Prob. 18.11.

Solution

f(t) = 11sin π t [u(t) - u(t - 2)]

$$e^{j(-\omega+\pi)t} - e^{-j(\omega+\pi)t}$$

(18.8): $$F(\omega) = \int_0^2 11\sin\pi t\, e^{-j\omega t}\,dt = \frac{11}{2j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{-j\omega t}\,dt$$

$$= \frac{11}{2j}\left[\int_0^2 \left(e^{+j(-\omega+\pi)t} - e^{-j(\omega+\pi)t}\right)dt\right]$$

$$= \frac{11}{2j}\left[\frac{1}{-j(\omega-\pi)}e^{-j(\omega-\pi)t}\Big|_0^2 - \frac{e^{-j(\omega+\pi)t}}{-j(\omega+\pi)}\Big|_0^2\right] = \frac{11}{2}\left[\frac{e^{-j(\omega-\pi)\cdot 2}-1}{\omega-\pi} - \frac{e^{-j(\omega+\pi)\cdot 2}-1}{\omega+\pi}\right]$$

$$= e^{-j2\omega+j2\pi} = e^{-j2\omega} \qquad = e^{-j2\omega\cdot j2\pi} = e^{-j2\omega}$$

$$= \frac{11}{2}\left(\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right) \qquad = \frac{11}{2}\left[\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right]$$

$$= \frac{11}{2}\left[\frac{(\pi+\omega)(1-e^{-j2\omega}) + (\pi-\omega)(1-e^{-j2\omega})}{\pi^2-\omega^2}\right]$$

$$= \frac{11}{2(\pi^2-\omega^2)}\left(2\pi - 2\pi e^{-j2\omega}\right)$$

$$= \frac{11}{2(\pi^2-\omega^2)}\left[\pi - \pi e^{-j2\omega} + \omega - \omega e^{-j2\omega} + \pi - \pi e^{-j2\omega} - \omega + \omega e^{-j2\omega}\right]$$

$$F(\omega) = \frac{11\pi}{\omega^2-\pi^2}\left(e^{-j\omega 2} - 1\right)$$

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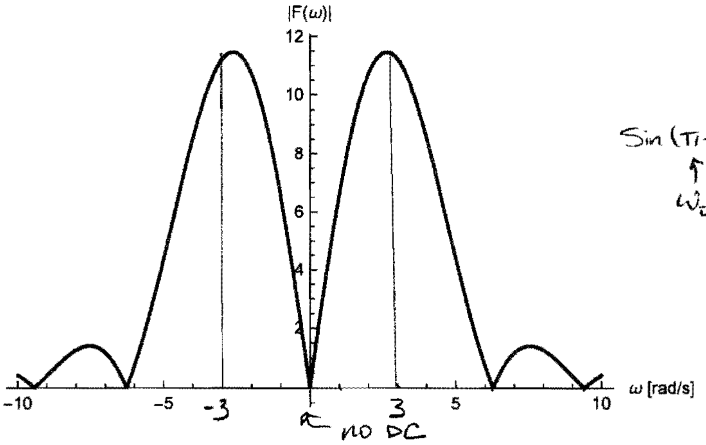
In[14]:=
```
F[ω_] := 11 * Pi / (ω^2 - Pi^2) * (Exp[-i * 2 * ω] - 1)
Plot[Abs[F[ω]], {ω, -10, 10}, AxesLabel → {"ω [rad/s]", "|F(ω)|"}]
```

Out[15]=



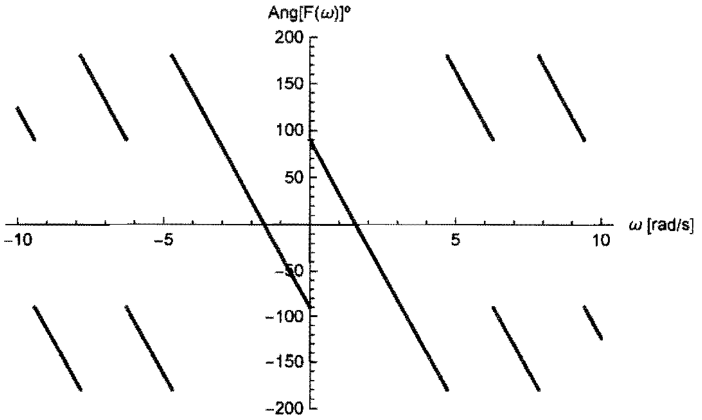
$\sin(\pi t)$

↑

$\omega_0 = \pi$

In[16]:=
```
Plot[Arg[F[ω]] * 180 / Pi, {ω, -10, 10}, AxesLabel → {"ω [rad/s]", "Ang[F(ω)]°"}]
```

Out[16]=



Solution 18.12

(a) $F_1(\omega) = \dfrac{10}{(3+j\omega)^2+100}$

(b) $F_2(\omega) = \dfrac{4+j\omega}{(4+j\omega)^2+100}$

(a) $f_1(t) = e^{-3t}\sin(10t)\,u(t)$

Table 18.2: $e^{-at}\sin(w_0 t)\,u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \dfrac{w_0}{(a+jw)^2+w_0^2}$

w/ $a=3$, $w_0=10 \Rightarrow$

$F_1(w) = \dfrac{10}{(3+jw)^2+100}$

(b) $f_2(t) = e^{-4t}\cos(10t)\,u(t)$

Table 18.2: $e^{-at}\cos(w_0 t)\,u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \dfrac{a+jw}{(a+jw)^2+w_0^2}$

w/ $a=4$, $w_0=10 \Rightarrow$

$F_2(w) = \dfrac{4+jw}{(4+jw)^2+100}$

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1/1

$$\mathcal{F}[f(t)] = F(\omega) = \frac{10}{(2+j\omega)(5+j\omega)}$$

Solution 18.23

$f(-3t)$ (a)

$f(3t)$ leads to $\frac{1}{3}\cdot\frac{10}{(2+j\omega/3)(5+j\omega/3)} = \frac{30}{(6+j\omega)(15+j\omega)}$

Scaling: $f(at) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$

$$F[f(-3t)] = \frac{30}{(6-j\omega)(15-j\omega)}$$

Reversal. $f(-t) \overset{\mathcal{F}}{\Longleftrightarrow} F^*(\omega)$

$f(2t-1)$ (b)

$f(2t) \longrightarrow \frac{1}{2}\cdot\frac{10}{(2+j\omega/2)(5+j\omega/2)} = \frac{20}{(4+j\omega)(10+j\omega)}$

Scaling: $f(2t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{2}F\left(\frac{\omega}{2}\right)$

$f(2t-1) = f[2(t-1/2)] \longrightarrow \frac{20e^{-j\omega/2}}{(4+j\omega)(10+j\omega)}$

Time Shift: $f(t-a) \overset{\mathcal{F}}{\Longleftrightarrow} e^{-j\omega a}F(\omega)$

$f(t)\cos(2t)$ (c)

$f(t)\cos(2t) \rightarrow \frac{1}{2}F(\omega+2) + \frac{1}{2}F(\omega+2)$

Modulation: $\cos(\omega_0 t) f(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{2}[F(\omega+\omega_0)+F(\omega-\omega_0)]$

$$= \frac{5}{[2+j(\omega+2)][5+j(\omega+2)]} + \frac{5}{[2+j(\omega-2)][5+j(\omega-2)]}$$

$\frac{d}{dt}f(t)$ (d)

$$F[f'(t)] = j\omega F(\omega) = \frac{j\omega 10}{(2+j\omega)(5+j\omega)}$$

Time differentiation: $\frac{df}{dt} \overset{\mathcal{F}}{\Longleftrightarrow} j\omega F(\omega)$

$\int_{-\infty}^{t} f(t)\,dt$

(e) $\int_{-\infty}^{t} f(\tau)\,d\tau \longrightarrow \frac{F(\omega)}{j(\omega)} + \pi F(0)\delta(\omega)$

Time integration: $\int_{-\infty}^{t} f(\tau)\,d\tau \overset{\mathcal{F}}{\Longleftrightarrow} \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

$$= \frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)\frac{10}{2 \cdot 5}$$

$$= \frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)$$

Solution 18.25

See below - (a) g(t) = 5e^{2t}u(t)
See below - (b) h(t) = 3e$^{-2|t|}$

(c) $X(\omega) = \frac{A}{s-1} + \frac{B}{s-2} = \frac{10}{(s-1)(s-2)}$ ← Partial fraction expansion

$s = j\omega$

$$A = \frac{10}{1-2} = -10, \quad B = \frac{10}{2-1} = 10$$

$$\therefore \quad X(\omega) = \frac{-10}{j\omega - 1} + \frac{10}{j\omega - 2}$$

Table 18.2 : $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{a+j\omega}$

x(t) = (–10e^{t}u(t)+10e^{2t})u(t)

(a) $G(\omega) = \frac{5}{j\omega - 2}$

Table 18.2: $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{a+j\omega}$

w/ $a = -2$

$g(t) = 5e^{2t}u(t)$

(b) $H(\omega) = \frac{12}{\omega^2 + 4}$

Table 18.2 : $\frac{2a}{a^2+\omega^2} \overset{\mathcal{F}}{\Longleftrightarrow} e^{-a|t|}$

w/ $a = 2$, $H(\omega) = \frac{3 \cdot 2 \cdot 2}{\omega^2 + 2^2}$

$\therefore$ $h(t) = 3e^{-2|t|}$

Solution 18.26

see below (a) $\underline{f(t) = e^{-(t-2)}u(t-2)}$

see below (b) $\underline{h(t) = te^{-4t}u(t)}$

(c) If $x(t) = u(t+1) - u(t-1) \longrightarrow X(\omega) = 2\frac{\sin\omega}{\omega}$ From Example 18.2 w/ $A = 1$ & $\tau = 2$

By using duality property, Table 18.1 $F(t) \overset{\mathcal{F}}{\Longleftrightarrow} 2\pi f(-\omega)$

$$G(\omega) = 2u(\omega+1) - 2u(\omega-1) \longrightarrow \underline{g(t) = \frac{2\sin t}{\pi t}}$$

(a) $F(\omega) = \dfrac{e^{-j2\omega}}{1+j\omega}$

- Time shift: $f(t-a) \overset{\mathcal{F}}{\Longleftrightarrow} e^{-j\omega a}F(\omega)$
- $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{a+j\omega}$

Let $G(\omega) = \frac{1}{1+j\omega}$, then $F(\omega) = e^{-j2\omega}G(\omega)$

$$\therefore \underline{\underline{f(t)}} = g(t-a) = e^{-t}u(t)\Big|_{t \to t-2} = \underline{\underline{e^{-(t-2)}u(t-2)}}$$

(b) $H(\omega) = \dfrac{1}{(j\omega+4)^2}$

- Table 18.2: $t^n e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{n!}{(a+j\omega)^{n+1}}$

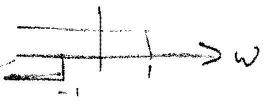
w/ $n = 1$ w/ $n = 1$

$$\underline{\underline{h(t) = te^{-4t}u(t)}}$$

(c) $G(\omega) = 2u(\omega+1) - 2u(\omega-1)$ Duality property, Table 18.1 $F(t) \overset{\mathcal{F}}{\Longleftrightarrow} 2\pi f(-\omega)$

From Example 18.2 w/ $A = 1$; $\tau = 2$:

$$x(t) = u(t+1) - u(t-1) \overset{\mathcal{F}}{\Longleftrightarrow} X(\omega) = 2\frac{\sin(\omega)}{\omega}$$



$$2\pi X(-\omega) = 2\pi[u(-\omega+1) - u(-\omega-1)] = 2\pi[u(\omega+1) - u(\omega-1)] = \pi \cdot G(\omega)$$

$$\therefore g(t) = \frac{1}{\pi} \cdot X(t) = \frac{2}{\pi}\frac{\sin(t)}{t}$$

Solution 18.27

(a) $j\omega \to s$ Let $F(s)=\dfrac{100}{s(s+10)} \overset{\text{Partial fraction expansion}}{=} \dfrac{A}{s}+\dfrac{B}{s+10}$, $s=j\omega$ $\quad F(\omega)=\dfrac{100}{j\omega(j\omega+10)}$

$$A=\frac{100}{10}=10,\ B=\frac{100}{-10}=-10$$

$$\therefore \quad F(\omega)=\frac{10}{j\omega}-\frac{10}{j\omega+10}$$

Table 18.2 $\cdot\ \mathrm{sgn}(t) \overset{\mathcal{F}}{\Longleftrightarrow} \dfrac{2}{j\omega}$ $\cdot\ e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \dfrac{1}{a+j\omega}$

$$f(t) = \mathbf{5\,sgn(t) - 10e^{-10t}u(t)}$$

(b) $j\omega \to s$ $G(s)=\dfrac{10s}{(2-s)(3+s)} \overset{\text{PFE}}{=} \dfrac{A}{2-s}+\dfrac{B}{s+3}$, $s=j\omega$ $\quad G(\omega)=\dfrac{10j\omega}{(-j\omega+2)(j\omega+3)}$

$$A=\frac{20}{5}=4,\quad B=\frac{-30}{5}=-6$$

$$\therefore \quad G(\omega)=\frac{4}{=-j\omega+2}-\frac{6}{j\omega+3}$$

Table 18.2 : $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \dfrac{1}{a+j\omega}$

Table 18.1 : Reversal $f(-t) \overset{\mathcal{F}}{\Longleftrightarrow} F(-\omega)$

$$g(t) = \mathbf{4e^{2t}u(-t) - 6e^{-3t}u(t)}$$

(c) $$H(\omega)=\frac{60}{(j\omega)^2+j40\omega+1300}=\frac{60=2\cdot 30}{(j\omega+20)^2+\underbrace{900}_{=30^2}}$$

Table 18.2 : $e^{-at}\sin(\omega_0 t)u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$

$$h(t) = \mathbf{2e^{-20t}\sin(30t)u(t)}$$

(d) $$y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\delta(\omega)e^{j\omega t}d\omega}{(2+j\omega)(j\omega+1)}=\frac{1}{2\pi}\cdot\frac{1}{2}=\frac{1}{4\pi}$$

$Y(\omega)=\dfrac{\delta(\omega)}{(j\omega+1)(j\omega+2)}$

(18.9)

$$=\frac{1}{2\pi}\left.\frac{e^{j\omega t}}{(2+j\omega)(j\omega+1)}\right|_{\omega=0}=\frac{1}{2\pi}\cdot\frac{1}{2\cdot 1}=\frac{1}{4\pi}$$

Solution 18.28

(a) $$\underline{\underline{f(t)}} \overset{(18.9)}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)\,e^{j\omega t}}{(5+j\omega)(2+j\omega)}d\omega$$

$$= \frac{1}{2}\frac{1}{(5)(2)} = \frac{1}{20} = \underline{\underline{\mathbf{0.05}}}$$

(b) $$f(t) \overset{(18.9)}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{10\delta(\omega+2)}{j\omega(j\omega+1)}e^{j\omega t}\,d\omega \overset{\omega=-2}{=} \frac{10}{2\pi}\frac{e^{-j2t}}{(-j2)(-j2+1)}$$

$$= \frac{j5}{2\pi}\frac{e^{-j2t}}{1-j2} = \frac{\mathbf{(-2+j)e^{-j2t}}}{\mathbf{2\pi}}$$

$$\frac{j5}{2\pi}\frac{e^{-j2t}}{1-j2}\cdot\frac{1+j2}{1+j2} = \frac{j5(1+j2)\,e^{-j2t}}{2\pi\cdot 5}$$

(c) $$f(t) \overset{(18.9)}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{20\delta(\omega-1)e^{j\omega t}}{(2+j\omega)(3+j\omega)}d\omega \overset{\omega=1}{=} \frac{20}{2\pi}\frac{e^{jt}}{(2+j)(3+j)}$$

$$(2+j)(3+j) = 6+3j+2j-1 = 5+j5$$

$$= \frac{20e^{jt}}{2\pi(5+5j)} = \frac{\mathbf{(1-j)e^{jt}}}{\boldsymbol{\pi}}$$

$$\frac{20e^{jt}}{2\pi(5+j5)}\frac{5-j5}{5-j5} = \frac{10e^{jt}(5-j5)}{\pi\,(25+25)} = \frac{50e^{jt}(1-j)}{50\pi}$$

(d) Let $$F(\omega) = \frac{5\pi\delta(\omega)}{(5+j\omega)} + \frac{5}{j\omega(5+j\omega)} = F_1(\omega) + F_2(\omega)$$

$$f_1(t) \overset{(18.9)}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{5\pi\delta(\omega)}{5+j\omega}e^{j\omega t}\,d\omega \overset{\omega=0}{=} \frac{5\pi}{2\pi}\cdot\frac{1}{5} = 0.5$$

$j\omega \to s$ $$F_2(s) = \frac{5}{s(5+s)} \overset{\text{P.F.E.}}{=} \frac{A}{s} + \frac{B}{s+5}, \quad A = 1, B = -1$$

$$\therefore\ F_2(\omega) = \frac{1}{j\omega} - \frac{1}{j\omega+5}$$

Table 18.2:
- $\mathrm{sgn}(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{2}{j\omega}$
- $e^{-at}u(t) \overset{\mathcal{F}}{\Longleftrightarrow} \frac{1}{a+j\omega}$

notice! Tricky

$$f_2(t) = \frac{1}{2}\mathrm{sgn}(t) - e^{-5t} = -\frac{1}{2} + u(t) - e^{5t}$$

$$\therefore\ \underline{\underline{f(t)}} = f_1(t) + f_2(t) = \underline{\underline{\mathbf{u(t) - e^{-5t}}}}$$

$\frac{1}{2}\mathrm{sgn}(t)$