Whites

EE 221 - Circuits II

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$$v(t) = \overbrace{30}^{A_0} + \overbrace{20}^{A_1}\cos(\underbrace{60\pi t}_{\omega_0} + 45°) + \overbrace{10}^{A_2}\cos(\underbrace{120\pi t}_{2\omega_0} - 45°) \text{ V}$$

Solution 17.43

$$= \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

(a) (17.49): $\underline{V_{rms}} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{30^2 + \frac{1}{2}(20^2 + 10^2)} = \underline{\mathbf{33.91\ V}}$

$i(t) = 6 + 4\cos(60\pi t + 10°) - 2\cos(120\pi t - 60°)$ A

(b) (17.49): $\underline{I_{rms}} = \sqrt{6^2 + \frac{1}{2}(4^2 + 2^2)} = \underline{\mathbf{6.782\ A}}$

(c) (17.46): $\underline{P} = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{2} V_n I_n \cos(\Theta_n - \Phi_n)$

$\Theta_1 = -45°$, $\Phi_1 = -10°$
$\Theta_2 = 45°$, $\Phi_2 = 60°$

$= 30x6 + 0.5[20x4\cos(45°-10°) - 10x2\cos(-45°+60°)]$

$= 180 + 32.76 - 9.659 = \underline{\mathbf{203.1\ W}}$

$\cos(45° - 60°) = \cos[-45° + 60°]$

$\cos[-45° - (-10°)] = \cos[45° - 10°]$

Fourier series:
$$i(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$$

Solution 17.47

$T = 2, \quad \omega_o = 2\pi/T = \pi$ rad/s

DC value

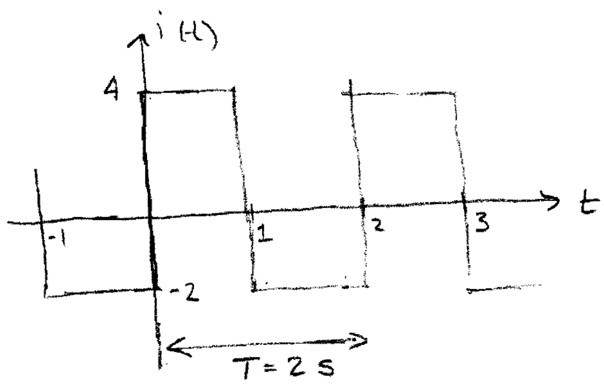
(17.6): $$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-2)dt\right] = \frac{1}{2}(4-2) = 1 \text{ A}$$

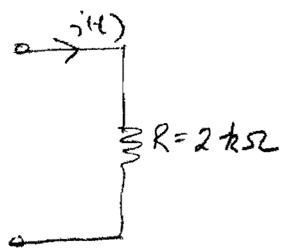
(17.51): (Frms = Irms) $$P = Ri_{rms}^2 = \frac{R}{T}\int_0^T f^2(t)dt = \frac{R}{2}\left[\int_0^1 4^2 dt + \int_1^2 (-2)^2 dt\right] = 10R$$

Definition of Rms²

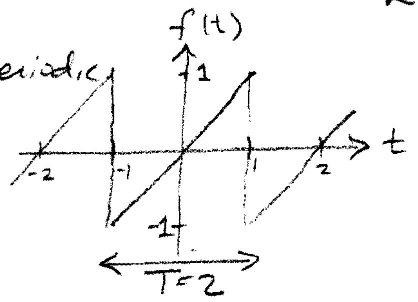
The average power dissipation caused by the dc component is

$$P_0 = Ra_o^2 = R = \underline{\underline{10\% \text{ of P}}}$$





$f(t) = t \quad -1 < t < 1$; periodic



Solution 17.50 $\quad f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \quad (17.58)$

(17.59): $\quad c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_0 nt}dt, \quad \omega_o = \frac{2\pi}{T} = \pi$

$$= \frac{1}{2}\int_{-1}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$dv = \frac{1}{2}e^{-jn\pi t}dt$ which leads to $v = -[1/(2jn\pi)]e^{-jn\pi t}$

$uv - \int v\,du$

$$\therefore \quad c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1}e^{-jn\pi t}dt$$

$= \frac{jt}{2n\pi}e^{-jn\pi t}\Big|_{-1}^{1}$

$= \frac{j}{2n\pi}(e^{-jn\pi} + e^{+jn\pi})$

$$= \frac{j}{2n\pi}\left[e^{-jn\pi} + e^{jn\pi}\right] + \frac{1}{2n^2\pi^2(-j)\cancel{\cdot}j}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\underbrace{\cos(n\pi)}_{=(-1)^n} + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})\frac{j2}{j2}$$

$$c_n = \frac{j(-1)^n}{n\pi} - \frac{2j}{2n^2\pi^2}\cancel{\sin(n\pi)}^{\,0} = \frac{j(-1)^n}{n\pi}$$

Thus

$$\underline{f(t)} = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \underline{\sum_{\substack{n=-\infty \\ (n\neq 0)}}^{\infty} (-1)^n \frac{j}{n\pi}e^{jn\pi t}}$$

$$\underline{c_0} = \frac{1}{T}\int_0^T f(t)\,dt = \underline{0}$$

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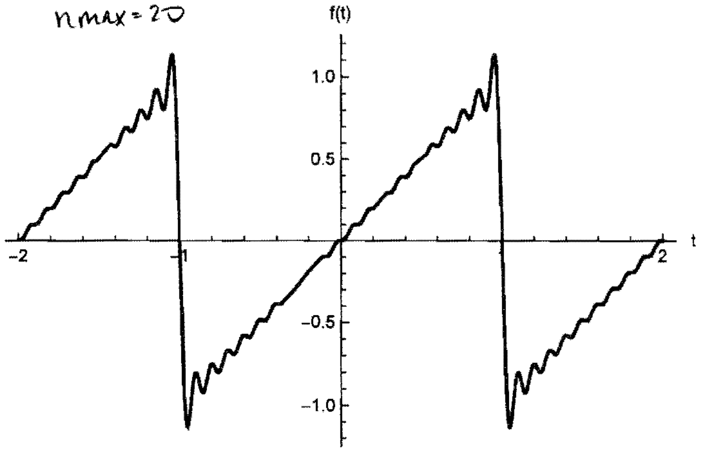
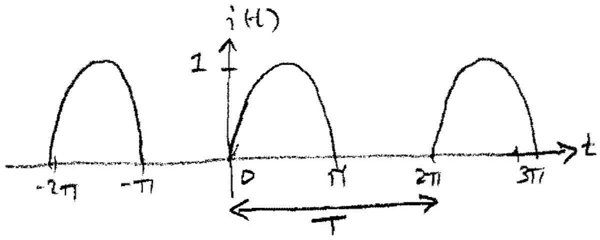
```
In[1]:= ω0 := Pi
        T := 2 * Pi / ω0
        f[nmax_, t_] := Sum[i * (-1)^n / (n * Pi) * Exp[i * n * Pi * t], {n, -nmax, -1, 1}] +
          Sum[i * (-1)^n / (n * Pi) * Exp[i * n * Pi * t], {n, 1, nmax, 1}]
        Plot[f[20, t], {t, -T, T}, AxesLabel → {"t", "f(t)"}]
```

nmax = 20

Out[4]=



$$f(t) = \sum_{n=-n_{max}}^{-1} c_n e^{jn\pi t} + \sum_{n=1}^{n_{max}} c_n e^{jn\pi t}$$



Solution 17.55

$T = 2\pi,\ \omega_o = 2\pi/T = 1$ rad/s

$$i(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} \quad (17.58)$$

(17.59): $$c_n = \frac{1}{T}\int_0^T i(t)e^{-jn\omega_o t}dt$$

But $$i(t) = \begin{cases} \sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$\therefore \quad c_n = \frac{1}{2\pi}\int_0^{\pi}\sin(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^{\pi}\frac{1}{2j}(e^{jt}-e^{-jt})e^{-jnt}dt = \frac{1}{4\pi j}\int_0^{\pi}\left[e^{jt(1-n)} - e^{-jt(1+n)}\right]dt$$

$n \neq \pm 1$ $$= \frac{1}{4\pi j}\left[\frac{e^{jt(1-n)}}{j(1-n)} + \frac{e^{-jt(1+n)}}{j(1+n)}\right]_0^{\pi}$$

$$= -\frac{1}{4\pi}\left[\frac{e^{j\pi(1-n)}-1}{1-n} + \frac{e^{-j\pi(n+1)}-1}{1+n}\right]$$

$$= -\frac{1}{4\pi}\left[\frac{e^{j\pi(1-n)}-1}{1-n}\cdot\frac{1+n}{1+n} + \frac{e^{-j\pi(n+1)}-1}{1+n}\cdot\frac{1-n}{1-n}\right]$$

$(1+n)(1-n) = 1-n^2$

$$= \frac{1}{4\pi(n^2-1)}\left[e^{j\pi(1-n)} - 1 + ne^{j\pi(1-n)} - n + e^{-j\pi(1+n)} - 1 - ne^{-j\pi(1+n)} + n\right]$$

But $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 = e^{-j\pi}$

$$= \frac{-2e^{-jn\pi}-2}{2\cdot 4\pi(n^2-1)}$$

$$c_n = \frac{1}{4\pi(n^2-1)}\left[-e^{-jn\pi} - e^{-jn\pi} - ne^{-jn\pi} + ne^{-jn\pi} - 2\right] = \frac{1+e^{-jn\pi}}{2\pi(1-n^2)}$$

Thus

$$i(t) = \sum_{\substack{n=-\infty \\ (n=\pm 1)}}^{\infty} \frac{1+e^{-jn\pi}}{2\pi(1-n^2)}e^{jnt}$$

$n=1$: $$\underline{C_1} = \frac{1}{2\pi}\int_0^{\pi}\sin t\, e^{-jnt}dt = \frac{1}{2\pi}\int_0^{\pi}\frac{1}{2j}(e^{jt}-e^{-jt})e^{-jt}dt = \frac{1}{j4\pi}\int_0^{\pi}(1-e^{-j2t})dt$$

$$= \frac{1}{j4\pi}\left[t\Big|_0^{\pi} - \frac{1}{-j2}e^{-j2t}\Big|_0^{\pi}\right] = \frac{1}{j4\pi}\left[\pi - \frac{j}{2}(\underbrace{e^{-j2\pi}}_{=1}-1)\right] = \underline{\frac{1}{j4}}$$

$n=-1$ $$\underline{C_{-1}} = -C_1 = \underline{\frac{-1}{j4}}$$

$$\underline{C_0} = \frac{1}{T}\int_0^{\pi}\sin t\, dt = \frac{1}{2\pi}(-\cos t)\Big|_0^{\pi} = \frac{-1}{2\pi}(\cos\pi - 1) = -\frac{1}{2\pi}(-2) = \underline{\frac{1}{\pi}}$$

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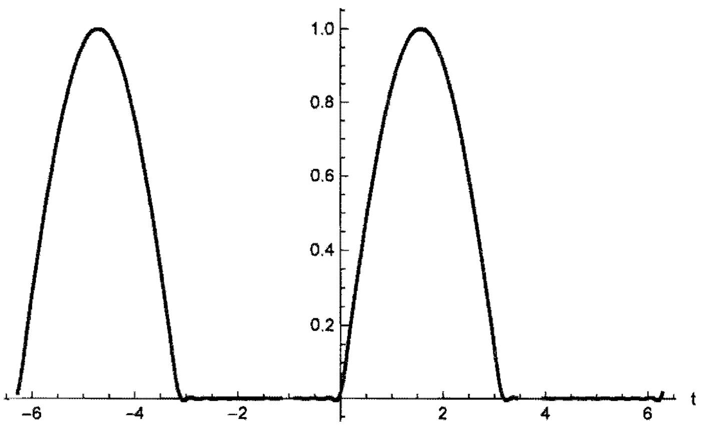
C_{-1} $\qquad\qquad\qquad\qquad$ C_0 $\qquad$ C_{-1}

```
In[59]:= ω0 := 1
         T := 2 * Pi / ω0
         f[nmax_, t_] := -1 / (ⅈ * 4) * Exp[-ⅈ * 1 * t] + 1 / Pi + 1 / (ⅈ * 4) * Exp[ⅈ * 1 * t] +
           Sum[(1 + Exp[-ⅈ * n * Pi]) / (2 * Pi * (1 - n^2)) * Exp[ⅈ * n * t], {n, -nmax, -2, 1}] +
           Sum[(1 + Exp[-ⅈ * n * Pi]) / (2 * Pi * (1 - n^2)) * Exp[ⅈ * n * t], {n, 2, nmax, 1}]
         Plot[f[20, t], {t, -T, T}, AxesLabel → {"t", "f(t)"}]
```

$n_{max} = 20$

Out[62]=



$$f(t) = \sum_{n=-n_{max}}^{-2} C_n e^{jnt} + C_{-1} e^{-jt} + C_0 + C_1 e^{jt} + \sum_{n=2}^{n_{max}} C_n e^{jnt}$$

Solution 17.59

For f(t), $T = 2\pi$, $\omega_o = 2\pi/T = \underline{1.}$

$$\underline{a_o} = \text{DC component} = (1\text{x}\pi + 0)/2\pi = \underline{0.5}$$

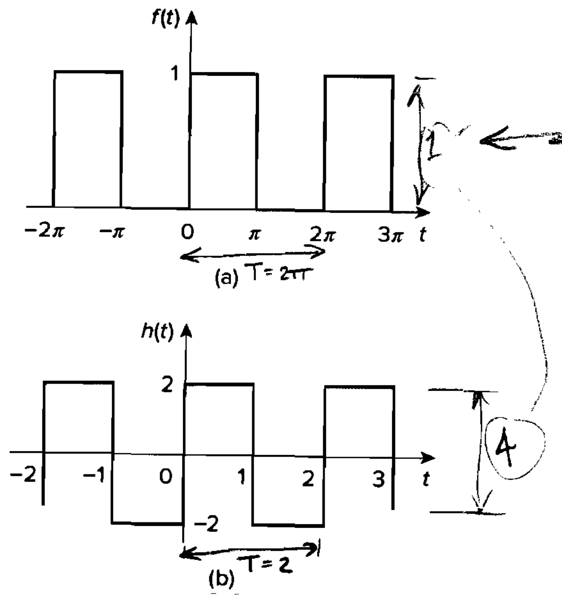
For h(t), $T = 2$, $\omega_o = 2\pi/T = \underline{\pi.}$

$$\underline{a_o} = (2\text{x}1 - 2\text{x}1)/2 = \underline{0}$$

Thus by replacing $\omega_o = 1$ with $\omega_o = \pi$ and multiplying the magnitude by four, we obtain

$\omega_o = \pi$

$$h(t) = -\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j4e^{-j(2n+1)\pi t}}{(2n+1)\pi}$$



$\omega_o = 1$

$$f(t) = \frac{1}{2} - \sum_{n=-\infty}^{\infty} \frac{je^{-j(2n+1)t}}{(2n+1)\pi}$$

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$i(t) = 4 + \underset{b_1}{6} \sin(100\pi t) + \underset{a_1}{8} \cos(100\pi t) - \underset{b_2}{3} \sin(200\pi t) - \underset{a_2}{4} \cos(200\pi t)$ A.

$n=1$ $n=2$

Solution 17.74

(a) (17.13b): $A_n = \sqrt{a_n^2 + b_n^2}$, $\phi = -\tan^{-1}(b_n/a_n)$

$A_1 = \sqrt{6^2 + 8^2} = 10$, $\phi_1 = -\tan^{-1}(6/8) = -36.87°$

$A_2 = \sqrt{3^2 + 4^2} = 5$, $\phi_2 = -\tan^{-1}(3/4) = -36.87°$

$i(t) = \{4 + 10\cos(100\pi t - 36.87°) - 5\cos(200\pi t - 36.87°)\}$ A

(b) $p = I_{DC}^2 R + 0.5\sum I_n^2 R$

$= 2[4^2 + 0.5(10^2 + 5^2)] =$ **157 W**

(17.3): $i(t) = a_0 + \sum_{n=1}^{2} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$, $\omega_0 = 100\pi$

(17.10): $i(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n)$

$$\left.\begin{aligned} A_n &= \sqrt{a_n^2 + b_n^2} \\ \phi_n &= -\tan^{-1}\left(\frac{b_n}{a_n}\right) \end{aligned}\right\} \text{(17.13b)}$$

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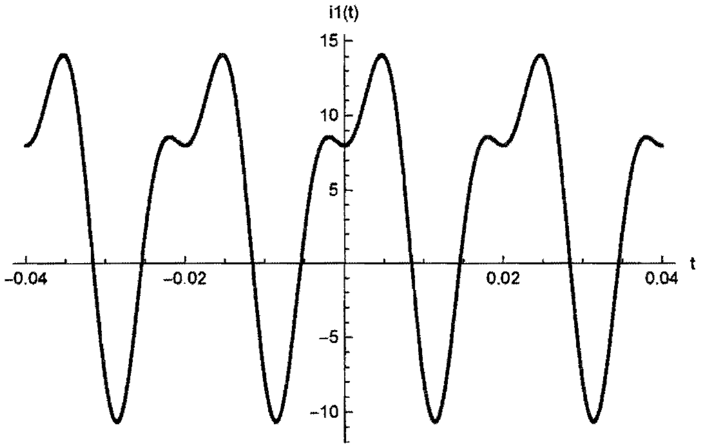
In[71]:=
```
ω0 := 100 * Pi
T := 2 * Pi / ω0
i1[t_] := 4 + 6 * Sin[ω0 * t] + 8 * Cos[ω0 * t] - 3 * Sin[2 * ω0 * t] - 4 * Cos[2 * ω0 * t]
Plot[i1[t], {t, -2 * T, 2 * T}, AxesLabel → {"t", "i1(t)"}]
```

Out[74]=



In[77]:=
```
i2[t_] := 4 + 10 * Cos[ω0 * t - 36.87 * Pi / 180] - 5 * Cos[2 * ω0 * t - 36.87 * Pi / 180.]
Plot[i2[t], {t, -2 * T, 2 * T}, AxesLabel → {"t", "i2(t)"}]
```

Out[78]=