Name________________________________ SN_______________________

EE 221 – *Circuits II*
Final Exam

December 8, 2017
10:00-11:15 AM
100 points

Turn off and store out of sight your mobile telephone, smart watch, and all electronic devices, other than your calculator. A calculator is the only electronic device you may operate during this exam. Write your name and student number where indicated above. This exam is to be an individual effort and is closed book, closed notes, and no formula sheets. Using pre-programmed equations (symbolic or otherwise) on your calculator is prohibited. Show all of your work on the supplied sheets of paper. **Do not write on the back of any sheet of paper**. This exam will not be returned to you.

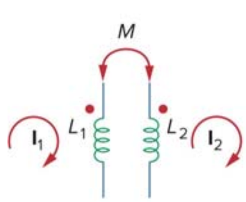
Prob. #1	Prob. #2	Prob. #3	Prob. #4	Prob. #5	**TOTAL**
20	20	20	20	20	100 pts.

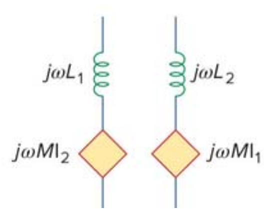
- $P_{\max} = \dfrac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}, \quad \dfrac{\partial P}{\partial X_L} = 0 \Rightarrow X_L = -X_{\text{Th}}, \quad \dfrac{\partial P}{\partial R_L} = 0 \Rightarrow R_L = \sqrt{R_{\text{Th}}^2 + \left(X_{\text{Th}} + X_L\right)^2}$
- $X_{\text{rms}} = \sqrt{\dfrac{1}{T}\displaystyle\int_{t_0}^{t_0+T} x^2(t)\,dt}$
- $\mathbf{S} = \dfrac{1}{2}\mathbf{V}\mathbf{I}^* = P + jQ$
- $\text{pf} = \cos\left(\theta_v - \theta_i\right)$
- $C = \dfrac{Q_C}{\omega V_{\text{rms}}^2} = \dfrac{P\left(\tan\theta_1 - \tan\theta_2\right)}{\omega V_{\text{rms}}^2}$
- $\mathbf{Z}_Y = \dfrac{\mathbf{Z}_\Delta}{3}$
- $\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p\mathbf{I}_p^* = \sqrt{3}V_L I_L \measuredangle\theta$

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}\,V_p\angle 30°$
	$\mathbf{V}_{bn} = V_p\angle -120°$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120°$
	$\mathbf{V}_{cn} = V_p\angle +120°$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120°$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0°$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120°$	
	$\mathbf{V}_{ca} = V_p\angle +120°$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30°}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120°$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120°$

[1]Positive or *abc* sequence is assumed.

- Mutual inductance: $M = k\sqrt{L_1L_2}$, $w = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 \pm Mi_1i_2$





- Ideal transformer: $\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1}$, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$, $\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}$

TABLE 14.4

Summary of the characteristics of resonant *RLC* circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

$$\mathfrak{L}\{f(t)\} = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

TABLE 15.1

Properties of the Laplace transform.

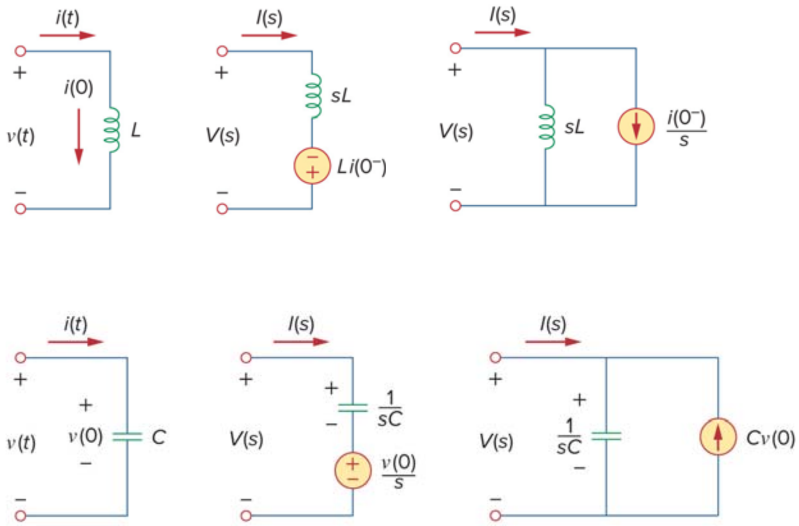
Property	$f(t)$	$F(s)$
Linearity	$a_1f_1(t) + a_2f_2(t)$	$a_1F_1(s) + a_2F_2(s)$
Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)dx$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

TABLE 15.2

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.



- $\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax)$, $\int x\sin(ax)\,dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$
- $f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\right] = a_0 + \sum_{n=1}^{\infty} A_n\cos(n\omega_0 t + \phi_n) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$

  $a_0 = \frac{1}{T}\int_0^T f(t)\,dt$, $a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$, $b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$,

  $c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}\,dt$, $A_n\measuredangle\phi_n = a_n - jb_n = 2c_n$
- Even function of t about $t = 0$: $a_0 = \frac{2}{T}\int_0^{T/2} f_e(t)\,dt$, $a_n = \frac{4}{T}\int_0^{T/2} f_e(t)\cos(n\omega_0 t)\,dt$, $b_n = 0$
- Odd function of t about $t = 0$: $a_0 = 0$, $a_n = 0$, $b_n = \frac{4}{T}\int_0^{T/2} f_o(t)\sin(n\omega_0 t)\,dt$
- With $v(t) = V_{\text{dc}} + \sum_{n=1}^{\infty} V_n\cos(n\omega_0 t - \theta_n)$ and $i(t) = I_{\text{dc}} + \sum_{n=1}^{\infty} I_n\cos(n\omega_0 t - \phi_n)$

  $P = V_{\text{dc}}I_{\text{dc}} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n\cos(\theta_n - \phi_n)$
- $F_{\text{rms}} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2} = \sqrt{|c_0|^2 + 2\sum_{n=1}^{\infty}|c_n|^2}$
- $P_{1\Omega} = F_{\text{rms}}^2$

- $F(\omega)=\mathfrak{F}\left[f(t)\right]=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\,,\;\; f(t)=\mathfrak{F}^{-1}\left[F(\omega)\right]=\dfrac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}dt$
- $W_{1\Omega}=\int_{-\infty}^{\infty} f^2(t)\,dt=\dfrac{1}{2\pi}\int_{-\infty}^{\infty}\left|F(\omega)\right|^2 d\omega$
- $\int x^2e^{ax}dx=\dfrac{e^{ax}}{a^3}\left(a^2x^2-2ax+2\right)$
- $\int\dfrac{dx}{\left(a+bx^2\right)^2}=\dfrac{x}{2a\left(a+bx^2\right)}+\dfrac{1}{2a}\dfrac{1}{\sqrt{ab}}\tan^{-1}\left(\dfrac{x\sqrt{ab}}{a}\right)\quad (ab>0)$

TABLE 18.1

Properties of the Fourier transform.

Property	$f(t)$	$F(\omega)$
Linearity	$a_1f_1(t)+a_2f_2(t)$	$a_1F_1(\omega)+a_2F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)$	$e^{-j\omega a}F(\omega)$
Frequency shift	$e^{j\omega_0 t}f(t)$	$F(\omega-\omega_0)$
Modulation	$\cos(\omega_0 t)f(t)$	$\frac{1}{2}[F(\omega+\omega_0)+F(\omega-\omega_0)]$
Time differentiation	$\frac{df}{dt}$	$j\omega F(\omega)$
	$\frac{d^nf}{dt^n}$	$(j\omega)^nF(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$
Frequency differentiation	$t^nf(t)$	$(j)^n\frac{d^n}{d\omega^n}F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in t	$f_1(t)*f_2(t)$	$F_1(\omega)F_2(\omega)$
Convolution in ω	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$

TABLE 18.2

Fourier transform pairs.

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$
$u(t+\tau) - u(t-\tau)$	$2\frac{\sin\omega\tau}{\omega}$
$\|t\|$	$\frac{-2}{\omega^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-at}\,u(t)$	$\frac{1}{a+j\omega}$
$e^{at}\,u(-t)$	$\frac{1}{a-j\omega}$
$t^n e^{-at}\,u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$
$e^{-at}\sin\omega_0 t u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
$e^{-at}\cos\omega_0\, t u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$