Name________________________________ SN______________________

EE 221 – *Circuits II*
Exam #3

December 1, 2017
11:00-11:50 PM
100 points

Turn off and store out of sight your mobile telephone, smart watch, and all electronic devices, other than your calculator. A calculator is the only electronic device you may operate during this exam. Write your name and student number where indicated above. This exam is to be an individual effort and is closed book, closed notes, and no formula sheets. Using pre-programmed equations (symbolic or otherwise) on your calculator is prohibited. Show all of your work on the supplied sheets of paper. **Do not write on the back of any sheet of paper**.

Prob. #1	Prob. #2	Prob. #3	Prob. #4	Prob. #5	**TOTAL**
20	20	20	20	20	100 pts.

- Ideal transformer: $\dfrac{\mathbf{V}_2}{\mathbf{V}_1}=\dfrac{N_2}{N_1}$, $\dfrac{\mathbf{I}_2}{\mathbf{I}_1}=\dfrac{N_1}{N_2}$, $\mathbf{Z}_{\text{in}}=\dfrac{\mathbf{Z}_L}{n^2}$
- $\int x\cos(ax)\,dx=\dfrac{1}{a^2}\cos(ax)+\dfrac{x}{a}\sin(ax)$, $\int x\sin(ax)\,dx=\dfrac{1}{a^2}\sin(ax)-\dfrac{x}{a}\cos(ax)$
- $f(t)=a_0+\sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t)+b_n\sin(n\omega_0 t)\right]=a_0+\sum_{n=1}^{\infty}A_n\cos(n\omega_0 t+\phi_n)=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_0 t}$

  $a_0=\dfrac{1}{T}\int_0^T f(t)\,dt$, $a_n=\dfrac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$, $b_n=\dfrac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$,

  $c_n=\dfrac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}\,dt$, $A_n\measuredangle\phi_n=a_n-jb_n=2c_n$
- Even function of *t* about *t* = 0: $a_0=\dfrac{2}{T}\int_0^{T/2} f_e(t)\,dt$, $a_n=\dfrac{4}{T}\int_0^{T/2} f_e(t)\cos(n\omega_0 t)\,dt$, $b_n=0$
- Odd function of *t* about *t* = 0: $a_0=0$, $a_n=0$, $b_n=\dfrac{4}{T}\int_0^{T/2} f_o(t)\sin(n\omega_0 t)\,dt$

- With $v(t) = V_{\text{dc}} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t - \theta_n)$ and $i(t) = I_{\text{dc}} + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t - \phi_n)$

  $$P = V_{\text{dc}} I_{\text{dc}} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

- $F_{\text{rms}} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2} = \sqrt{\left|c_0\right|^2 + 2\sum_{n=1}^{\infty}\left|c_n\right|^2}$
- $P_{1\Omega} = F_{\text{rms}}^2$

$$\mathfrak{L}\{f(t)\} = F(s) = \int_{0-}^{\infty} f(t) e^{-st} dt$$

TABLE 15.1

Properties of the Laplace transform.

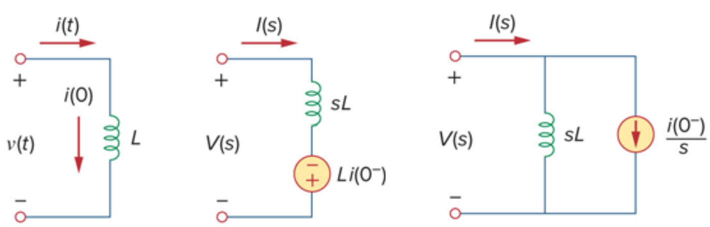
Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as} F(s)$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)dx$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^{\infty} F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

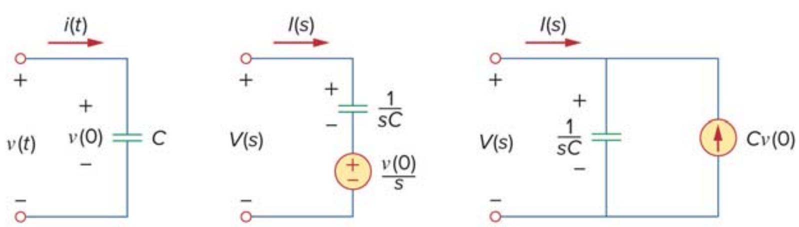
TABLE 15.2

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.





$$F(\omega) = \mathfrak{F}\left[f(t)\right] = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\,,\ \ f(t) = \mathfrak{F}^{-1}\left[F(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}dt$$

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|F(\omega)\right|^2 d\omega$$

TABLE 18.1

Properties of the Fourier transform.

Property	$f(t)$	$F(\omega)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t) f(t)$	$\frac{1}{2}[F(\omega + \omega_0) + F(\omega - \omega_0)]$
Time differentiation	$\frac{df}{dt}$	$j\omega F(\omega)$
	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
Frequency differentiation	$t^n f(t)$	$(j)^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in t	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$

TABLE 18.2

Fourier transform pairs.

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$
$u(t+\tau) - u(t-\tau)$	$2\frac{\sin \omega\tau}{\omega}$
$\|t\|$	$\frac{-2}{\omega^2}$
$\text{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-at}\,u(t)$	$\frac{1}{a + j\omega}$
$e^{at}\,u(-t)$	$\frac{1}{a - j\omega}$
$t^n e^{-at}\,u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$
$e^{-a\|t\|}$	$\frac{2a}{a^2 + \omega^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$\cos \omega_0 t$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$e^{-at}\sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
$e^{-at}\cos \omega_0\, t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$